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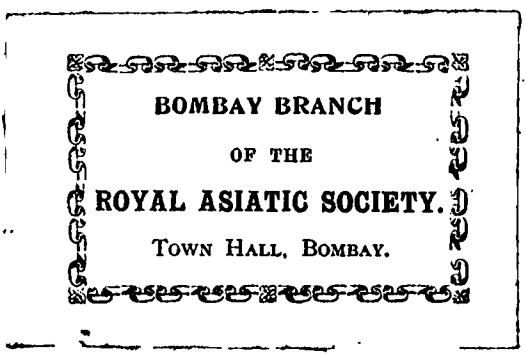

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OBSERVATIONS
ON
REVERSIONARY PAYMENTS;

ON

SCHEMES for providing (ANNUITIES) for WIDOWS, and for Perſons in OLD AGE;

ON

THE METHOD OF CALCULATING THE VALUES OF ASSURANCES ON LIVES;

AND ON

THE NATIONAL DEBT.

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ALSO,

ESSAYS on different Subjects in the Doctrine of LIFE-ANNUITIES and POLITICAL ARITHMETIC;

A Collection of NEW TABLES, and a POSTSCRIPT on the POPULATION of the KINGDOM.

BY RICHARD PRICE, D.D. F.R.S.

TO WHICH ARE ADDED,

ALGEBRAICAL NOTES, the SOLUTION of ſeveral NEW PROBLEMS in the DOCTRINE of ANNUITIES,

And a GENERAL INTRODUCTION.

BY WILLIAM MORGAN, F.R.S.

FIFTH EDITION.

VOL. II.

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GENERAL INTRODUCTION:

CONTAINING

An Account of the NEW TABLES *of the Duration of Human Life at* Cheſter, Warrington, *the Kingdom of* Sweden, Stockholm, London, *&c. inſerted in the following Collection of Tables.*

I HAVE, in the preceding Volume, p. 352, and in the Poſtſcript to the Second Eſſay, p. 308, given an account of the improvement which was made in the former edition of this work, of the Table of Obſervations for *Northampton*, and of my reaſons for wiſhing to diſcard the tables of the values of ſingle and joint lives, founded on Mr. *De Moivre*'s hypotheſis, and ſubſtituting in their room the tables in the following collection.— I was farther enabled to improve this work, in that edition of it, by inſerting tables

formed from a regiſter of mortality eſtabliſhed near twenty years ago at CHESTER.—This regiſter was formed on the plan propoſed in the preceding Volume, p. 367; and, therefore, is more comprehenſive than any regiſter of the ſame kind that has been hitherto eſtabliſhed.

Cheſter is a healthy town, of moderate ſize, where the births had, for many years, a little exceeded the burials; and the regiſter to which I refer had the particular advantage of being under the direction of Dr. *Haygarth*, its founder * as well as conductor. As it gives an accurate account of the diſtempers of which all the inhabitants die in every ſeaſon, and at every age, it contains much phyſical inſtruction; but my views lead me only to take notice of that part of it which gives the law according to which human life waſtes in all its different ſtages, both among males and females.

A ſummary of this part of the regiſter is given in the introduction to the CHESTER tables, in the following collection of tables.

* This able and ingenious phyſician has given another proof of his zeal to render his profeſſional character as uſeful as poſſible, by inſtituting a plan, which he has been carrying on at *Cheſter*, for preventing the ſpread of the ſmall-pox by infection, and thus gradually exterminating it.

Concerning

Concerning theſe tables it is neceſſary I ſhould make the following obſervations.

The table for females muſt be conſidered as particularly correct, becauſe the number of females *born* and *buried* in *Cheſter* are very nearly equal.——On the contrary; the number of males *born* being about an 8th greater than the number *buried*, it follows that, in the table of decrements for males, the numbers of the living, and conſequently the probabilities of living at every age, for at leaſt 10 or 15 of the firſt years of life, muſt be given too low.

The expectation of a female at birth is, according to theſe tables, nearly $33\frac{1}{4}$ years; and of a male $28\frac{1}{8}$. The number of females, therefore, at Cheſter, is to the number of males as $33\frac{1}{4}$ to $28\frac{1}{8}$, or in the proportion of 8000 to 6771, which is the proportion diſcovered by a ſurvey in 1774, when the females in this city were found to be 8016, and the males 6697 (*a*).

(*a*) It appeared from this ſurvey (made with great care under the direction of Dr. *Haygarth*), that in 1774 there were in the ten pariſhes of *Cheſter*, including the ſuburbs,

Families.	Inhabitants.	Males.	Females.
3428.	14713	6697	8016
Married.	Widowers.	Widows.	Under 15.
4881	258	736	4486
Above 70.	Recovered of the ſmall-pox in 1774.		Dead of the ſmall-pox in 1774.
625	1183		202
Ill of the ſmall-pox in Jan. 1775.		Not had the ſmall-pox in Jan. 1775.	
19		1060	

Theſe tables are farther confirmed by the proportion which they give of the number of males and females living under 15 to the whole number. This proportion is by the tables nearly that of 4486 to 14,888, and the *actual* numbers found by the enumeration in 1774, were 4486 and 14,713.

In like manner; the number of the living above 70 was, by the ſame ſurvey, found to be 625; and the tables give this number nearly the ſame.

The expectation at birth, taking males and females together, is at *Cheſter*, by the tables, near 31; and therefore one in 31 ought to die annually. But the quotient ariſing from dividing the number of inhabitants (14,713) by 409 (the medium of annual burials from 1772 and 1781), will ſhew that in reality no more than one in 36 die annually.——The reaſon of this difference is, firſt, that the births exceed the burials; and that, conſequently, a table which takes the burials for its *radix*, muſt give the expectations of life too low.——A ſecond reaſon is, the emigration of males from *Cheſter*; in conſequence of which, though more males than females are born, and though males are alſo more ſhort-lived; yet fewer die at *Cheſter*, many dying in the army, navy, militia, &c. The effect of the firſt of theſe cauſes will be particularly exemplified hereafter, in the caſe of the kingdom of SWEDEN.

Obſerva-

Obſervations ſimilar to theſe may be made on the tables in the following collection, formed from a regiſter of mortality at *Warrington* in *Lancaſhire*, founded and conducted by the ingenious Dr. *Aikin*, (then the phyſician there, but now phyſician at YARMOUTH in *Norfolk*) to whoſe kindneſs and communicativeneſs, as well as to Dr. *Haygarth*'s, I have been much obliged. See Tables 39th and 40th.

The expectation of a male juſt born, at WARRINGTON, is, by theſe tables, $20\frac{4}{5}$; of a female $25\frac{1}{3}$; and of males and females taken together, $23\frac{1}{10}$ nearly.

In the beginning of 1781 Dr. *Aikin* procured an enumeration of the houſes and inhabitants in *Warrington* and its vicinity, conſiſting of the town of *Warrington*, the townſhip as far as the lays are collected, *Poulton*, *Fearnhead*, and *Woolſton*. The number of houſes, including 74 uninhabited, was 2000; of inhabitants 9501, or $4\frac{55}{100}$ to a houſe.——The number of inhabitants divided by 302 (the annual average of burials for 9 years from 1773 to 1781) gives $31\frac{2}{3}$, but divided by 321, the annual average of burials for five years from 1777 to 1781 (which, in this caſe, ſeems the faireſt average) gives $29\frac{3}{5}$. There is, therefore, in this town, a greater difference between the proportions dying annually, as determined by enumeration and by calculation from the regiſter, than there is at *Cheſter*; and the

reaſon is, that the two cauſes juſt mentioned operate more here. The births in particular (the annual average of which for the 5 years juſt mentioned was 411) exceed the burials much more at *Warrington*; and therefore the burials are much more below the true average, and the probabilities of living exhibited by the table of decrements, much more below the true probabilities. Every one muſt be ſtruck with the difference, in reſpect of longevity, which theſe tables exhibit between the inhabitants of *Warrington* and *Cheſter*; and it will appear more remarkable when it is conſidered, that about an 8th or 9th of the inhabitants included in the *Warrington* bills, are inhabitants of the country for a mile or two round *Warrington*.——*Cheſter* appears, indeed, to be an extraordinary exception to the hurtful effects of towns on the duration of life. The probabilities of living in it, though lower than in country pariſhes, are conſiderably higher than in any other city where obſervations have been made. I am not qualified to explain the cauſes which give it this diſtinction. A probable account of them has been given by Dr. *Haygarth*, in a paper printed at *Cheſter*, and containing *Obſervations on the Population and Diſeaſes of Cheſter in* 1774 (*a*).

It is farther obſervable, that theſe tables agree in exhibiting, in a ſtriking light, the difference between the probabilities of living among males and females. But this difference will

will appear more evidently from the Tables for *Sweden*, of which I am next to give an account.

There are two ſorts of *data* for forming tables of the probabilities of the duration of human life at every age. One is furniſhed by regiſters of mortality ſhewing the numbers dying at all ages. The other, by the proportions of deaths at all ages to the numbers living at thoſe ages diſcovered by ſurveys or enumerations.——Tables formed from the former of theſe *data*, are correct only when there is no conſiderable fluctuation among the inhabitants of a place, and the births and burials are equal. When there are more removals *from* than *to* a place, and the births exceed the burials, as is almoſt always the caſe in country pariſhes and villages, tables ſo formed give the probabilities of living too low. When the contrary happens, as is generally the caſe in towns, they give the probabilities of living too high. But tables formed from the latter of theſe *data*, are ſubject to no errors. They muſt be correct, whatever the fluctuations are in a place, and how great ſoever the inequalities may be between the births and burials.——I know of no obſervations extant which furniſh the means of forming *ſuch* tables, except thoſe publiſhed by the late Mr. *Wargentin* in the Memoirs of the Academy

of Sciences at STOCKHOLM, in 1776; an abſtract of which I have given in an Eſſay at the end of this volume; and a continuation of which, from 1763 to 1776, Mr. *Wargentin* with the greateſt goodneſs, communicated to me ſome time before his death. Theſe obſervations are more curious than any that have been yet publiſhed, and leave us little to wiſh for on this ſubject, except that ſimilar obſervations were made in other kingdoms under the direction of men equally able and ingenious with Mr. *Wargentin*.—It is from the reſult of all theſe obſervations taken together, that I have conſtructed Tables 42d, 43d, &c. in the following collection.

The tables for SWEDEN at large, compared with thoſe for STOCKHOLM the capital, confirm, in a very ſtriking manner, all that I have ſaid in the 1ſt Eſſay, Vol. I. and other parts of this work, of the difference between the duration of life in great towns, and in the country.——They likewiſe furniſh the moſt indiſputable evidence for the ſhorter duration of the lives of males than of females; and it deſerves particular notice, that the tables for *Sweden* at large differ, in this reſpect, but little from the tables formed from Dr. *Haygarth*'s Obſervations at *Cheſter*. Theſe obſervations give ſufficient *data* for calculating, with ſome correctneſs, diſtinct tables of the values of lives among males and females, taken ſeparately

parately and conjunctly; but I have preferred for this purpoſe the SWEDEN obſervations, becauſe (as has been juſt obſerved) more correct *in their nature*; and becauſe alſo (being made on the inhabitants of a whole kingdom for 21 years, and the enumeration which gives them their chief value having been repeated at *ſeven* different periods) they are much more to be depended on, and muſt give a juſter valuation of lives among mankind at large, including all town and country inhabitants.

I have, for my own ſatisfaction, conſtructed tables for SWEDEN and STOCKHOLM from the former of the *data* I have mentioned (or the numbers dying every year in every ſtage of life, as given by Mr. *Wargentin*); but being afraid of crowding this volume too much with tables, I have not inſerted them. The reader, if he chuſes to make ſuch tables for himſelf, is furniſhed with ſufficient means of doing it in the firſt Eſſay at the end of this volume: and he will find, on comparing them with Tables 42d, &c. all the errors exemplified ariſing from the common methods of conſtructing tables of obſervation. In particular; he will find that though it appears from the tables for *Sweden* in the following collection, that the *true* expectation of a child juſt born in that kingdom, taking males and females together, is $35\frac{1}{10}$; yet,

yet, a table formed from the numbers dying in every ſtage of life in the method deſcribed in the laſt Eſſay in the former Volume, will, (in conſequence of the births exceeding the burials near a *third* of the burials) give this expectation only 25 years and three quarters; in connexion with which, he will alſo find, that in all the firſt ſtages of life it gives the probabilities of living much too low.

I muſt add, that ſuch a table formed for *Stockholm*, and compared with the correct table (or Table 44th), will exhibit all the errors in the common tables for *London*, deſcribed in the Eſſay juſt referred to (*a*).

For

(*a*) In a table thus conſtructed (that is, on the ſuppoſition that all who die at *Stockholm* were born there) the numbers in the column of the living will be,

	Males.	Females.
at age 0	10,000	10,000
1	7,082	7,260
2	6,522	6,648
5	5,699	5,809
10	5,302	5,422
15	5,108	5,290
19	4,915	5,180
20	4,865	5,145
25	4,480	4,854
30	3,958	4,449
40	2,807	3,498
50	1,796	2,629
60	1,036	1,918
70	478	1,171
80	138	412
85	53	179
90	15	39
Totals, including the numbers omitted — —	242,100	285,367

Theſe

For inſtance. According to the correct table, the expectation of a male at birth in *Stockholm* is only $14\frac{1}{4}$; and of a female 18. But in a table formed from the deaths only,

Theſe totals divided by 10,000, and the quotients diminiſhed by half unity, give 23.71 the expectation of a *male* at birth in *Stockholm*, and 28 the expectation of a female. The expectation, therefore, at birth of males and females conjointly, is, by this table, 25.85 (or $25\frac{4}{5}$) which agrees almoſt exactly with the expectation at birth by a table formed in the ſame manner for *London*. See the former volume, p. 337; and Table 13th, in the following collection.——It deſerves particular notice, that there is a like agreement between theſe tables at every age between birth and the utmoſt extent of life, as will ſufficiently appear from the following compariſon.

EXPECTATIONS of males and females conjointly, by a table of obſervations conſtructed from the bills, on the ſuppoſition that all who *die* were *born*

	at STOCKHOLM.		at LONDON.
Age 10	$36\frac{4}{5}$	—	37
20	29	—	$29\frac{1}{3}$
30	$23\frac{1}{3}$	—	$24\frac{1}{10}$
40	$19\frac{1}{2}$	—	$19\frac{1}{2}$
50	$15\frac{4}{5}$	—	$15\frac{9}{10}$
60	$11\frac{3}{5}$	—	$11\frac{7}{10}$
70	$7\frac{3}{4}$	—	8

With theſe expectations compare the true expectations at *Stockholm*, deduced from Table 44th.

Age 10	$33\frac{2}{5}$
20	$26\frac{2}{5}$
30	$22\frac{1}{5}$
40	$17\frac{1}{4}$
50	$13\frac{1}{2}$
60	$9\frac{1}{2}$
70	$5\frac{4}{5}$

in

in the ſame manner with Table 13th for *London*, the former expectation comes out no leſs than $23\frac{3}{4}$, and the latter 28.——Again. The correct table makes 62 hundredths die annually of the males living between birth and five years of age; one in $3\frac{1}{2}$ of the males living between 5 and 10; one in 65, between 8 and 16; one in 69, between 10 and 20; one in 40, between 20 and 30; one in $29\frac{1}{2}$ between 30 and 40; one in 22, between 40 and 50; one in 16, between 50 and 60; one in 11, between 60 and 70; and one in 7 between 70 and 80. But the other table, would make only 43 hundredths die between birth and five years of age; one in 70, between 5 and 10; one in 120, between 8 and 16; one in 117, between 10 and 20; one in 50, between 20 and 30; one in 30, between 30 and 40; one in 23, between 40 and 50; one in $18\frac{1}{2}$, between 50 and 60; one in $13\frac{1}{2}$, between 60 and 70; and one in 9 between 70 and 80.

Of FEMALES, the *correct* table makes fifty-nine hundredths die annually of the living between birth and five years of age; one in $3\frac{9}{10}$ of the living between 5 and 10; one in 90, between 8 and 16; one in 107, between 10 and 20; one in 68, between 20 and 30; one in 41, between 30 and 40; one in 30, between 40 and 50; one in $24\frac{1}{2}$, between 50 and 60; one in 15, between 60 and 70; and one in

$7\frac{1}{2}$

$7\frac{1}{2}$, between 70 and 80. But the other table would make only forty-two hundredths (*a*) of females die between birth and five years of age; one in 72, between 5 and 10; one in 180, between 8 and 16; one in 191, between 10 and 20; one in 70, between 20 and 30; one in 42 between 30 and 40; one in 35, between 40 and 50; one in 32, between 50 and 60; one in 21, between 60 and 70; and one in $10\frac{1}{2}$, between 70 and 80.

Farther. The correct table makes the number of inhabitants (taking males and females together) dying annually at *Stockholm*, to be nearly a 16th and a half. The other would make it a 26th part of the inhabitants; whereas, the number actually dying is nearly a 19th.——The former table gives this proportion too great, becauſe, in conſequence of giving the true order in which a given number born will die, it gives only the expectation at *birth* in *Stockholm* (*b*); and therefore, cannot include the expecta-

(*a*) Compare the laſt note with the correct Table, or Table 44th.

(*b*) And this too on the ſuppoſition, that the probabilities of living, at every particular age, among the inhabitants born in *Stockholm*, are the ſame that they are among the whole body of inhabitants at that age, conſiſting of *natives* and *foreigners*; whereas the truth is, that the mortality of great towns falls more on the newcomers, than on thoſe who have been ſeaſoned to it by having lived in it ſome time.

tion

tion at *entrance* of thoſe who begin their reſidence in *Stockholm* after infancy.——The other muſt give this proportion too little, for the reaſons explained in the preceding Volume, p. 337, &c.

In order to make a table conſtructed for *Stockholm* in the manner mentioned in the note p. 10, a juſt repreſentation of the inhabitants, the numbers of the living (the decrements continuing the ſame) ſhould be diminiſhed at *every* age by a number equal to the annual average of new-comers at and after that age. After this diminution, the table will exhibit the ſame probabilities of life at every age with Table 44th; and if the ſum of the remaining numbers is divided by the ſum of the decrements, the quotient leſſened by half unity will, agreeably to the rule in p. 341 of the former Volume, give the number which I have called the *expectation at entrance*, and conſequently the true proportion of inhabitants dying annually. ——But there being no obſervations which make a ſubtraction of this kind at every particular age practicable; it is neceſſary to be ſatisfied with ſuch a ſubtraction at the beginning of mature life as that directed in the preceding Volume, p. 339, &c. The *Stockholm* obſervations happily give a proof of the neceſſity and uſe of this ſubtraction, by informing us of the *true* probabilities of living at *Stockholm*, as exhibited in table 44th; and at the ſame time furniſhing us with the means

means of conſtructing a table (like the 13th for *London*) of the probabilities of living in this town, on the ſuppoſition that all who die were born there. Let therefore, (ſince the exceſs of the burials above the births is nearly the ſame (*a*) in both cities) the correction be applied to this laſt table which has been applied to Table 13th for *London*. That is; let it be ſuppoſed that one quarter of all males and females who die at *Stockholm*, begin their reſidence in their 20th year; and in conformity to this ſuppoſition, let 2500, or a quarter of the *radix*, be ſubtracted from all the numbers living at every age before 20, preſerving the decrements the ſame. The reſult will be a table which, when compared with Table 44th, will appear to exhibit more nearly the true probabilities of living in all the ſtages of life. By giving them, however, too high, it will appear that the correction (*b*) has not been ſufficient; and that, conſequently, the expectation at entrance will come out, though much *nearer*, yet ſtill *above* the truth.

I have

(*a*) In nine years before 1764, the births at *Stockholm*, excluſive of the ſtill-born; were 7,907, and the burials 11,344.

(*a*) After this correction, the numbers in the note p. 10, will be

Males

I have thought it worth while to make theſe obſervations, in order to ſhew, from an unqueſtionable fact, what judgment ought to be formed of the tables for *London* in the following collection; and it ſeems impoſſible not to be convinced by them that though theſe tables give the probabilities of the duration of life in *London* (and conſequently the values of life-annuities) ſtrikingly lower than in other ſituations, yet they do not give them low enough; and that, in particular, the number by which the annual deaths ought to be multiplied to find the number of inhabitants, and which Table 14th determines to be $20\frac{3}{4}$, is not probably ſo much as 20.

		Living			Living
Males at age	0	7,500	Females at age	0	7,500
	1	4,582		1	4,760
	2	4,022		2	4,148
	5	3,199		5	3,309
	10	2,802		10	2,909
	15	2,608		15	2,790
	19	2,415		19	2,680
	20	4,865		20	5,145
	25	4,480		25	4,854
	&c.	&c.		&c.	&c.
Totals (including the numbers omitted) after deducting 5000 - -		187,100			230,367

Therefore the expectation at *entrance* of *males* is $18\frac{71}{100}$, of *females* is $23\frac{93}{100}$; of both conjointly $20\frac{87}{100}$; but theſe expectations are really (as appears from the obſervations) 16.80——20.93, and 18.89 reſpectively.

In

In ſhort. From the agreement in almoſt every particular between the *London* and *Stockholm* bills, and between two tables formed on the ſame principles from the deaths *only* in both towns, it ſeems a neceſſary concluſion that, ſince one of theſe tables (even after the correction explained in the fourth eſſay) gives certainly too favourable a repreſentation of human life, the other muſt do the ſame.

The following fact has ſome tendency to confirm this concluſion.

It appears from the midwifery reports of the general *Weſtminſter* Infirmary, that of 1618 married men, and 1618 married women, examined by Dr. *Bland* the phyſician to this Infirmary, only 329 of the *men* and 495 of the *women*, had been born in *London* (*a*); that is, a *fifth* of the men, and ſomewhat more than a *quarter* of the women. But the correction I have been conſidering implies, that a number equal to *half* of all turned of 20 in *London*, are natives of *London*; and therefore, if we may judge at all from this fact, it muſt be an inſufficient correction.

(*a*) See Dr. *Bland*'s account in the Philoſophical Tranſactions, Vol. 71ſt, Part II. p. 370 ——Of the whole number (3236) four-ſevenths, or 1870, were born in the different counties of *England* and *Wales*; 209 in *Scotland*; 280 in *Ireland*; and 53 were foreigners.

TABLE I.

The preſent Value of 1*l.* to be received at the End of any Number of Years, not exceeding 100; diſcounting at the Rates of 3, 3½, 4, 4½, 5, and 6 *per cent.* Compound Intereſt.

Yrs	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
1	,970874	,966184	,961538	,956938	,952381	,943396
2	,942596	,933511	,924556	,915730	,907029	,889996
3	,915142	,901943	,888996	,876297	,863838	,839619
4	,888487	,871442	,854804	,838561	,822702	,792094
5	,862609	,841973	,821927	,802451	,783526	,747258
6	,837484	,813501	,790315	,767896	,746215	,704961
7	,813092	,785991	,759918	,734828	,710681	,665057
8	,789409	,759412	,730690	,703185	,676839	,627412
9	,766417	,733731	,702587	,672904	,644609	,591898
10	,744094	,708919	,675564	,643928	,613913	,558395
11	,722421	,684946	,649581	,616199	,584679	,526788
12	,701380	,661783	,624597	,589664	,556837	,496969
13	,680951	,639404	,600574	,564272	,530321	,468839
14	,661118	,617782	,577475	,539973	,505068	,442301
15	,641862	,596891	,555265	,516720	,481017	,417265
16	,623167	,576706	,533908	,494469	,458112	,393646
17	,605016	,557204	,513373	,473176	,436297	,371364
18	,587395	,538361	,493628	,452800	,415521	,350344
19	,570286	,520156	,474642	,433302	,395734	,330513
20	,553676	,502566	,456387	,414643	,376889	,311805
21	,537549	,485571	,438834	,396787	,358942	,294155
22	,521893	,469151	,421955	,379701	,341850	,277505
23	,506692	,453286	,405726	,363350	,325571	,261797
24	,491934	,437957	,390121	,347703	,310068	,246979
25	,477606	,423147	,375117	,332731	,295303	,232999
26	,463695	,408838	,360689	,318402	,281241	,219810
27	,450189	,395012	,346817	,304691	,267848	,207368
28	,437077	,381654	,333477	,291571	,255094	,195630
29	,424346	,368748	,320651	,279015	,242946	,184557
30	,411987	,356278	,308319	,267000	,231377	,174110
31	,399987	,344230	,296460	,255502	,220359	,164255
32	,388337	,332590	,285058	,244500	,209866	,154957

TABLE I. continued.

Yr.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
33	,377026	,321343	,274094	,233971	,199873	,146186
34	,366045	,310476	,263552	,223896	,190355	,137912
35	,355383	,299977	,253415	,214254	,181290	,130105
36	,345032	,289833	,243669	,205028	,172657	,122741
37	,334983	,280032	,234297	,196199	,164436	,115793
38	,325226	,270562	,225285	,187750	,156605	,109239
39	,315754	,261413	,216621	,179665	,149148	,103056
40	,306557	,252572	,208289	,171929	,142046	,097222
41	,297628	,244031	,200278	,164525	,135282	,091719
42	,288959	,235779	,192575	,157440	,128840	,086527
43	,280543	,227806	,185168	,150661	,122704	,081630
44	,272372	,220102	,178046	,144173	,116861	,077009
45	,264439	,212659	,171198	,137964	,111297	,072650
46	,256737	,205468	,164614	,132023	,105997	,068538
47	,249259	,198520	,158283	,126338	,100949	,064658
48	,241999	,191806	,152195	,120898	,096142	,060998
49	,234950	,185320	,146341	,115692	,091564	,057546
50	,228107	,179053	,140713	,110710	,087204	,054288
51	,221463	,172998	,135301	,105942	,083051	,051215
52	,215013	,167148	,130097	,101380	,079096	,048316
53	,208750	,161496	,125093	,097014	,075330	,045582
54	,202670	,156035	,120282	,092837	,071743	,043001
55	,196767	,150758	,115656	,088839	,068326	,040567
56	,191036	,145660	,111207	,085013	,065073	,038271
57	,185472	,140734	,106930	,081353	,061974	,036105
58	,180070	,135975	,102817	,077849	,059023	,034061
59	,174825	,131377	,098863	,074497	,056212	,032133
60	,169733	,126934	,095060	,071289	,053536	,030314
61	,164789	,122642	,091404	,068219	,050986	,028598
62	,159990	,118495	,087889	,065281	,048558	,026980
63	,155330	,114487	,084508	,062470	,046246	,025453
64	,150806	,110616	,081258	,059780	,044044	,024012
65	,146413	,106875	,078133	,057206	,041946	,022653
66	,142149	,103261	,075128	,054743	,039949	,021370
67	,138009	,099769	,072238	,052385	,038047	,020161
68	,133989	,096395	,069460	,050129	,036235	,019020
69	,130086	,093136	,066788	,047971	,034509	,017943
70	,126297	,089986	,064219	,045905	,032866	,016927

TABLE I. continued.

Yrs.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
71	,122619	,086943	,061749	,043928	,031301	,015969
72	,119047	,084003	,059374	,042037	,029811	,015065
73	,115580	,081162	,057091	,040226	,028391	,014213
74	,112214	,078418	,054895	,038494	,027039	,013408
75	,108945	,075766	,052784	,036836	025752	,012649
76	,105772	,073204	,050754	,035250	,024525	,011933
77	,102691	,070728	,048801	,033732	,023357	,011258
78	,099700	,068336	,046924	,032280	,022245	,010620
79	,096796	,066026	,045120	,030890	,021186	,010019
80	,093977	,063793	,043384	,029559	,020177	,009452
81	,091240	,061636	,041716	,028287	,019216	,008917
82	,088582	,059551	,040111	,027069	,018301	,008412
83	,086002	,057538	,038569	,025903	,017430	,007936
84	,083497	,055592	,037085	,024787	,016600	,007487
85	,081065	,053712	,035659	,023720	,015809	,007063
86	,078704	,051896	,034287	,022699	,015056	,006663
87	,076412	,050141	,032969	,021721	,014339	,006286
88	,074186	,048445	,031701	,020786	,013657	,005930
89	,072026	,046807	,030481	,019891	,013006	,005595
90	,069928	,045224	,029309	,019034	,012387	,005278
91	,067891	,043695	,028182	,018215	,011797	,004979
92	,065914	,042217	,027098	,017430	,011235	,004697
93	,063994	,040789	,026056	,016680	,010700	,004432
94	,062130	,039410	,025053	,015961	,010191	,004181
95	,060320	,038077	,024090	,015274	,009705	,003944
96	,058563	,036790	,023163	,014616	,009243	,003721
97	,056858	,035546	,022272	,013987	,008803	,003510
98	,055202	,034344	,021416	,013385	,008384	,003312
99	,053594	,033182	,020592	,012808	,007985	,003124
100	,052033	,032060	,019800	,012257	,007604	,002947

TABLE II.

The present Value of an Annuity of One Pound for any Number of Years not exceeding 100, at the several Rates of 3, $3\frac{1}{2}$, 4, 5, and 6 *l. per cent.*

Year	3 per Ct.	$3\frac{1}{2}$ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
1	.9708	.9662	.9615	.9523	.9433
2	1.9134	1.8997	1.8861	1.8594	1.8333
3	2.8286	2.8016	2.7751	2.7232	2.6730
4	3.7170	3.6731	3.6299	3.5459	3.4651
5	4.5797	4.5151	4.4518	4.3294	4.2123
6	5.4171	5.3286	5.2421	5.0756	4.9173
7	6.2302	6.1145	6.0020	5.7863	5.5823
8	7.0196	6.8740	6.7327	6.4632	6.2097
9	7.7861	7.6077	7.4353	7.1078	6.8016
10	8.5302	8.3166	8.1109	7.7217	7.3600
11	9.2526	9.0015	8.7605	8.3064	7.8868
12	9.9540	9.6633	9.3850	8.8632	8.3838
13	10.6349	10.3027	9.9856	9.3935	8.8526
14	11.2960	10.9205	10.5631	9.8986	9.2949
15	11.9379	11.5174	11.1184	10.3796	9.7122
16	12.5611	12.0941	11.6523	10.8377	10.1058
17	13.1661	12.6513	12.1656	11.2740	10.4772
18	13.7535	13.1897	12.6593	11.6895	10.8276
19	14.3238	13.7098	13.1339	12.0853	11.1581
20	14.8774	14.2124	13.5903	12.4622	11.4699
21	15.4150	14.6980	14.0291	12.8211	11.7640
22	15.9369	15.1671	14.4511	13.1630	12.0415
23	16.4436	15.6204	14.8568	13.4885	12.3033
24	16.9355	16.0584	15.2469	13.7986	12.5503
25	17.4131	16.4815	15.6220	14.0939	12.7833

TABLE II. continued.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
26	17.8768	16.8904	15.9827	14.3751	13.0031
27	18.3270	17.2854	16.3295	14.6430	13.2105
28	18.7641	17.6670	16.6630	14.8981	13.4061
29	19.1884	18.0358	16.9837	15.1410	13.5907
30	19.6004	18.3920	17.2920	15.3724	13.7648
31	20.0004	18.7363	17.5884	15.5928	13.9290
32	20.3887	19.0689	17.8735	15.8026	14.0840
33	20.7657	19.3902	18.1476	16.0025	14.2302
34	21.1318	19.7007	18.4111	16.1929	14.3681
35	21.4872	20.0007	18.6646	16.3741	14.4982
36	21.8322	20.2905	18.9082	16.5468	14.6209
37	22.1672	20.5705	19.1425	16.7112	14.7367
38	22.4924	20.8411	19.3678	16.8678	14.8460
39	22.8082	21.1025	19.5844	17.0170	14.9490
40	23.1147	21.3551	19.7927	17.1590	15.0462
41	23.4124	21.5991	19.9930	17.2943	15.1380
42	23.7013	21.8349	20.1856	17.4232	15.2245
43	23.9819	22.0627	20.3707	17.5459	15.3061
44	24.2542	22.2828	28.5488	17.6627	15.3831
45	24.5187	22.4955	20.7200	17.7740	15.4558
46	24.7754	22.7009	20.8846	17.8800	15.5243
47	25.0247	22.8994	21.0429	17.9810	15.5890
48	25.2667	23.0912	21.1951	18.0771	15.6500
49	25.5016	23.2766	21.3414	18.1687	15.7075
50	25.7297	23.4556	21.4821	18.2559	15.7618
51	25.9512	23.6286	21.6174	18.3389	15.8130
52	26.1662	23.7958	21.7475	18.4180	15.8613
53	26.3749	23.9573	21.8726	18.4934	15.9069
54	26.5776	24.1133	21.9929	18.5651	15.9499
55	26.7744	24.2641	22.1086	18.6334	15.9905

TABLE II. continued.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
56	26.9654	24.4097	22.2198	18.6985	16.0288
57	27.1509	24.5504	22.3267	18.7605	16.0649
58	27.3310	24.6864	22.4299	18.8195	16.0989
59	27.5058	24.8178	22.5284	18.8757	16.1311
60	27.6755	24.9447	22.6234	18.9292	16.1614
61	27.8403	25.0674	22.7148	18.9802	16.1900
62	28.0003	25.1859	22.8027	19.0288	16.2170
63	28.1556	25.3004	22.8872	19.0750	16.2424
64	28.3064	25.4110	22.9685	19.1191	16.2664
65	28.4528	25.5178	23.0466	19.1610	16.2891
66	28.5950	25.6211	23.1218	19.2010	16.3104
67	28.7330	25.7209	23.1940	19.2390	16.3306
68	28.8670	25.8173	23.2635	19.2753	16.3496
69	28.9971	25.9104	23.3302	19.3098	16.3676
70	29.1234	26.0004	23.3945	19.3426	16.3845
71	29.2460	26.0873	23.4562	19.3739	16.4005
72	29.3650	26.1713	23.5156	19.4037	16.4155
73	29.4806	26.2525	23.5727	19.4321	16.4297
74	29.5928	26.3309	23.6276	19.4592	16.4431
75	29.7018	26.4067	23.6804	19.4849	16.4558
76	29.8076	26.4799	23.7311	19.5094	16.4677
77	29.9102	26.5506	23.7799	19.5328	16.4790
78	30.0099	26.6190	23.8268	19.5550	16.4896
79	30.1067	26.6850	23.8720	19.5762	16.4996
80	30.2007	26.7483	23.9153	19.5964	16.5091
81	30.2920	26.8104	23.9571	19.6156	16.5180
82	30.3805	26.8700	23.9972	19.6339	16.5264
83	30.4665	26.9275	24.0357	19.6514	16.5343
84	30.5500	26.9831	24.0728	19.6680	16.5418
85	30.6311	27.0368	24.1085	19.6838	16.5489

TABLE II. continued.

Year	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
86	30.7098	27.0887	24.1428	19.6988	16.5556
87	30.7862	27.1388	24.1757	19.7132	16.5618
88	30.8604	27.1873	24.2074	19.7268	16.5678
89	30.9324	27.2341	24.2379	19.7398	16.5734
90	31.0024	27.2793	24.2672	19.7522	16.5787
91	31.0703	27.3230	24.2954	19.7640	16.5836
92	31.1362	27.3652	24.3225	19.7752	16.5883
93	31.2002	27.4060	24.3486	19.7859	16.5928
94	31.2623	27.4454	24.3736	19.7961	16.5969
95	31.3226	27.4835	24.3977	19.8058	16.6009
96	31.3812	27.5203	24.4209	19.8151	16.6046
97	31.4380	27.5558	24.4431	19.8239	16.6081
98	31.4932	27.5902	24.4646	19.8323	16.6114
99	31.5468	27.6234	24.4852	19.8403	16.6145
100	31.5989	27.6554	24.5050	19.8479	16.6175
Perpetuity.	33.3333	28.5714	25.0000	20.0000	16.6666

TABLE III.

Shewing the Sum to which 1 *l.* Principal will increaſe at Compound Intereſt in any Number of Years not exceeding a hundred.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	1.030,000	1.035,000	1.040,000	1.050,000	1.060,000
2	1.060,900	1.071,225	1.081,600	1.102,500	1.123,600
3	1.092,727	1.108,717	1.124,864	1.157,625	1.191,016
4	1.125,508	1.147,523	1.169,858	1.215,506	1.262,476
5	1.159,274	1.187,686	1.216,652	1.276,281	1.338,225
6	1.194,052	1.229,255	1.265,319	1.340,095	1.418,519
7	1.229,873	1.272,279	1.315,931	1.407,100	1.503,630
8	1.266,770	1.316,809	1.368,569	1.477,455	1.593,848
9	1.304,773	1.362,897	1.423,311	1.551,328	1.689,478
10	1.343,916	1.410,598	1.480,244	1.628,894	1.790,847
11	1.384,233	1.459,969	1.539,454	1.710,339	1.898,298
12	1.425,760	1.511,068	1.601,032	1.795,856	2.012,196
13	1.468.533	1.563,956	1.665,073	1.885,649	2.132,928
14	1.512,589	1.618,694	1.731,676	1.979,931	2.260,903
15	1.557.967	1.675,348	1.800,943	2.078,928	2.396,558
16	1.604,706	1.733,986	1.872,981	2.182,874	2.540,351
17	1.652,847	1.794,675	1.947,900	2.292,018	2.692,772
18	1.702,433	1.857,489	2,025,816	2.406,619	2.854,339
19	1.753,506	1.922,501	2.106,849	2.526,950	3.025,599
20	1.806,111	1.989,788	2.191,123	2.653,297	3.207,135
21	1.860,294	2.059,431	2.278,768	2.785,962	3.399,563
22	1.916,103	2.131,511	2.369,918	2.925,260	3.603,537
23	1.973,586	2.206,114	2.464,715	3.071,523	3.819,749
24	2.032,794	2.283,328	2.563,304	3.225,099	4.048,934
25	2.093,777	2.363,244	2.665,836	3.386,354	4.291,870
26	2.156,591	2.445,958	2.772,469	3.555,672	4.549,382
27	2,221,289	2.531,567	2.883,368	3.733,456	4.822,345
28	2.287,927	2.620,171	2.998,703	3.920,129	5.111,686
29	2.356,565	2.711,877	3.118,651	4.116,135	5.418,387
30	2.427,262	2.806,793	3.243,397	4.321,942	5.743,491
31	2.500,080	2.905,031	3.373,133	4.538,039	6.088,100
32	2.575,082	3.006,707	3.508,058	4.764,941	6.453,386
33	2.652,335	3.111,942	3.648,381	5.003,188	6.840,589
34	2.731,905	3.220,860	3.794,316	5.253,347	7.251,025
35	2.813,862	3.333,590	3.946,088	5.516,015	7.686,086
36	2.898,278	3.450,266	4.103,932	5.791,816	8.147,252

TABLE III. continued.

Yrs	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
37	2.985,226	3.571,025	4.268,089	6.081,406	8.636,087
38	3.074,783	3.696,011	4.438,813	6.385,477	9.154,252
39	3.167,026	3.825,371	4.616,365	6.704,751	9.703,507
40	3.262,037	3.959,259	4.801,020	7.039,988	10.285,717
41	3.359,898	4.097,833	4.993,061	7.391,988	10.902,861
42	3.460,695	4.241,257	5.192,783	7.761,587	11.557,032
43	3.564,516	4.389,702	5.400,495	8.149,666	12.250,454
44	3.671,452	4.543,341	5.616,515	8.557,150	12.985,481
45	3.781,595	4.702,358	5.841,175	8.985,007	13.764,610
46	3.895,043	4.866,941	6.074,822	9.434,258	14.590,487
47	4.011,895	5.037,284	6.317,815	9.905,971	15.465,916
48	4.132,251	5.213,588	6.570,528	10.401,269	16.393,871
49	4.256,219	5.396,064	6.833,349	10.921,333	17.377,504
50	4.383,906	5.584,926	7.106,683	11.467,399	18.420,154
51	4.515,423	5.780,399	7.390,950	12.040,769	19.525,363
52	4.650,885	5.982,713	7.686,588	12.642,808	20.696,885
53	4.790,412	6.192,108	7.994,052	13.274,948	21.938,698
54	4.934,124	6.408,832	8.313,814	13.938,696	23.255,020
55	5.082,148	6.633,141	8.646,366	14.635,630	24.650,321
56	5.234,613	6.865,301	8.992,221	15.367,412	26.129,340
57	5.391,651	7.105,586	9.351,910	16.135,783	27.697,101
58	5.553,400	7.354,282	9.725,986	16.942,572	29.358,927
59	5.720,003	7.611,682	10.115,026	17.789,700	31.120,463
60	5.891,603	7.878,090	10.519,627	18.679,185	32.987,690
61	6.068,351	8.153,824	10.940,412	19.613,145	34.966,952
62	6.250,401	8.439,207	11.378,029	20.593,802	37.064,969
63	6.437,913	8.734,580	11.833,150	21.623,492	39.288,867
64	6.631,051	9.040,290	12.306,476	22.704,667	41.646,199
65	6.829,982	9.356,700	12.798,735	23.839,900	44.144,971
66	7.034,882	9 684,185	13.310,684	25.031,895	46.793,669
67	7.245,928	10.023,131	13.843,112	26.283,490	49.601,290
68	7.463,306	10.373,941	14.396,836	27.597,664	52.577,367
69	7.687,205	10,737,029	14.972,709	28.977,548	55.732,009
70	7.917,821	11,112,825	15.571,618	30.426,425	59.075,930
71	8.155,356	11.501,774	16.194,483	31.947,746	62.620,485
72	8.400,017	11.904,336	16.842,262	33.545,134	66.377,715
73	8.652,017	12.320,988	17.515,952	35.222,390	70.360,378
74	8.911,578	12.752,222	18.216,591	36.983,510	74.582,000
75	9,178,925	13.198,550	18.945,254	38.832,685	79.056,920
76	9.454,293	13.660,499	19.703,064	40.774,320	83.800,336
77	9.737,922	14.138,617	20.491,187	42.813,036	88.828,356

TABLE III. continued.

rs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
78	10.030,059	14.633,468	21.310,834	44.953,688	94.158,057
79	10.330,961	15.145,640	22.163,268	47.201,372	99.807,541
80	10.640,890	15.675,737	23.049,799	49.561,441	105.795,993
81	10.960,117	16.224,388	23.971,791	52.039,513	112.143,753
82	11.288,920	16.792,241	24.930,662	54.641,488	118.872,378
83	11.627,588	17.379,970	25.927,889	57.373,563	126.004,720
84	11.976,416	17.988,269	26.965,004	60.242,241	133.565,004
85	12.335,708	18.617,858	28.043,604	63.254,353	141.578,904
86	12.705,779	19.269,483	29.165,349	66 417,071	150.073,638
87	13.086,953	19.943,915	30.331,963	69.737,924	159.078,057
88	13.479,561	20.641,952	31.545,241	73.224,820	168.622,740
89	13.883,948	21.364,421	32.807,051	76.886,061	178.740,104
90	14.300,467	22,112,175	34.119,333	80.730,365	189.464,511
91	14.729,481	22.886,102	35,484,106	84.766,883	200.832,381
92	15.171,365	23.687,115	36.903,470	89.005,227	212.882,324
93	15.626,506	24.516,164	38.379,609	93.455,488	225.655,264
94	16.095,301	25.374,230	39.914,794	98.128,263	239.194,580
95	16.578,160	26.262,328	41.511,385	103.034,676	253.546,254
96	17.075,505	27.181,510	43.171,841	108.186,410	268.759,030
97	17.587,770	28.132,862	44.898,715	113.595,730	284.884,572
98	18.115,403	29.117,513	46.694,663	119.275,517	301.977,646
99	18.658,866	30 136,626	48.562,450	125.239,293	320.096,305
100	19.218,631	31.191,407	50.504,948	131.501,257	339.302,083

TABLE IV.

Shewing the Sum to which 1 *l. per ann.* will increaſe at Compound Intereſt in any Number of Years not exceeding a hundred.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
1	1.000,000	1.000,000	1.000,000	1.000,000	1,000,000
2	2.030,000	2.035,000	2.040,000	2.050,000	2.060,000
3	3.090,900	3.106,225	3.121,600	3.152,500	3.183,600
4	4.183,627	4.214,942	4.246,464	4.310,125	4.374,616
5	5.309,135	5.362,465	5.416,322	5.525,631	5.637,092
6	6.468,409	6.550,152	6.632,975	6.801,912	6.975,318
7	7.662,462	7.779,407	7.898,294	8.142,008	8.393,837
8	8.892,336	9.051,686	9.214,226	9.549,108	9.897,467
9	10.159,106	10.368,495	10.582,795	11.026,564	11.491,315
10	11.463,879	11.731,393	12.006,107	12.577,892	13.180,794
11	12.807,795	13.141,991	13.486,351	14.206,787	14.971,642
12	14.192,029	14.601,961	15.025,805	15.917,126	16.869,941
13	15.617,790	16.113,030	16.626,837	17.712,982	18.882,137
14	17.086,324	17.676,986	18.291,911	19.598,631	21.015,065
15	18.598,913	19.295,680	20.023,587	21.578,563	23.275,969
16	20.156,881	20.971,029	21.824,531	23.657,491	25.672,528
17	21.761,587	22.705,015	23.697,512	25.840,366	28.212,879
18	23.414,435	24.499,691	25.645,412	28.132,384	30.905,652
19	25.116,868	26.357,180	27.671,229	30.539,003	33.759,991
20	26.870,374	28.279,681	29.778,078	33.065,954	36.785,591
21	28.676,485	30.269,470	31.969,201	35.719,251	39.992,726
22	30.536,780	32.328,902	34.247,969	38.505,214	43.392,290
23	32.452,883	34.460,413	36.617,888	41.430,475	46.995,827
24	34.426,470	36.666,528	39.082,604	44.501,998	50.815,577
25	36.459,264	38.949,856	41.645,908	47.727,098	54.864,512
26	38.553,042	41.313,101	44.311,744	51.113,453	59,156,382
27	40.709,633	43,759,060	47.084,214	54.669,126	63.705,765
28	42.930,922	46,290,627	49.967,582	58.402,582	68.528,111
29	45.218,850	48,910,799	52.966,286	62.322,711	73.639,798
30	47.575,415	51.622,677	56.084,937	66.438,847	79.058,186
31	50.002,678	54.429,470	59.328,335	70.760,789	84.801,677
32	52.502,758	57.334.502	62.701,468	75.298,829	90.889,778
33	55.077,841	60.341,210	66.209,527	80.063,770	97.343,164
34	57.730,176	63.453,152	69.857,908	85,066,959	104.183,754
35	60.462,081	66.674,012	73.652,224	90.320,307	111.434,779
36	63.275,944	70.007,603	77.598,313	95.836,322	119.120,866

TABLE IV. continued.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
37	66.174,222	73.457,869	81.702,246	101.628,138	127.268,118
38	69.159,449	77.028,894	85.970,336	107.709,545	135.904,205
39	72.234,232	80.724,906	90.409,149	114.095,023	145.058,458
40	75.401,259	84.550,277	95.025,515	120.799,774	154.761,965
41	78.663,297	88.509,537	99.826,536	127.839,762	165.047,683
42	82.023,196	92.607,371	104.819,597	135.231,751	175.950,544
43	85.483,892	96.848,629	110.012,381	142.993,338	187.507,577
44	89.048,409	101.238,331	115.412,876	151.143,005	199.758,031
45	92.719,861	105.781,672	121.029,392	159.700,155	212.743,513
46	96.501,457	110.484,031	126.870,567	168.685,163	226.508,124
47	100.396,500	115.350,972	132.945,390	178.119,421	241.098,612
48	104.408,395	120.388,256	139.263,206	188.025,392	256.564,528
49	108.540,647	125.601,845	145.833,734	198.426,662	272.958,400
50	112.796,867	130.997,910	152.667,083	209.347,995	290.335,904
51	117.180,773	136.582,837	159.773,767	220.815,395	308.756,058
52	121.696,196	142.363,236	167.164,717	232.856,165	328.281,422
53	126.347,082	148.345,949	174.851,306	245.498,973	348.978,307
54	131.137,494	154.538,057	182.845,358	258.773,922	370.917,006
55	136.071,619	160.946,889	191.159,173	272.712,618	394.172,026
56	141.153,768	167.580,030	199.805,539	287.348,249	418.822,348
57	146.388,381	174.445,332	208.797,761	302.715,661	444.951,689
58	151.780,032	181.550,918	218.149,672	318.851,444	472.648,790
59	157.333,433	188.905,200	227.875,658	335.794,017	502.007,717
60	163.053,436	196.516,882	237.990,685	353.583,717	533.128,180
61	168.945,039	204.394,973	248.510,312	372.262,903	566.115,871
62	175.013,391	212.548,797	259.450,725	391.876,048	601.082,824
63	181.263,792	220.988,005	270.828,754	412.469,851	638.147,793
64	187.701,706	229.722,585	282.661,904	434.093,343	677.436,661
65	194.332,757	238.762,876	294.968,380	456.798,011	719.082,860
66	201.162,740	248.119,577	307.767,115	480.637,911	763.227,832
67	208.197,622	257.803,762	321.077,800	505.669,807	810.021,502
68	215.443,551	267.826,894	334.920,912	531.953,297	859.622,792
69	222.906,858	278.200,835	349.317,748	559.550,962	912.200,160
70	230.594,063	288.937,864	364.290,458	588.528,510	967.932,169
71	238.511,885	300.050,689	379.862,077	618.954,936	1027.008,099
72	246.667,242	311.552,463	396.056,560	650.902,683	1089.628,585
73	255.067,259	323.456,800	412.898,822	684.447,817	1156.006,300
74	263.719,277	335.777,788	430.414,775	719.670,208	1226.366,679
75	272.630,855	348.530,010	448.631,366	756.653,718	1300.948,679
76	281.809,781	361.728,561	467.576,621	795.486,404	1380.005,600
77	291.264,074	375.389,060	487.279,686	836.260,724	1463.805,936

TABLE IV. continued.

Yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
78	301.001,996	389.527,677	507,770,873	879.073,760	1552.634,292
79	311.032,056	404.161,146	529.081,708	924.027,448	1646.792,350
80	321.363,018	419.306.786	551.244,976	971.228,821	1746.599,891
81	332.003,909	434.982,524	574.294,775	1020.790,262	1852.395,884
82	342.964,026	451.206,912	598.266,566	1072.829,775	1964.539,637
83	354.252,947	467.999,154	623.197,229	1127.471,264	2083.412,016
84	365.880,535	485.379,125	649.125,118	1184.844,827	2209.416,737
85	377.856,951	503.367,394	676.090,123	1245.087,068	2342.981,741
86	390.192,660	521.985,253	704.133,728	1308.341,422	2484.560,645
87	402.898,440	541.254,737	733.299,077	1374.758,493	2634.634,284
88	415.985,393	561.198,652	763.631,040	1444.496,418	2793.712,341
89	429.464,955	581.840,605	795.176,282	1517.721,238	2962.335,082
90	443.348,903	603.205,027	827.983,333	1594.607,300	3141,075,187
91	457.649,370	625.317,202	862.102,667	1675.337,665	3330.539,698
92	472.378,851	648.203,305	897,586,773	1760.104,549	3531.372,080
93	487.550,217	671.890,420	934.490,244	1849.109,776	3744.254,405
94	503.176,723	696.406,585	972.869,854	1942.565,265	3969.909.669
95	519.272,025	721.780,815	1012.784,648	2040.693,528	4209.104,249
96	535.850,186	748.043,144	1054.296,034	2143.728,205	4462.650,504
97	552.925,692	775.224,654	1097.467,875	2251.914,615	4731.409,534
98	570.513,462	803.357,517	1142.366,590	2365.510,346	5016.294,106
99	588.628,866	832.475,030	1189.061,254	2484.785,863	5318.271,753
100	607.287,732	862.611,656	1237.623,704	2610.025,156	5638.368,058

CONSTRUCTION *of the four preceding Tables.*

THESE Tables may be met with in moſt of the books which treat of compound intereſt and annuities; but there has been, in this work, ſo much occaſion for referring to them, that it was neceſſary to ſave the reader the trouble of turning to other books for them.

The 1ſt, 2d, 3d, &c. numbers in the firſt table, are the quotients of unity divided by the 1ſt, 2d, 3d, &c. powers reſpectively of 1 *l.* increaſed by its intereſt for a year; that is, $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. r ſignifying 1 *l.* increaſed by its intereſt for a year; or 1.03, 1.035, 1.04, 1.045, 1.05, 1.06, as the intereſt is 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, or 6 *per cent.*

The 2d, 3d, 4th, &c. numbers in the ſecond table, are the *ſums* of the 1ſt and 2d; of the 1ſt, 2d, and 3d; of the 1ſt, 2d, 3d, and 4th, &c. &c. numbers reſpectively in the firſt Table.

The numbers in the 3d Table are the powers of 1 *l.* increaſed by its intereſt for a year; that is, r, r^2, r^3, &c.

The 2d, 3d, 4th, &c. numbers in the 4th Table, are the ſums of the 1ſt and 2d; of the 1ſt, 2d, and 3d; of the 1ſt, 2d, 3d, and 4th, &c. numbers in the 3d Table, with unity added.

N. B. At the cloſe of this collection there is a continuation of theſe Tables for the intereſts of 2, $2\frac{1}{2}$, 7, 8, 9, and 10 *per cent.*

USES

USES *of the preceding Tables.*

Queſtion I. To what *ſum* or *annuity* will any given *ſum* or *annuity* increaſe in a given number of years, at a given rate of compound intereſt?

Anſ. Multiply the number in Table 3d under the given rate and oppoſite to the given number of years, by the given *ſum* or *annuity*, and the product will be the anſwer.

EXAMPLE. The product of 40*l.* into 2.0258 (that is, *l.* 81.032) is the ſum to which 40*l.* principal will increaſe in 18 years, reckoning intereſt at 4 *per cent.*; and the ſame product is likewiſe the *annuity* to which an *annuity* of 40*l.* will increaſe in the ſame time, reckoning the ſame intereſt.

Queſt. II. To what ſum will a given *annuity* amount at a given rate of compound intereſt for a given number of years?

Anſ. Multiply the number in the *fourth* Table under the rate and oppoſite to the given number of years, by the given annuity, and the *product* will be the anſwer.

EXAMPLE. The product of 40*l.* into 25.6454 (that is, *l.* 1025.826) is the ſum to which 40*l.* *per ann.* will amount in 18 years, reckoning intereſt at 4 *per cent.*

Queſt.

Queſt. III. In what number of years will a given *ſum* or *annuity* increaſe to another given *ſum* or *annuity* in conſequence of being improved at a given rate of intereſt?

Anſ. Divide the latter *ſum* or *annuity* by the former. Find the quotient (or the number neareſt to it) in the *third* Table, under the given rate, and the years oppoſite to it will be the anſwer.

EXAMPLE. The quotient of 1025.826*l.* divided by 40, is 25.6454, which number, under 4 *per cent.* in the third Table, is oppoſite to 18 years; which, therefore, is the number of years in which 40*l.* will increaſe to 1025.826*l.* if improved at 4 *per cent.* compound intereſt.

Queſt. IV. In what time will a given *annuity* amount to a given *ſum* at a given rate of intereſt?

Anſ. Divide the given *ſum* by the given *annuity*. Find the quotient (or the numneareſt to it) in the *fourth* Table under the given rate, and the number of years correſponding to it will be the anſwer.

EXAMPLE. A perſon owes 1000*l.* and reſolves to appropriate 10*l. per ann.* of his income towards diſcharging it. In what time will ſuch an appropriation, intereſt being at 4 *per cent.* amount to a ſum equal to the debt?——1000*l.* divided by 10*l.* gives 100*l.* The number in the *fourth*

Table, under 4 *per cent.* and neareſt to this quotient, is 99.8265, which correſponds to 41 years; and this, therefore, is the time in which ſuch an appropriation would ſink the debt. In like manner, it may be found that an appropriation of a million *per ann.* would, in the ſame time, ſink a public debt of a *hundred millions*, carrying 4 *per cent.* intereſt; and, in 56 years a debt of *two hundred millions*; and in 82 years, a debt of *ſix hundred millions.*

Queſt. V. In what time will a *given principal* be annihilated by taking out of it, at the end of a year, a given ſum; and after that, the ſame ſum annually, together with its growing intereſts?

Anſ. In the ſame time in which an equal annuity would amount to the *given principal.*

A perſon, therefore, poſſeſſed of 1000 *l.* capital, bearing intereſt at 4 *per cent.* would, by Queſt. IV. reduce it to nothing in 41 years, by taking out of it 10 *l.* at the beginning of the firſt year, and as much more every following year as would be neceſſary, together with the intereſt of the remaining capital, to make his annual income conſtantly 50 *l.*

TABLE V.

Shewing the Probabilities of the Duration of Life, as deduced by Dr. *Halley* from Obſervations on the Bills of Mortality of BRESLAW.

Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living	Decr. of Life.
1	1000	145	31	523	8	61	232	10
2	855	57	32	515	8	62	222	10
3	798	38	33	507	8	63	212	10
4	760	28	34	499	9	64	202	10
5	732	22	35	490	9	65	192	10
6	710	18	36	481	9	66	182	10
7	692	12	37	472	9	67	172	10
8	680	10	38	463	9	68	162	10
9	670	9	39	454	9	69	152	10
10	661	8	40	445	9	70	142	11
11	653	7	41	436	9	71	131	11
12	646	6	42	427	10	72	120	11
13	640	6	43	417	10	73	109	11
14	634	6	44	407.	10	74	98	10
15	628	6	45	397	10	75	88	10
16	622	6	46	387	10	76	78	10
17	616	6	47	377	10	77	68	10
18	610	6	48	367	10	78	58	9
19	604	6	49	357	11	79	49	8
20	598	6	50	346	11	80	41	7
21	592	6	51	335	11	81	34	6
22	586	7	52	324	11	82	28	5
23	579	6	53	313	11	83	23	4
24	573	6	54	302	10	84	19	4
25	567	7	55	292	10	85	15	4
26	560	7	56	282	10	86	11	3
27	553	7	57	272	10	87	8	3
28	546	7	58	262	10	88	5	2
29	539	8	59	252	10	89	3	2
30	531	8	60	242	10	90	1	1

TABLE VI.

Shewing the Probabilities of the Duration of Human Life at all Ages, formed from the Regiſter of Mortality at *Northampton*, for 46 Years from 1735 to 1780.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decrem.
0	11650	1340	31	4310	75	65	1632	80
3 months	10310	554	32	4235	75	66	1552	80
6 months	9756	553	33	4160	75	67	1472	80
9 months	9203	553	34	4085	75	68	1392	80
1 Year	8650	1367	35	4010	75	69	1312	80
2 Years	7283	502	36	3935	75	70	1232	80
3	6781	335	37	3860	75	71	1152	80
4	6446	197	38	3785	75	72	1072	80
5	6249	184	39	3710	75	73	992	80
6	6065	140	40	3635	76	74	912	80
7	5925	110	41	3559	77	75	832	80
8	5815	80	42	3482	78	76	752	77
9	5735	60	43	3404	78	77	675	73
10	5675	52	44	3326	78	78	602	68
11	5623	50	45	3248	78	79	534	65
12	5573	50	46	3170	78	80	469	63
13	5523	50	47	3092	78	81	406	60
14	5473	50	48	3014	78	82	346	57
15	5423	50	49	2936	79	83	289	55
16	5373	53	50	2857	81	84	234	48
17	5320	58	51	2776	82	85	186	41
18	5262	63	52	2694	82	86	145	34
19	5199	67	53	2612	82	87	111	28
20	5132	72	54	2530	82	88	83	21
21	5060	75	55	2448	82	89	62	16
22	4985	75	56	2366	82	90	46	12
23	4910	75	57	2284	82	91	34	10
24	4835	75	58	2202	82	92	24	8
25	4760	75	59	2120	82	93	16	7
26	4685	75	60	2038	82	94	9	5
27	4610	75	61	1956	82	95	4	3
28	4535	75	62	1874	81	96	1	1
29	4460	75	63	1793	81			
30	4385	75	64	1712	80	Total	299198	11650

N. B.

N. B. The decrements in this Table for the four quarters of the firſt year of life, are given nearly in conformity to the *Cheſter* regiſter of mortality (ſee Table 41ſt in this collection); and the ſame is true of the decrements at 3 and 4 years of age, the *Northampton* regiſter affording no direction at theſe ages, becauſe it gives only the totals of deaths under two years of age, and between two and five. Many more obſervations on the method I have purſued in forming this Table, may be found in the Poſtſcript to the Second Eſſay in the preceding Volume, p. 308, &c. and in the Fourth Eſſay, p. 352, &c.

It is proper to add, that it has been taken to be the foundation and guide of the buſineſs tranſacted by the Society in CHATHAM-PLACE, for *Equitable Aſſurances* on *Lives and Survivorſhips*; and that the Tables of this Society, which will be given hereafter, together with the Tables of the values of ſingle and joint lives from Table XVIII. to Table XXXII. have been all calculated from it.

TABLE VII.

Shewing the EXPECTATIONS of Human Life at every Age, deduced from the *Northampton* Table of Obſervations.

Ages.	Expectat.	Ages.	Expectat.	Ages	Expectat.	Ages.	Expectat.
0	25.18	25	30.85	50	17.99	75	6.54
1	32.74	26	30.33	51	17.50	76	6.18
2	37.79	27	29.82	52	17.02	77	5.83
3	39.55	28	29.30	53	16.54	78	5.48
4	40.58	29	28.79	54	16.06	79	5.11
5	40.84	30	28.27	55	15.58	80	4.75
6	41.07	31	27.76	56	15.10	81	4.41
7	41.03	32	27.24	57	14.63	82	4.09
8	40.79	33	26.72	58	14.15	83	3.80
9	40.36	34	26.20	59	13.68	84	3.58
10	39.78	35	25.68	60	13.21	85	3.37
11	39.14	36	25.16	61	12.75	86	3.19
12	38.49	37	24.64	62	12.28	87	3.01
13	37.83	38	24.12	63	11.81	88	2.86
14	37.17	39	23.60	64	11.35	89	2.66
15	36.51	40	23.08	65	10.88	90	2.41
16	35.85	41	22.56	66	10.42	91	2.09
17	35.20	42	22.04	67	9.96	92	1.75
18	34.58	43	21.54	68	9.50	93	1.37
19	33.99	44	21.03	69	9.05	94	1.05
20	33.43	45	20.52	70	8.60	95	0.75
21	32.90	46	20.02	71	8.17	96	0.50
22	32.39	47	19.51	72	7.74		
23	31.88	48	19.00	73	7.33		
24	31.36	49	18.49	74	6.92		

TABLE VIII.

Shewing the Probabilities of Life at NORWICH. See page 353, Vol. I.

Ages.	Persons living.	Decr. of Life	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life
0	1185	320	32	392	6	63	174	9
1	865	160	33	386	6	64	165	9
2	705	60	34	380	6	65	156	9
3	645	32	35	374	6	66	147	9
4	613	23	36	368	6	67	138	9
5	590	20	37	362	6	68	129	9
6	570	16	38	356	6	69	120	9
7	554	13	39	350	7	70	111	9
8	541	11	40	343	6	71	102	8
9	530	9	41	337	6	72	94	8
10	521	7	42	331	6	33	86	8
11	514	6	43	325	7	74	78	8
12	508	6	44	318	7	75	70	8
13	502	5	45	311	7	76	62	7
14	497	5	46	304	7	77	55	7
15	492	5	47	297	7	78	48	6
16	487	5	48	290	7	79	42	5
17	482	5	49	283	7	80	37	5
18	477	5	50	276	7	81	32	4
19	472	5	51	269	7	82	28	4
20	467	6	52	262	7	83	24	4
21	461	6	53	255	8	84	20	3
22	455	6	54	247	8	85	17	3
23	449	6	55	239	8	86	14	3
24	443	6	56	231	8	87	11	2
25	437	6	57	223	8	88	9	2
26	431	7	58	215	8	89	7	2
27	424	7	59	207	8	90	5	2
28	417	7	60	199	8	91	3	2
29	410	6	61	191	8	92	1	1
30	404	6	62	183	9	93	1	1
31	398	6						

TABLE IX.

Shewing the Probability of the Duration of Life in LONDON, deduced by Mr. *Simpſon* from Obſervations on the Bills of Mortality in LONDON for 10 Years, from 1728 to 1737.

Ages.	Perſons living.	Decre. of Life.	Ages.	Perſons living.	Decre. of Life.	Ages.	Perſons living.	Decr. of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	
26	327	6	53	141	6			

TABLE X.

Shewing the *Expectations* of life in LONDON, according to the preceding Table. See Mr. *Simpſon's Select Exerciſes*, p. 255.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
0	19.2	27	25.1	54	14.5
1	27.0	28	24.6	55	14.2
2	32.0	29	24.1	56	13.8
3	34.0	30	23.6	57	13.4
4	35.6	31	23.1	58	13.1
5	36.0	32	22.7	59	12.7
6	36.0	33	22.3	60	12.4
7	35.8	34	21.9	61	12.0
8	35.6	35	21.5	62	11.6
9	35.2	36	21.1	63	11.2
10	34.8	37	20.7	64	10.8
11	34.3	38	20.3	65	10.5
12	33.7	39	19.9	66	10.1
13	33.1	40	19.6	67	9.8
14	32.5	41	19.2	68	9.4
15	31.9	42	18.8	69	9.1
16	31.3	43	18.5	70	8.8
17	30.7	44	18.1	71	8.4
18	30.1	45	17.8	72	8.1
19	29.5	46	17.4	73	7.8
20	28.9	47	17.0	74	7.5
21	28.3	48	16.7	75	7.2
22	27.7	49	16.3	76	6.8
23	27.2	50	16.0	77	6.4
24	26.6	51	15.6	78	6.0
25	26.1	52	15.2	79	5.5
26	25.6	53	14.9	80	5.0

TABLE II.

Shewing the Value of an Annuity on *One* Life, according to the Probabilities of Life in LONDON. See Mr. *Simpson's Select Exercises*, p. 200.

Age.	Yrs. purchase at 3 per Cent	Yrs. purchase at 4 per Cent.	Yrs purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.	Age.	Yrs. purchase at 3 per Cent.	Yrs. purchase at 4 per Cent.	Yrs. purchase at 5 per Cent.
6	18.8	16.2	14.1	31	14.8	12.9	11.4	56	10.1	9.1	8.4
7	18.9	16.3	14.2	32	14.6	12.7	11.3	57	9.9	8.9	8.2
8	19.0	16.4	14.3	33	14.4	12.6	11.2	58	9.6	8.7	8.1
9	19.0	16.4	14.3	34	14.2	12.4	11.0	59	9.4	8.6	8.0
10	19.0	16.4	14.3	35	14.1	12.3	10.9	60	9.2	8.4	7.9
11	19.0	16.4	14.3	36	13.9	12.1	10.8	61	8.9	8.2	7.7
12	18.9	16.3	14.2	37	13.7	11.9	10.6	62	8.7	8.1	7.6
13	18.7	16.2	14.1	38	13.5	11.8	10.5	63	8.5	7.9	7.4
14	18.5	16.0	14.0	39	13.3	11.6	10.4	64	8.3	7.7	7.3
15	18.3	15.8	13.9	40	13.2	11.5	10.3	65	8.0	7.5	7.1
16	18.1	15.6	13 7	41	13.0	11.4	10.2	66	7.8	7.3	6.9
17	17.9	15.4	13.5	42	12.8	11.2	10.1	67	7.6	7.1	6.7
18	17.6	15.2	13.4	43	12.6	11.1	10.0	68	7.4	6.9	6.6
19	17.4	15.0	13.2	44	12.5	11.0	9.9	69	7.1	6.7	6.4
20	17.2	14.8	13.0	45	12.3	10.8	9.8	70	6.9	6.5	6.2
21	17.0	14.7	12.9	46	12.1	10.7	9.7	71	6.7	6.3	6.0
22	16.8	14.5	12.7	47	11.9	10.5	9.5	72	6.5	6.1	5.8
23	16.5	14.3	12.6	48	11.8	10.4	9.4	73	6.2	5.9	5.6
24	16.3	14.1	12.4	49	11.6	10.2	9.3	74	5.9	5.6	5.4
25	16.1	14.0	12.3	50	11.4	10.1	9.2	75	5.6	5.4	5.2
26	15.9	13.8	12.1	51	11.2	9.9	9.0				
27	15.6	13.6	12 0	52	11.0	9.8	8.9				
28	15 4	13.4	11.8	53	10.7	9.6	8.8				
29	15.2	13.2	11.7	54	10.5	9.4	8.6				
30	15.0	13.1	11.6	55	10.3	9.3	8.5				

TABLE XII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives according to the Probabilities of Life in LONDON. See Mr. *Simpſon's Select Exerciſes*, p. 266.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent	Value at 4 per Cent.	Value at 5 per Cent.
	10	14.7	13.0	11.6		20	12.8	11.3	10.1
	15	14.3	12.7	11.3		25	12.2	10.8	9.7
	20	13.8	12.2	10.8		30	11.6	10.3	9.2
	25	13.1	11.6	10.2		35	10.9	9.8	8.8
	30	12.3	10.9	9.7		40	10.2	9.2	8.4
10	35	11.5	10.2	9.1	20	45	9.5	8.6	7.9
	40	10.7	9.6	8.6		50	8.8	8.0	7.4
	45	10.0	9.0	8.1		55	8.1	7.5	6.9
	50	9.3	8.4	7.6		60	7.4	6.9	6.4
	55	8.6	7.8	7.1		65	6.7	6.3	5.9
	60	7.8	7.2	6.6		70	6.0	5.7	5.4
	65	6.9	6.5	6.1		75	5.2	5.0	4.8
	70	6.1	5.8	5.5					
	75	5.3	5.1	4.9		25	11.8	10.5	9.4
						30	11.3	10.1	9.0
	15	13.9	12.3	11.0		35	10.7	9.6	8.6
	20	13.3	11.8	10.5		40	10.0	9.1	8.2
	25	12.6	11.2	10.1		45	9.4	8.5	7.8
	30	11.9	10.6	9.5	25	50	8.7	7.9	7.3
	35	11.2	10.0	9.0		55	8.0	7.4	6.8
	40	10.4	9.4	8.5		60	7.3	6.8	6.3
15	45	9.6	8.8	8.0		65	6.6	6.2	5.8
	50	8.9	8.2	7.5		70	5.9	5.6	5.3
	55	8.2	7.6	7.0		75	5.1	4.9	4.7
	60	7.5	7.0	6.5					
	65	6.8	6.4	6.0		30	10.8	9.6	8.6
	70	6.0	5.7	5.4	30	35	10.3	9.2	8.3
	75	5.2	5.0	4.8		40	9.7	8.8	8.0

TABLE XII. continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
30	45	9.1	8.3	7.6
	50	8.5	7.8	7.2
	55	7.9	7.3	6.7
	60	7.2	6.7	6.2
	65	6.5	6.1	5.7
	70	5.8	5.5	5.2
	75	5.1	4.9	4.7
35	35	9.9	8.8	8.0
	40	9.4	8.5	7.7
	45	8.9	8.1	7.4
	50	8.3	7.6	7.0
	55	7.7	7.1	6.6
	60	7.1	6.5	6.1
	65	6.4	6.0	5.6
	70	5.7	5.4	5.1
	75	5.0	4.8	4.6
40	40	9.1	8.1	7.3
	45	8.7	7.8	7.1
	50	8.2	7.4	6.8
	55	7.6	6.9	6.4
	60	7.0	6.4	6.0
	65	6.4	5.9	5.5
	70	5.7	5.4	5.1
	75	5.0	4.8	4.6
45	45	8.3	7.4	6.7
	50	7.9	7.1	6.5
	55	7.4	6.7	6.2
	60	6.8	6.3	5.8
45	65	6.3	5.8	5.4
	70	5.6	5.3	5.0
	75	4.9	4.7	4.5
50	50	7.6	6.8	6.2
	55	7.2	6.5	6.0
	60	6.7	6.1	5.7
	65	6.2	5.7	5.3
	70	5.5	5.2	4.9
	75	4.8	4.6	4.4
55	55	6.9	6.2	5.7
	60	6.5	5.9	5.5
	65	6.0	5.6	5.2
	70	5.4	5.1	4.8
	75	4.7	4.5	4.3
60	60	6.1	5.6	5.2
	65	5.7	5.3	4.9
	70	5.2	4.9	4.6
	75	4.6	4.4	4.2
65	65	5.4	5.0	4.7
	70	4.9	4.6	4.4
	75	4.4	4.2	4.0
70	70	4.6	4.4	4.2
	75	4.2	4.0	3.9
75	75	3.8	3.7	3.6

TABLE XIII.

Shewing the Probabilities of Life in LONDON, on the Supposition that all who die in LONDON were born there. Formed from the Bills, for 10 Years, from 1759 to 1768. See Vol. I. p. 343, &c.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages	Persons living.	Decr. of Life.
0	1000	240	31	404	9	62	132	7
1	760	99	32	395	9	63	125	7
2	661	42	33	386	9	64	118	7
3	619	29	34	377	9	65	111	7
4	590	21	35	368	9	66	104	7
5	569	13	36	359	9	67	97	7
6	556	10	37	350	9	68	90	7
7	546	7	38	341	9	69	83	7
8	539	5	39	332	10	70	76	6
9	534	4	40	322	10	71	70	6
10	530	4	41	312	10	72	64	6
11	526	4	42	302	10	73	58	5
12	522	4	43	292	10	74	53	5
13	518	3	44	282	10	75	48	5
14	515	3	45	272	10	76	43	5
15	512	3	46	262	10	77	38	5
16	509	3	47	252	10	78	33	4
17	506	3	48	242	9	79	29	4
18	503	4	49	233	9	80	25	3
19	499	5	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

TABLE XIV.

Shewing the true Probabilities of Life in London till the Age of 19. See Vol. I. p. 347, &c.

Age.	Perſons living.	Decrements of Life.
0	750	240
1	510	99
2	411	42
3	369	29
4	340	21
5	319	13
6	306	10
7	296	7
8	289	5
9	284	4
10	280	4
11	276	4
12	272	3
13	269	3
14	266	3
15	263	3
16	260	3
17	257	4
18	253	4
19	249	5
20	494	

TABLE XV.

Shewing the *true* Probabilities of Life in LONDON for all Ages, formed from the Bills for 10 Years, from 1759 to 1768. See Vol. I. p. 341. &c.

Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.	Ages.	Persons living.	Decr. of Life.
0	1518	486	31	404	9	62	132	7
1	1032	200	32	395	9	63	125	7
2	832	85	33	386	9	64	118	7
3	747	59	34	377	9	65	111	7
4	688	42	35	368	9	66	104	7
5	646	23	36	359	9	67	97	7
6	623	20	37	350	9	68	90	7
7	603	14	38	341	9	69	83	7
8	589	12	39	332	10	70	76	6
9	577	10	40	322	10	71	70	6
10	567	9	41	312	10	72	64	6
11	558	9	42	302	10	73	58	5
12	549	8	43	292	10	74	53	5
13	541	7	44	282	10	75	48	5
14	534	6	45	272	10	76	43	5
15	528	6	46	262	10	77	38	5
16	522	7	47	252	10	78	33	4
17	515	7	48	242	9	79	29	4
18	508	7	49	233	9	80	25	3
19	501	7	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

TABLE XVI.

Shewing the Probabilities of the Duration of Human Life in LONDON; and formed from the Bills for ten Years, from 1771 to 1780.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	28452	9018	34	7949	190	68	1831	130
1	19434	3000	35	7759	190	69	1701	130
2	16434	1536	36	7569	190	70	1571	130
3	14898	1200	37	7379	190	71	1441	120
4	13698	800	38	7189	190	72	1321	120
5	12898	500	39	6999	200	73	1201	120
6	12398	318	40	6799	210	74	1081	110
7	12080	210	41	6589	210	75	971	110
8	11870	160	42	6379	210	76	861	100
9	11710	130	43	6169	210	77	761	100
10	11580	130	44	5959	210	78	661	90
11	11450	130	45	5749	200	79	571	80
12	11320	130	46	5549	200	80	491	70
13	11190	130	47	5349	200	81	421	60
14	11060	130	48	5149	200	82	361	52
15	10930	130	49	4949	193	83	309	48
16	10800	130	50	4756	190	84	261	44
17	10670	130	51	4566	190	85	217	40
18	10540	135	52	4376	180	86	177	35
19	10405	135	53	4196	180	87	142	30
20	10270	140	54	4016	180	88	112	25
21	10130	150	55	3836	170	89	87	20
22	9980	155	56	3666	170	90	67	15
23	9825	155	57	3496	165	91	52	12
24	9670	160	58	3331	160	92	40	10
25	9510	160	59	3171	160	93	30	8
26	9350	160	60	3011	160	94	22	7
27	9190	170	61	2851	150	95	15	6
28	9020	170	62	2701	150	96	9	5
29	8850	171	63	2551	150	97	4	3
30	8679	180	64	2401	150	98	1	1
31	8499	180	65	2251	140			
32	8319	180	66	2111	140	Tot.	572781	28452
33	8139	190	67	1971	140			

REMARKS *on the preceding Table.*

According to this Table, the numbers dying in every decad of life from 20 to old age, are the very numbers given by the bills. For inſtance. The ſum of the decrements in the Table between 20 and 30, between 30 and 40, between 40 and 50, between 50 and 60, between 60 and 70, between 70 and 80, between 80 and 90, and above 90, are 1591, 1880, 2043, 1745, 1440, 1080, 423, and 68, reſpectively; and theſe are the average numbers which, according to the bills, have died annually in *London*, in theſe ſeveral diviſions of life, from 1771 to 1780. The ſum of all theſe numbers is 10,270, which, therefore, agreeably to the directions in the 4th Eſſay, p. 339, &c. is given in the Table as the number of the living at the age of 20.

The *proportions* of the decrements *before* 20, are likewiſe exactly the ſame with thoſe given by the bills. For inſtance. The number (*deducting the abortive and ſtill-born*) given by the bills as having died annually under two years of age from 1771 to 1780, is 7000; and the numbers given as having died between 2 and 5, between 5 and 10, and between 10 and 20, are 2060, 768, and 763. Theſe decrements, according to the Table, are 12018, 3535, 1318, and 1310: which numbers are in the ſame proportion to one another with the former numbers; and the

numbers of the living correſponding to theſe decrements are ſo adjuſted, as to make the number dying annually between 8 and 16, as *ſmall* as is conſiſtent with any degree of credibility; that is, they have been ſo adjuſted as to make this laſt number only an 86th part of the whole number living, which is a ſmaller proportion than Mr. *Wales* ſays have for 20 years died of children of the ſame ages in *Chriſt's-Hoſpital*, though near a third reſide in the country. See the note p. 343, Vol. I.

It ſhould be obſerved here, that the number living *at* 20, and the proportions of the decrements *before* 20, and the probabilities of living in one diviſion of life being obtained or aſſumed, all the numbers in the ſecond column of this Table, are ſo far determined as to render it not poſſible to fall into any material error in fixing them.—It is neceſſary to add, that though the particular decrements under two years of age, between 2 and 5, &c. are given by the bills too ſmall; this affords no reaſon for concluding that their *proportions* are not given right. On the contrary; the reaſons mentioned in the note p. 354, Vol. I. ſeem to prove they may be depended on.

The account now given ſhews, that moſt *probably* the preceding Table exhibits the probabilities of living conſiderably too high before the age of 20; and it does this *certainly* from 20 to 35 or 40, for the reaſons explained

explained in p. 339, 340, &c. Vol. I.; and in old age it gives the probabilities of living rather higher than they are in ſituations the moſt healthful. We may, therefore, ſafely conclude that it exhibits the ſtate of human life in *London* as upon the whole more favourable than it is. According to this Table, however, one half of all born in *London* die in the firſt four years; and the expectation of a child at birth is only 19½.—It is farther obſervable, that for all ages after 20, it agrees ſo nearly with Table 9th formed from the bills from 1728 to 1737, and with Table 15th formed from the bills from 1759 to 1768, as to demonſtrate that, for the laſt 50 years, there has been no change in the ſtate of *London* which has *greatly* affected its influence on the duration of human life. This will appear from the following compariſon.

Expectations of Life at	By Table 9th	By Table 15th	By Table 16th
20	28.9	29.3	29.6
25	26.1	26.6	26.7
30	23.6	24.1	24.1
35	21.5	21.7	21.6
40	19.6	19.5	19.3
45	17.8	17.6	17.4
50	16.0	15.9	15.5
55	14.2	13.9	13.6
60	12.4	11.7	11.7
65	10.5	9.7	9.8
70	8.8	8.0	7.9
75			
80			

It cannot but be reckoned remarkable, that the duration of human life in *London* ſhould come out by the bills ſo nearly the ſame at the three periods for which the Table mentioned in this compariſon were formed. A ſmall difference, indeed, appears from the age of 20 to 30 in favour of *London* in its preſent ſtate; but it muſt not be depended on as a reaſon for concluding that *London* is now leſs prejudicial to health than it was; for Mr. *Simpſon*, in forming Table 9th, did not take, as I have done, the decrements of life between 20 and 30 exactly from the bills, but extended his corrections very properly to this diviſion of life as well as thoſe preceding it; and had I done the ſame, the expectations for 20 and 25, deduced from Tables 15th and 16th, would have been leſs than they are.——With reſpect to all ages before 20, nothing certain can be collected from theſe Tables. The laſt makes, indeed, one half of the children born to ſurvive 4 years of age, whereas the other Tables make one half live only to three years of age; but it ſhould be recollected, that this difference has been occaſioned by the act of parliament paſſed in 1767, and mentioned in the notes, p. 251, 354, Vol. I. requiring all pariſh children to be ſent into the country for ſix years. If only a thouſand burials of infants under two years of age, and born in *London*, have by this act been taken out of the bills, which uſed to be, and ought

to

to be, included in them, it will follow that one half of the children born in *London* do not live to three years of age; and a table conſtructed in the manner of the laſt table, would have ſhewn this as well as the other tables.——Mr. *Howlett* tells us, that this deficiency amounts to 2100; and were this true, it would follow that *London* is now more fatal to children than ever it was. But I have learnt not to rely on Mr. *Howlett*'s accounts. See the note in Vol. I. p. 251.

This Table would have been very nearly the ſame, had it been formed from the bills for the laſt *five* years from 1777 to 1781, inſtead of being formed as it is from the bills for ten years from 1771 to 1780.

TABLE XVII.

Shewing the Value of an Annuity on a single Life at every Age, according to the Probabilities of the Duration of Human Life at NORTHAMPTON. See Table VI. p. 36.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.	Value at 7 per Ct.	Value at 8 per Ct.
Birth		10.327	8.863			
½ year		13.008	11.274			
1	16.024	13.465	11.563	10.107	8.963	8.046
2	18.599	15.633	13.420	11.724	10.391	9.321
3	19.575	16.462	14.135	12.348	10 941	9.812
4	20.210	17.010	14.613	12.769	11.315	10 147
5	20.473	17.248	14.827	12.962	11.489	10.304
6	20.727	17.482	15.041	13.156	11.666	10 466
7	20.853	17.611	15.166	13.275	11.777	10.570
8	20.885	17.662	15.226	13.337	11 840	10.631
9	20.812	17.625	15.210	13.335	11.846	10.641
10	20.663	17.523	15.139	13.285	11 809	10.614
11	20.480	17.393	15.043	13.212	11.752	10.569
12	20.283	17.251	14.937	13.130	11.687	10.517
13	20.081	17.103	14.826	13.044	11.618	10.461
14	19.872	16.950	14.710	12.953	11.545	10.401
15	19.657	16.791	14.588	12.857	11.467	10.337
16	19.435	16.625	14.460	12.755	11.384	10 268
17	19.218	16.462	14.334	12.655	11.302	10.200
18	19.013	16.309	14.217	12.562	11.226	10.137
19	18.820	16.167	14.108	12.477	11.157	10.081
20	18.638	16.033	14.007	12.398	11.094	10.030
21	18.470	15.912	13.917	12.329	11.042	9.986
22	18.311	15.797	13.833	12.265	10.993	9.947
23	18.148	15.680	13.746	12.200	10.942	9.907
24	17.983	15.560	13.658	12.132	10.890	9.865
25	17.814	15.438	13.567	12.063	10.836	9 823
26	17.642	15.312	13.473	11.992	10.780	9.778
27	17.467	15.184	15.377	11.917	10.723	9.732
28	17.289	15.053	13.278	11.841	10.663	9.685
29	17.107	14.918	13.177	11.763	10 602	9.635
30	16.922	14.781	13.072	11.682	10.539	9.584
31	16.732	14.639	12.965	11.598	10.473	9.531
32	16.540	14.495	12.854	11.512	10.404	9.476
33	16.343	14 347	12 740	11.423	10.333	9 418
34	16.142	14.195	12 623	11.331	10.260	9.359

TABLE XVII. continued.

Age	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per C.	Value at 7 per Ct.	Value at 8 per Ct.
35	15.938	14.039	12.502	11.236	10.183	9.296
36	15.729	13.880	12.377	11.137	10.104	9.231
37	15.515	13.716	12.249	11.035	10.021	9.164
38	15.298	13.548	12.116	10.929	9.935	9.093
39	15.075	13.375	11.979	10.819	9.845	9.019
40	14.848	13.197	11.837	10.705	9.752	8.941
41	14.620	13.018	11.695	10.589	9.657	8.863
42	14.391	12.838	11.551	10.473	9.562	8.783
43	14.162	12.657	11.407	10.356	9.466	8.703
44	13.929	12.472	11.258	10.235	9.366	8.620
45	13.692	12.283	11.105	10.110	9.262	8.533
46	13.450	12.089	10.947	9.980	9.154	8.443
47	13.203	11.890	10.784	9.846	9.042	8.348
48	12.951	11.685	10.616	9.707	8.925	8.249
49	12.693	11.475	10.443	9.563	8.804	8.146
50	12.436	11.264	10.269	9.417	8.681	8.041
51	12.183	11.057	10.097	9.273	8.559	7.937
52	11.930	10.849	9.925	9.129	8.437	7.833
53	11.674	10.637	9.748	8.980	8.311	7.725
54	11.414	10.421	9.567	8.827	8.181	7.614
55	11.150	10.201	9.382	8.670	8.047	7.499
56	10.882	9.977	9.193	8.509	7.909	7.379
57	10.611	9.749	8.999	8.343	7.766	7.256
53	10.337	9.516	8.801	8.173	7.619	7.128
59	10.058	9.280	8.599	7.999	7.468	6.996
60	9.777	9.039	8.392	7.820	7.312	6.860
61	9.493	8.795	8.181	7.637	7.152	6.719
62	9.205	8.547	7.966	7.449	6.988	6 574
63	8.910	8.291	7.742	7.253	6.815	6.421
64	8.611	8.030	7.514	7.052	6.637	6.262
65	8.304	7.761	7.276	6.841	6.449	6.095
66	7.994	7.488	7.034	6.625	6.256	5.922
67	7.682	7.211	6.787	6.405	6.058	5.743
68	7.367	6.930	6.536	6.179	5.855	5.559
69	7.051	6.647	6.281	5.949	5.646	5 370
70	6.734	6.361	6.023	5.716	5.434	5.176
71	6.418	6.075	5.764	5.479	5.21	4.978
72	6.103	5.790	5.504	5.241	5.000	4.778
73	5.794	5.507	5.245	5.004	4.781	4 576
74	5.491	5.230	4.990	4.769	4.565	4.375

TABLE XVII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.	Value at 7 per Ct.	Value at 8 per Ct.
75	5.199	4.962	4.744	4.542	4.354	4.180
76	4.925	4.710	4.511	4.326	4.154	3.994
77	4.652	4.457	4.277	4.109	3.952	3.806
78	4.372	4.197	4.035	3.884	3.742	3.609
79	4.077	3.921	3.776	3.641	3.514	3.394
80	3.781	3.643	3.515	3.394	3.281	3.174
81	3.499	3.377	3.263	3.156	3.055	2 960
82	3 229	3 122	3.020	2.926	2.836	2.751
83	2.982	2.887	2.797	2.713	2.632	2.557
84	2.793	2.708	2.627	2.551	2.479	2.410
85	2.620	2.543	2.471	2.402	2.337	2.275
86	2.462	2.393	2.328	2.266	2.207	2.151
87	2.312	2.251	2.193	2.138	2.085	2.035
88	2.185	2.131	2.080	2.031	1.984	1.939
89	2.013	1.967	1.924	1.882	1.842	1.803
90	1.794	1.758	1.723	1.689	1.656	1.625
91	1.501	1.474	1.447	1.422	1.398	1.374
92	1.190	1.171	1.153	1.136	1.118	1.102
93	0.839	0.827	0.816	0.806	0.795	0.785
94	0.536	0.530	0.524	0.518	0 512	0.507
95	0.242	0.240	0.238	0.236	0.234	0.232
96	0.000	0.000	0.000	0.000	0.000	0.000

The values of annuities in the preceding Table (and in all the other Tables in this collection), ſuppoſe the payments to be made *yearly*, and to begin at the end of a year; except in the ſingle inſtance of an annuity on a life aged *half* a year, the value of which is given in the preceding Table, on the ſuppoſitions that the firſt payment is to be a half-yearly one made at the end of half a year, and that all the ſubſequent payments are yearly ones.

If all the payments are to be *half-yearly* payments, and to be made at the end of every *half* year from the time of purchaſe, their value will be increaſed about *one fifth* of a year's purchaſe. When the *tabular* value (that is, the value of an annuity to commence at the end of a year, and payable yearly) is greater than 11 or 12 years purchaſe, this addition will give ſomewhat *more*, and when leſs it will give ſomewhat *leſs* than the value of the ſame annuity payable *half-yearly*; but in no inſtance will the error exceed a 20th of a year's purchaſe.

TABLE XVIII.

Shewing the Value of an Annuity on the *joint Continuance* of Two Lives, having the ſame common Age, according to the *Northampton* Table of Obſervations. See Table VI. p. 36.

Difference of Age 0.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1- 1	9.491	8.252	7.287	6.515
2- 2	12.789	11.107	9.793	8.741
3- 3	14.196	12.325	10.862	9.689
4- 4	15.181	13.185	11.621	10.365
5- 5	15.638	13.591	11.984	10.691
6- 6	16.099	14.005	12.358	11.031
7- 7	16.375	14.224	12.596	11.251
8- 8	16.510	14.399	12.731	11.382
9- 9	16.483	14.396	12.744	11.404
10-10	16.339	14.277	12.665	11.345
11-11	16.142	14.133	12.546	11.249
12-12	15.926	13.966	12.411	11.139
13-13	15.702	13.789	12.268	11.023
14-14	15.470	13.604	12.118	10.899
15-15	15.229	13.411	11.960	10.767
16-16	14.979	13.212	11.793	10.626
17-17	14.737	13.019	11.630	10.489
18-18	14.516	12.841	11.483	10.365
19-19	14.316	12.679	11.351	10.255
20-20	14.133	12.535	11.232	10.156

TABLE XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
21-21	13.974	12.409	11.131	10.074
22-22	13.830	12.293	11.043	10.002
23-23	13.683	12.179	10.951	9.928
24-24	13.534	12.062	10.858	9.853
25-25	13.383	11.944	10.764	9.776
26-26	13.230	11.822	10.667	9.697
27-27	13.074	11.699	10.567	9.616
28-28	12.915	11.573	10.466	9.533
29-29	12.754	11.445	10.362	9.448
30-30	12.589	11.313	10.255	9.360
31-31	12.422	11.179	10.146	9.270
32-32	12.252	11.042	10.034	9.178
33-33	12.079	10.902	9.919	9.082
34-34	11.902	10.759	9.801	8.984
35-35	11.722	10.612	9.680	8.883
36-36	11.539	10.462	9.555	8.778
37-37	11.351	10.307	9.427	8.670
38-38	11.160	10.149	9.294	8.558
39-39	10.964	9.986	9.158	8.442
40-40	10.764	9.820	9.016	8.322
41-41	10.565	9.654	8.876	8.202
42-42	10.369	9.491	8.737	8.083
43-43	10.175	9.326	8.599	7.965
44-44	9.978	9.160	8.457	7.843
45-45	9.776	8.990	8.312	7.718
46-46	9.571	8.815	8.162	7.589
47-47	9.362	8.637	8.008	7.455
48-48	9.149	8.453	7.849	7.316

TABLE XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
49-49	8.931	8.266	7.686	7.173
50-50	8.714	8.081	7.522	7.030
51-51	8.507	7.900	7.366	6.893
52-52	8.304	7.723	7.213	6.758
53-53	8.099	7.544	7.056	6.620
54-54	7.891	7.362	6.897	6.480
55-55	7.681	7.179	6.735	6.336
56-56	7.470	6.993	6.571	6.190
57-57	7.256	6.805	6.404	6.041
58-58	7.041	6.614	6.234	5.890
59-59	6.824	6.421	6.062	5.735
60-60	6.606	6.226	5.888	5.579
61-61	6.387	6.030	5.712	5.420
62-62	6.166	5.831	5.533	5.259
63-63	5.938	5.626	5.347	5.089
64-64	5.709	5.417	5.158	4.917
65-65	5.471	5.201	4.960	4.736
66-66	5.231	4.982	4.759	4.551
67-67	4.990	4.760	4.555	4.363
68-68	4.747	4.537	4.348	4.171
69-69	4.504	4.312	4.140	3.977
70-70	4.261	4.087	3.930	3.781
71-71	4.020	3.862	3.719	3.584
72-72	3.781	3.639	3.510	3.387
73-73	3.548	3.421	3.304	3.193
74-74	3.324	3.211	3.105	3.005
75-75	3.114	3.015	2.917	2.827
76-76	2.920	2.833	2.750	2.668

TABLE XVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
77-77	2.741	2.656	2.583	2.511
78-78	2.550	2.470	2.410	2.346
79-79	2.338	2.271	2.217	2.161
80-80	2.122	2.068	2.018	1.969
81-81	1.917	1.869	1.827	1.786
82-82	1.719	1.681	1.642	1.606
83-83	1.538	1.510	1.472	1.441
84-84	1.416	1.387	1.357	1.330
85-85	1.309	1.339	1.256	1.232
86-86	1.218	1.195	1.171	1.149
87-87	1.141	1.124	1.098	1.078
88-88	1.103	1.030	1.063	1.044
89-89	1.036	1.015	1.001	0.984
90-90	0.938	0.922	0.909	0.895
91-91	0.769	0.756	0.748	0.737
92-92	0.591	0.583	0.576	0.569
93-93	0.369	0.365	0.361	0.357
94-94	0.203	0.201	0.199	0.197
95-95	0.060	0.060	0.059	0.058
96-96	0.000	0.000	0.000	0.000

TABLE XIX.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Observations. See Table VI. p. 36.

Difference of Age *five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1- 6	12.347	10.741	9.479	8.467
2- 7	14.461	12.581	11.100	9.911
3- 8	15.300	13.319	11.755	10.498
4- 9	15.809	13.775	12.165	10.869
5-10	15.974	13.933	12.315	11.010
6-11	16.110	14.068	12.447	11.136
7-12	16.137	14.111	12.498	11.192
8-13	16.089	14.089	12.492	11.197
9-14	15.957	13.992	12.421	11.144
10-15	15.762	13.841	12.302	11.048
11-16	15.538	13.664	12.158	10.929
12-17	15.308	13.480	12.009	10.805
13-18	15.086	13.303	11.864	10.685
14-19	14.870	13.130	11.723	10.568
15-20	14.660	12.961	11.585	10.453
16-21	14.457	12.799	11.452	10.342
17-22	14.265	12.646	11.327	10.239
18-23	14.082	12.500	11.209	10.140
19-24	13.908	12.361	11.096	10.048
20-25	13.741	12.229	10.989	9.960
21-26	13.584	12.105	10.890	9.879
22-27	13.433	11.987	10.796	9.803

TABLE XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
23-28	13.280	11.866	10.699	9.724
24-29	13.124	11.743	10.600	9.643
25-30	12.966	11.618	10.499	9.561
26-31	12.805	11 489	10.396	9.476
27-32	12.641	11.359	10.289	9.389
28-33	12.474	11.225	10.181	9.299
29-34	12.304	11.088	10.069	9.207
30-35	12.131	10.948	9.954	9.112
31-36	11.955	10.805	9.837	9.014
32-37	11.775	10.659	9.716	8.913
33-38	11.592	10.508	9.591	8.808
34-39	11.404	10.354	9.463	8.701
35-40	11.213	10.196	9.331	8.589
36-41	11.021	10.037	9.198	8.476
37-42	10.828	9.877	9.062	8.362
38-43	10.635	9.716	8.927	8.246
39-44	10.437	9.550	8.787	8.127
40-45	10.236	9.381	8.643	8.003
41-46	10.033	9.210	8.497	7.878
42-47	9.829	9.037	8.350	7.751
43-48	9.624	8.862	8.200	7.621
44-49	9.414	8.683	8.046	7.488
45-50	9.204	8.503	7.891	7.353
46-51	8.997	8.326	7.737	7.219
47-52	8.790	8.147	7.582	7.084
48-53	8.579	7.965	7.424	6.945
49-54	8.366	7.780	7.262	6.802
50-55	8.152	7.593	7.098	6.658

TABLE XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
51-56	7.941	7.409	6.936	6.515
52-57	7.730	7.225	6.774	6.371
53-58	7.518	7.039	6.609	6.225
54-59	7.304	6.850	6.442	6.076
55-60	7.088	6.659	6.272	5.924
56-61	6.870	6.465	6.100	5.770
57-62	6.651	6.270	5.925	5.613
58-63	6.427	6.070	5.744	5.450
59-64	6.201	5.867	5.561	5.284
60-65	5.970	5.658	5.372	5.112
61-66	5.737	5.447	5.180	4.938
62-67	5.503	5.285	4.986	4.760
63-68	5.265	5.017	4.786	4.576
64-69	5.025	4.798	4.585	4.390
65-70	4.783	4.573	4.378	4.199
66-71	4.540	4.349	4.169	4.005
67-72	4.298	4.124	3.960	3.811
68-73	4.059	3.901	3.752	3.616
69-74	3.825	3.683	3.547	3.423
70-75	3.599	3.471	3.347	3.236
71-76	3.386	3.270	3.159	3.059
72-77	3.176	3.070	2.971	2.882
73-78	2.963	2.869	2.780	2.701
74-79	2.743	2.659	2.580	2.511
75-80	2.526	2.448	2.381	2.323
76-81	2.325	2.258	2.195	2.147
77-82	2.131	2.077	2.013	1.975
78-83	1.947	1.899	1.838	1.810

TABLE XIX. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
79-84	1.793	1.751	1.750	1.672
80-85	1.645	1.608	1.573	1.539
81-86	1.511	1.478	1.447	1.417
82-87	1.385	1.356	1.329	1.303
83-88	1.284	1.259	1.235	1.212
84-89	1,183	1.164	1,145	1.124
85-90	1.074	1.054	1.038	1.021
86-91	0.921	0.902	0.892	0.879
87-92	0.756	0.738	0.734	0.725
88-93	0.562	0.554	0.547	0.541
89-94	0.377	0.373	0.369	0.365
90-95	0,179	0.177	0,175	0.174
91-96	0.000	0.000	0.000	0.000

TABLE XX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *ten* Years.

Ages.	Value at 3 per Cent	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cenr.
1-11	12.346	10.782	9.544	8.547
2-12	14.239	12.438	11.010	9.857
3-13	14.895	13.019	11.528	10.324
4-14	15.287	13.374	11.850	10.617
5-15	15.391	13.479	11.954	10.716
6-16	15.486	13.578	12.052	10.812
7-17	15.490	13.599	12.083	10.849
8-18	15.436	13.569	12.070	10.847
9-19	15.316	13.482	12.006	10.799
10-20	15.151	13.355	11.906	10.719
11-21	14.974	13.217	11.797	10.631
12-22	14.795	13.078	11.686	10.541
13-23	14.612	12.934	11.570	10.446
14-24	14.424	12.784	11.450	10.348
15-25	14.230	12.630	11.324	10.244
16-26	14.030	12.470	11.193	10.135
17-27	13.832	12.311	11.063	10.027
18-28	13.642	12.158	10.939	9.924
19-29	13.461	12.013	10.820	9.826
20-30	13.286	11.873	10.707	9.732
21-31	13.121	11.742	10.600	9.644

TABLE XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
22-32	12.561	11.615	10.498	9.561
23-33	12.798	11.485	10.393	9.474
24-34	12.632	11.352	10.285	9.386
25-35	12.463	11.217	10.175	9.295
26-36	12.291	11.078	10.062	9.201
27-37	12.116	10.936	9.946	9.105
28-38	11.937	10.791	9.826	9.005
29-39	11.755	10.642	9.703	8.902
30-40	11.568	10.490	9.576	8.795
31-41	11.382	10.336	9.448	8.688
32-42	11.195	10.182	9.320	8.580
33-43	11.007	10.027	9.190	8.471
34-44	10.817	9.869	9.058	8.358
35-45	10.622	9.706	8.921	8.242
36-46	10.424	9.540	8.781	8.122
37-47	10.221	9.370	8.636	7.998
38-48	10.014	9.195	8.487	7.870
39-49	9.803	9.015	8.333	7.737
40-50	9.590	8.834	8.177	7.602
41-51	9.383	8.658	8.025	7.470
42-52	9.179	8.483	7.875	7.340
43-53	8.975	8.308	7.724	7.208
44-54	8.767	8.130	7.569	7.073
45-55	8.557	7.948	7.411	6.935
46-56	8.344	7.763	7.249	6.793
47-57	8.127	7.574	7.084	6.648
48-58	7.907	7.382	6.915	6.498
49-59	7.684	7.186	6.742	6.344

TABLE XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
50-60	7.461	6.989	6.568	6.189
51-61	7.240	6.795	6.395	6.035
52-62	7.021	6.600	6.222	5.880
53-63	6.795	6.399	6.042	5.719
54-64	6.568	6.196	5.860	5.555
55-65	6.334	5.986	5.671	5.384
56-66	6.098	5.774	5.479	5.209
57-67	5.860	5.559	5.283	5.031
58-68	5.621	5.341	5.084	4.849
59-69	5.380	5.121	4.883	4.665
60-70	5.139	4.900	4.680	4.478
61-71	4.898	4.679	4.476	4.289
62-72	4.659	4.458	4.272	4.099
63-73	4.420	4.236	4.066	3.908
64-74	4.186	4.019	3.864	3.719
65-75	3.958	3.806	3.665	3.533
66-76	3.743	3.606	3.477	3.357
67-77	3.529	3.405	3.289	3.180
68-78	3.310	3.199	3.095	2.996
69-79	3.077	2.979	2.887	2.799
70-80	2.843	2.757	2.675	2.598
71-81	2.618	2.542	2.470	2.402
72-82	2.401	2.334	2.271	2.211
73-83	2.199	2.141	2.085	2.032
74-84	2.043	1.991	1.941	1.894
75-85	1.903	1.856	1.811	1.769
76-86	1.781	1.739	1.699	1.661
77-87	1.670	1.633	1.597	1.562

TABLE XX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
78-88	1.580	1.546	1.514	1.483
79-89	1.456	1.427	1.400	1.373
80-90	1.302	1.278	1.255	1.234
81-91	1.096	1.078	1.061	1.044
82-92	0.877	0.864	0.852	0.840
83-93	0.622	0.614	0.606	0.599
84-94	0.408	0.403	0.398	0.394
85-95	0.189	0.187	0.185	0.183
86-96	0.000	0.000	0.000	0.000

TABLE XXI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *fifteen* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-16	11.864	10.406	9.243	8.301
2-17	13.659	11.981	10.642	9.555
3-18	14.277	12.531	11.134	9.998
4-19	14.657	12.876	11.447	10.284
5-20	14.776	12.993	11.561	10.391
6-21	14.904	13.121	11.685	10.510
7-22	14.950	13.178	11.748	10.576
8-23	14.929	13.178	11.761	10.597
9-24	14.834	13.112	11.715	10.566
10-25	14.683	12.998	11.627	10.497
11-26	14.508	12.861	11.519	10.410
12-27	14.323	12.715	11.402	10.314
13-28	14.132	12.564	11.280	10.215
14-29	13.936	12.408	11.153	10.110
15-30	13.734	12.246	11.021	10.001
16-31	13.527	12.078	10.883	9.886
17-32	13.320	11.911	10.746	9.771
18-33	13.121	11.750	10.613	9.660
19-34	12.930	11.595	10.486	9.554
20-35	12.744	11.445	10.363	9.451
21-36	12.567	11.302	10.246	9.354
22-37	12.394	11.163	10.132	9.260
23-38	12.218	11.020	10.015	9.163
24-39	12.03[illegible]	10.874	9.895	9.063
25-40	11.854	10.725	9.771	8.960

TABLE XXI. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
26-41	11.670	10.574	9.647	8.855
27-42	11.486	10.423	9.522	8.751
28-43	11.302	10.272	9.396	8.645
29-44	11.114	10.117	9.267	8.536
30-45	10.923	9.959	9.135	8.424
31-46	10.728	9.797	8.998	8.309
32-47	10.530	9.631	8.858	8.189
33-48	10.327	9.461	8.714	8.066
34-49	10.120	9.286	8.565	7.938
35-50	9.912	9.110	8.415	7.809
36-51	9.707	8.937	8.267	7.681
37-52	9.503	8.763	8.119	7.553
38-53	9.296	8.586	7.966	7.421
39-54	9.085	8.406	7.810	7.286
40-55	8.870	8.221	7.651	7.146
41-56	8.655	8.035	7.489	7.005
42-57	8.439	7.848	7.326	6.862
43-58	8.222	7.660	7.162	6.718
44-59	8.003	7.469	6.994	6.570
45-60	7.781	7.274	6.822	6.418
46-61	7.556	7.076	6.648	6.263
47-62	7.328	6.875	6.469	6.104
48-63	7.093	6.667	6.283	5.937
49-64	6.854	6.454	6.093	5.767
50-65	6.611	6.236	5.897	5.590
51-66	6.369	6.019	5.701	5.412
52-67	6.127	5.801	5.504	5.233
53-68	5.884	5.580	5.303	5.050
54-69	5.638	5.357	5.100	4.864

TABLE XXI. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
55-70	5.391	5.132	4.893	4.674
56-71	5.145	4.905	4.685	4.482
57-72	4.899	4.679	4.477	4.289
58-73	4.656	4.455	4.269	4.096
59-74	4.418	4.234	4.064	3.906
60-75	4.189	4.021	3.866	3.721
61-76	3.974	3.821	3.679	3.546
62-77	3.760	3.621	3.492	3.371
63-78	3.538	3.414	3.297	3.188
64-79	3.303	3.192	3.088	2.990
65-80	3.063	2.965	2.873	2.786
66-81	2.833	2.746	2.664	2.587
67-82	2.610	2.533	2.461	2.393
68-83	2.403	2.336	2.272	2.211
69-84	2.244	2.183	2.126	2.071
70-85	2.097	2.042	1.991	1.941
71-86	1.963	1.914	1.867	1.823
72-87	1.838	1.794	1.753	1.713
73-88	1.736	1.697	1.600	1.625
74-89	1.603	1.570	1.538	1.508
75-90	1.440	1.413	1.387	1.361
76-91	1.221	1.200	1.180	1.160
77-92	0.985	0.970	0.955	0.942
78-93	0.706	0.697	0.688	0.679
79-94	0.458	0.453	0.448	0.443
80-95	0.210	0.208	0.206	0.204
81-96	0.000	0.000	0.000	0.000

TABLE XXII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *twenty* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-21	11.413	10.053	8.961	8.070
2-22	13.172	11.605	10.344	9.313
3-23	13.794	12.161	10.843	9.764
4-24	14.178	12.511	11.163	10.057
5-25	14.301	12.633	11.281	10.170
6-26	14.420	12.754	11.400	10.285
7-27	14.451	12.798	11.452	10.341
8-28	14.417	12.786	11.455	10.354
9-29	14.310	12.710	11.401	10.315
10-30	14.150	12.586	11.304	10.239
11-31	13.965	12.441	11.188	10.144
12-32	13.770	12.286	11.062	10.042
13-33	13.570	12.125	10.932	9.934
14-34	13.363	11.959	10.796	9.822
15-35	13.151	11.787	10.655	9.703
16-36	12.932	11.609	10.507	9.579
17-37	12.714	11.430	10.358	9.454
18-38	12.502	11.257	10.214	9.333
19-39	12.297	11.089	10.074	9.215
20-40	12.096	10.924	9.937	9.100
21-41	11.906	10.768	9.809	8.992
22-42	11.723	10.619	9.685	8.889
23-43	11.540	10.470	9.562	8.785
24-44	11.354	10.317	9.435	8.670

TABLE XXII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
25-45	11.164	10.160	9.304	8.569
26-46	10.970	10.000	9.170	8.455
27-47	10.773	9.836	9.032	8.338
28-48	10.572	9.667	8.890	8.217
29-49	10.366	9.495	8.744	8.092
30-50	10.160	9.321	8.596	7.966
31-51	9.957	9.151	8.451	7.841
32-52	9.756	8.980	8.306	7.716
33-53	9.550	8.806	8.157	7.588
34-54	9.342	8.629	8.005	7.457
35-55	9.131	8.448	7.849	7.322
36-56	8.916	8.264	7.690	7.183
37-57	8.699	8.076	7.527	7.041
38-58	8.477	7.884	7.360	6.894
39-59	8.253	7.689	7.189	6.744
40-60	8.025	7.490	7.015	6.590
41-61	7.796	7.290	6.838	6.434
42-62	7.567	7.088	6.660	6.276
43-63	7.332	6.881	6.477	6.112
44-64	7.095	6.671	6.289	5.944
45-65	6.850	6.453	6.094	5.769
46-66	6.602	6.230	5.894	5.588
47-67	6.351	6.004	5.690	5.403
48-68	6.096	5.774	5.481	5.213
49-69	5.839	5.541	5.268	5.019
50-70	5.582	5.306	5.054	4.822
51-71	5.328	5.074	4.841	4.626
52-72	5.077	4.845	4.630	4.430

TABLE XXII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
53-73	4.829	4.614	4.417	4.234
54-74	4.585	4.389	4.208	4.040
55-75	4.350	4.171	4.006	3.852
56-76	4.129	3.966	3.815	3.674
57-77	3.908	3.761	3.623	3.494
58-78	3.682	3.549	3.424	3.308
59-79	3.440	3.322	3.210	3.105
60-80	3.197	3.092	2.992	2.899
61-81	2.964	2.870	2.782	2.699
62-82	2.739	2.656	2.578	2.504
63-83	2.530	2.457	2.387	2.321
64-84	2.371	2.305	2.242	2.182
65-85	2.223	2.163	2.107	2.053
66-86	2.089	2.035	1.984	1.936
67-87	1.963	1.915	1.870	1.826
68-88	1.860	1.817	1.777	1.737
69-89	1.722	1.685	1.650	1.616
70-90	1.545	1.515	1.486	1.459
71-91	1.303	1.280	1.259	1.238
72-92	1.044	1.028	1.012	0.997
73-93	0.743	0.733	0.723	0.714
74-94	0.480	0.474	0.469	0.464
75-95	0.219	0.217	0.215	0.213
76-96	0.000	0.000	0.000	0.000

TABLE XXIII.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *twenty-five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-26	11.037	9.770	8.742	7.897
2-27	12.722	11.264	10.080	9.104
3-28	13.307	11.790	10.555	9.537
4-29	13.661	12.116	10.855	9.813
5-30	13.762	12.220	10.959	9.913
6-31	13.859	12.322	11.062	10.015
7-32	13.871	12.350	11.100	10.060
8-33	13.820	12.323	11.090	10.061
9-34	13.698	12.234	11.024	10.012
10-35	13.525	12.098	10.916	9.925
11-36	13.328	11.941	10.788	9.820
12-37	13.120	11.773	10.651	9.707
13-38	12.906	11.600	10.509	9.588
14-39	12.686	11.420	10.360	9.464
15-40	12.459	11.234	10.205	9.333
16-41	12.229	11.044	10.046	9.198
17-42	12.002	10.856	9.889	9.065
18-43	11.785	10.677	9.739	8.938
19-44	11.574	10.502	9.592	8.814
20-45	11.367	10.330	9.448	8.692
21-46	11.167	10.165	9.310	8.574
22-47	10.969	10.001	9.173	8.458
23-48	10.768	9.833	9.031	8.338

TABLE XXIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct
24-49	10.562	9.661	8.886	8.214
25-50	10.356	9.488	8.739	8.089
26-51	10.154	9.318	8.595	7.966
27-52	9.952	9.148	8.451	7.842
28-53	9.748	8.975	8.304	7.716
29-54	9.540	8.799	8.153	7.586
30-55	9.329	8.619	7.999	7.453
31-56	9.115	8.436	7.841	7.316
32-57	8.897	8.250	7.680	7.175
33-58	8.677	8.060	7.515	7.031
34-59	8.454	7.866	7.346	6.884
35-60	8.227	7.669	7.174	6.732
36-61	7.997	7.469	6.998	6.577
37-62	7.765	7.265	6.819	6.418
38-63	7.525	7.053	8.631	6.252
39-64	7.281	6.838	6.440	6.081
40-65	7.030	6.614	6.240	5.901
41-66	6.776	6.388	6.037	5.718
42-67	6.522	6.159	5.831	5.532
43-68	6.266	5.929	5.622	5.343
44-69	6.008	5.696	5.411	5.150
45-70	5.749	5.460	5.195	4.953
46-71	5.488	5.222	4.978	4.753
47-72	5.228	4.983	4.758	4.551
48-73	4.970	4.746	4.539	4.348
49-74	4.716	4.511	4.322	4.146
50-75	4.472	4.285	4.112	3.951
51-76	4.245	4.074	3.916	3.768

TABLE XXIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
52-77	4.019	3.864	3.720	3.586
53-78	3.787	3.648	3.518	3.396
54-79	3.540	3.416	3.299	3.189
55-80	3.291	3.180	3.076	2.978
56-81	3.051	2.953	2.861	2.774
57-82	2.820	2.733	2.651	2.574
58-83	2.608	2.530	2.457	2.388
59-84	2.446	2.376	2.310	2.247
60-85	2.297	2.234	2.174	2.118
61-86	2.162	2.105	2.051	2.000
62-87	2.036	1.985	1.937	1.891
63-88	1.932	1.886.	1.843	1.802
64-89	1.790	1.751	1.714	1.678
65-90	1.606	1.575	1.544	1.515
66-91	1.354	1.330	1.307	1.285
67-92	1.083	1.067	1.050	1.035
68-93	0.770	0.760	0.750	0.740
69-94	0.497	0.491	0.485	0.480
70-95	0.227	0.224	0.222	0.220
71-96	0.000	0.000	0.000	0.000

TABLE XXIV.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obfervations, p. 36.

Difference of Age *thirty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-31	10.605	9.438	8.483	7.691
2-32	12.203	10.865	9.767	8.855
3-33	12.743	11.355	10.213	9.263
4-34	13.061	11.651	10.488	9.518
5-35	13.136	11.732	10.572	9.602
6-36	13.207	11.812	10.656	9.687
7-37	13.195	11.819	10.676	9.715
8-38	13.122	11.772	10.648	9.701
9-39	12.981	11.665	10.565	9.637
10-40	12.791	11.513	10.442	9.537
11-41	12.580	11.342	10.302	9.420
12-42	12.363	11.165	10.156	9.298
13-43	12.144	10.985	10.007	9.173
14-44	11.918	10.799	9.852	9.042
15-45	11.687	10.607	9.690	8.905
16-46	11.448	10.408	9.522	8.762
17-47	11.210	10.208	9.353	8.617
18-48	10.975	10.011	9.186	8.473
19-49	10.746	9.818	9.021	8.332
20-50	10.523	9.630	8.861	8.195
21-51	10.313	9.454	8.712	8.067
22-52	10.111	9.284	8.568	7.944

TABLE XXIV. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
23-53	9.905	9.111	8.421	7.818
24-54	9.696	8.934	8.270	7.688
25-55	9.484	8.754	8.116	7.555
26-56	9.269	8.570	7.958	7.419
27-57	9.051	8.383	7.797	7.279
28-58	8.830	8.193	7.632	7.135
29-59	8.605	7.999	7.464	6.988
30-60	8.378	7.802	7.292	6.837
31-61	8.147	7.601	7.116	6.682
32-62	7.914	7.397	6.937	6.524
33-63	7.673	7.186	6.750	6.359
34-64	7.429	6.971	6.559	6.189
35-65	7.177	6.747	6.360	6.010
36-66	6.922	6.520	6.156	5.827
37-67	6.663	6.288	5.948	5.639
38-68	6.401	6.052	5.735	5.446
39-69	6.137	5.813	5.518	5.249
40-70	5.871	5.571	5.298	5.047
41-71	5.605	5.329	5.076	4.844
42-72	5.341	5.087	4.854	4.640
43-73	5.081	4.848	4.634	4.436
44-74	4.826	4.613	4.417	4.235
45-75	4.580	4.386	4.206	4.040
46-76	4.348	4.171	4.006	3.853
47-77	4.115	3.954	3.805	3.666
48-78	3.875	3.731	3.596	3.469
49-79	3.619	3.490	3.369	3.256
50-80	3.362	3.247	3.140	3.039

TABLE XXIV. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
51-81	3.117	3.015	2.920	2.829
52-82	2.882	2.792	2.707	2.627
53-83	2.665	2.585	2.510	2.438
54-84	2.501	2.428	2.360	2.295
55-85	2.349	2.284	2.222	2.164
56-86	2.211	2.153	2.097	2.044
57-87	2.082	2.030	1.980	1.932
58-88	1.975	1.928	1.883	1.841
59-89	1.828	1.788	1.750	1.713
60-90	1.641	1.608	1.577	1.547
61-91	1.382	1.358	1.334	1.311
62-92	1.105	1.088	1.071	1.055
63-93	0.785	0.774	0.764	0.754
64-94	0.506	0.500	0.494	0.489
65-95	0.230	0.228	0.226	0.224
66-96	0.000	0.000	0.000	0.000

TABLE XXV.

Shewing the Value of an Annuity on the *joint Continuance* of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *thirty-five* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-36	10.104	9.047	8.173	7.442
2-37	11.600	10.392	9.390	8.551
3-38	12.087	10.838	9.800	8.928
4-39	12.362	11.097	10.043	9.157
5-40	12.405	11.150	10.102	9.219
6-41	12.446	11.203	10.163	9.283
7-42	12.412	11.190	10.165	9.296
8-43	12.325	11.130	10.124	9.270
9-44	12.174	11.012	10.031	9.197
10-45	11.976	10.851	9.900	9.088
11-46	11.756	10.697	9.774	8.962
12-47	11.525	10.481	9.592	8.827
13-48	11.288	10.284	9.425	8.686
14-49	11.045	10.080	9.252	8.538
15-50	10.799	9.872	9.076	8.386
16-51	10.554	9.665	8.899	8.234
17-52	10.313	9.461	8.724	8.083
18-53	10.076	9.260	8.552	7.934
19-54	9.845	9.063	8.383	7.788
20-55	9.617	8.869	8.216	7.643
21-56	9.394	8.679	8.053	7.502
22-57	9.174	8.491	7.891	7.362
23-58	8.951	8.299	7.725	7.218
24-59	8.725	8.104	7.556	7.070
25-60	8.495	7.906	7.383	6.919
26-61	8.263	7.704	7.207	6.764
27-62	8.028	7.499	7.027	6.605
28-63	7.785	7.286	6.839	6.439

TABLE XXV. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
29-64	7.539	7.069	6.648	6.268
30-65	7.286	6.844	6.447	6.089
31-66	7.028	6.615	6.243	5.905
32-67	6.768	6.382	6.033	5.717
33-68	6.504	6.146	5.820	5.524
34-69	6.239	5.906	5.603	5.326
35-70	5.971	5.663	5.382	5.125
36-71	5.703	5.419	5.159	4.920
37-72	5.435	5.174	4.934	4.714
38-73	5.169	4.930	4.710	4.507
39-74	4.908	4.690	4.488	4.301
40-75	4.656	4.457	4.272	4.101
41-76	4.420	4.238	4.069	3.912
42-77	4.184	4.019	3.865	3.722
43-78	3.942	3.794	3.655	3.525
44-79	3.685	3.552	3.428	3.312
45-80	3.426	3.308	3.197	3.093
46-81	3.176	3.072	2.973	2.881
47-82	2.936	2.843	2.756	2.673
48-83	2.714	2.632	2.554	2.481
49-84	2.544	2.470	2.400	2.334
50-85	2.388	2.322	2.258	2.198
51-86	2.248	2.188	2.131	2.077
52-87	2.117	2.063	2.012	1.963
53-88	2.008	1.960	1.914	1.870
54-89	1.858	1.817	1.778	1.740
55-90	1.666	1.633	1.601	1.570
56-91	1.402	1.377	1.353	1.330
57-92	1.120	1.102	1.085	1.069
58-93	0.794	0.784	0.773	0.763
59-94	0.511	0.505	0.499	0.494
60-95	0.233	0.230	0.228	0.226
61-96	0.000	0.000	0.000	0.000

TABLE XXVI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *forty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-41	9.523	8.585	7.800	7.135
2-42	10.907	9.839	8.942	8.182
3-43	11.343	10.242	9.315	8.528
4-44	11.578	10.468	9.531	8.733
5-45	11.597	10.500	9.571	8.778
6-46	11.610	10.528	9.609	8.823
7-47	11.550	10.491	9.589	8.815
8-48	11.435	10.404	9.524	8.767
9-49	11.260	10.263	9.409	8.673
10-50	11.044	10.085	9.260	8.548
11-51	10.816	9.894	9.100	8.411
12-52	10.582	9.698	8.934	8.270
13-53	10.344	9.497	8.763	8.123
14-54	10.100	9.290	8.586	7.970
15-55	9.851	9.077	8.403	7.812
16-56	9.595	8.858	8.214	7.648
17-57	9.340	8.639	8.024	7.481
18-58	9.089	8.422	7.835	7.316
19-59	8.841	8.207	7.648	7.153
20-60	8.597	7.995	7.463	6.990
21-61	8.357	7.787	7.281	6.830
22-62	8.119	7.580	7.100	6.670
23-63	7.874	7.365	6.910	6.503
24-64	7.626	7.147	6.717	6.331
25-65	7.370	6.920	6.515	6.151
26-66	7.110	6.689	6.309	5.966
27-67	6.847	6.454	6.098	5.776

TABLE XXVI. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
28-68	6.581	6.215	5.883	5.581
29-69	6.313	5.973	5.664	5.383
30-70	6.043	5.729	5.442	5.180
31-71	5.772	5.483	5.218	4.974
32-72	5.502	5.236	4.992	4.767
33-73	5.235	4.991	4.766	4.559
34-74	4.973	4.749	4.543	4.353
35-75	4.720	4.516	4.327	4.152
36-76	4.481	4.295	4.123	3.962
37-77	4.242	4.073	3.916	3.770
38-78	3.996	3.844	3.702	3.570
39-79	3.734	3.598	3.471	3.352
40-80	3.469	3.349	3.236	3.130
41-81	3.216	3.109	3.009	2.914
42-82	2.973	2.878	2.789	2.705
43-83	2.750	2.666	2.587	2.511
44-84	2.581	2.505	2.433	2.365
45-85	2.424	2.356	2.291	2.230
46-86	2.282	2.221	2.162	2.107
47-87	2.148	2.093	2.041	1.991
48-88	2.036	1.987	1.941	1.895
49-89	1.882	1.840	1.800	1.761
50-90	1.685	1.651	1.619	1.596
51-91	1.417	1.391	1.367	1.343
52-92	1.130	1.113	1.095	1.079
53-93	0.801	0.790	0.780	0.770
54-94	0.515	0.509	0.503	0.498
55-95	0.234	0.232	0.230	0.228
56-96	0.000	0.000	0.000	0.000

TABLE XXVII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *forty-five* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-46	8.888	8.071	7.379	6.787
2-47	10.147	9.221	8.435	7.760
3-48	10.515	9.566	8.759	8.063
4-49	10.697	9.744	8.932	8.230
5-50	10.679	9.742	8.941	8.248
6-51	10.664	9.745	8.956	8.271
7-52	10.586	9.690	8.919	8.248
8-53	10.458	9.591	8.841	8.188
9-54	10.276	9.442	8.718	8.085
10-55	10.055	9.256	8.560	7.951
11-56	9.814	9.052	8.386	7.801
12-57	9.566	8.839	8.203	7.643
13-58	9.312	8.622	8.015	7.479
14-59	9.053	8.399	7.821	7.310
15-60	8.790	8.170	7.622	7.135
16-61	8.521	7.935	7.416	6.953
17-62	8.252	7.700	7.208	6.770
18-63	7.981	7.462	6.998	6.583
19-64	7.714	7.226	6.789	6.396
20-65	7.444	6.986	6.576	6.205
21-66	8.177	6.749	6.364	6.015
22-67	6.911	6.512	6.151	5.824
23-68	6.643	6.271	5.934	5.628
24-69	6.372	6.027	5.713	5.427
25-70	6.099	5.780	5.489	5.223

TABLE XXVII. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
26-71	5.826	5.532	5.263	5.016
27-72	5.554	5.283	5.035	4.807
28-73	5.284	5.036	4.808	4.597
29-74	5.019	4.792	4.583	4.390
30-75	4.764	4.557	4.365	4.188
31-76	4.523	4.335	4.160	3.997
32-77	4.282	4.111	3.952	3.804
33-78	4.035	3.881	3.737	3.602
34-79	3.771	3.633	3.505	3.384
35-80	3.506	3.383	3.268	3.160
36-81	3.251	3.142	3.040	2.944
37-82	3.005	2.909	2.818	2.733
38-83	2.779	2.694	2.613	2.537
39-84	2.607	2.530	2.457	2.388
40-85	2.448	2.379	2.313	2.251
41-86	2.304	2.241	2.182	2.126
42-87	2.168	2.113	2.060	2.009
43-88	2.055	2.006	1.959	1.914
44-89	1.901	1.859	1.818	1.779
45-90	1.702	1.668	1.635	1.604
46-91	1.431	1.405	1.380	1.356
47-92	1.140	1.122	1.105	1.089
48-93	0.808	0.797	0.786	0.776
49-94	0.519	0.512	0.507	0.501
50-95	0.235	0.233	0.231	0.229
51-96	0.000	0.000	0.000	0.000

TABLE XXVIII.

Shewing the Value of an Annuity on the *joint* Continuance of Two Lives, according to the *Northampton* Table of Observations, p. 36.

Difference of Age *fifty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-51	8.171	7.479	6.885	6.370
2-52	9.300	8.520	7.848	7.264
3-53	9.611	8.815	8.128	7.529
4-54	9.751	8.957	8.269	7.668
5-55	9.707	8.931	8.256	7.665
6-56	9.659	8.902	8.241	7.662
7-57	9.549	8.817	8.176	7.612
8-58	9.395	8.691	8.073	7.527
9-59	9.191	8.519	7.927	7.403
10-60	8.952	8.314	7.750	7.250
11-61	8.696	8.092	7.557	7.081
12-62	8.433	7.863	7.357	6.905
13-63	8.161	7.625	7.147	6.719
14-64	7.884	7.381	6.931	6.527
15-65	7.597	7.127	6.705	6.325
16-66	7.304	6.866	6.472	6.115
17-67	7.012	6.604	6.236	5.903
18-68	6.721	6.343	6.001	5.689
19-69	6.434	6.084	5.766	5.476
20-70	6.149	5.826	5.532	5.262
21-71	5.870	5.572	5.300	5.050
22-72	5.595	5.321	5.070	4.840
23-73	5.323	5.072	4.841	4.628

TABLE XXVIII. continued.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
24-74	5.056	4.827	4.615	4.419
25-75	4.799	4.589	4.396	4.216
26-76	4.556	4.365	4.188	4.024
27-77	4.313	4.140	3.979	3.829
28-78	4.064	3.908	3.762	3.626
29-79	3.798	3.659	3.528	3.406
30-80	3.530	3.406	3.290	3.181
31-81	3.274	3.164	3.060	2.963
32-82	3.027	2.929	2.838	2.751
33-83	2.800	2.713	2.632	2.555
34-84	2.627	2.549	2.476	2.406
35-85	2.468	2.398	2.331	2.268
36-86	2.323	2.260	2.200	2.143
37-87	2.187	2.130	2.077	2.026
38-88	2.072	2.022	1.974	1.929
39-89	1.915	1.872	1.832	1.792
40-90	1.713	1.679	1.646	1.614
41-91	1.439	1.413	1.388	1.364
42-92	1.146	1.128	1.111	1.094
43-93	0.811	0.800	0.790	0.779
44-94	0.521	0.515	0.509	0.503
45-95	0.236	0.234	0.232	0.230
46-96	0.000	0.000	0.000	0.000.

TABLE XXIX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *fifty-five* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-56	7.412	6.843	6.346	5.911
2-57	8.392	7.756	7.199	6.709
3-58	8.630	7.986	7.421	6.922
4-59	8.712	8.075	7.514	7.017
5-60	8.629	8.011	7.466	6.982
6-61	8.542	7.944	7.415	6.945
7-62	8.400	7.828	7.319	6.865
8-63	8.214	7.669	7.184	6.750
9-64	7.984	7.470	7.010	6.598
10-65	7.718	7.236	6.803	6.414
11-66	7.437	6.987	6.581	6.215
12-67	7.149	6.730	6.351	6.009
13-68	6.857	6.468	6.116	5.796
14-69	6.562	6.202	5.876	5.578
15-70	6.264	5.933	5.631	5.355
16-71	5.964	5.660	5.382	5.127
17-72	5.667	5.389	5.133	4.899
18-73	5.378	5.123	4.889	4.673
19-74	5.098	4.866	4.651	4.453
20-75	4.831	4.619	4.424	4.242
21-76	4.583	4.391	4.212	4.046
22-77	4.339	4.164	4.001	3.850
23-78	4.087	3.930	3.783	3.646

TABLE XXIX. continued.

Ages,	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at. 6 per Ct.
24-79	3.820	3.679	3.548	3.424
25-80	3.550	3.425	3.308	3.198
26-81	3.292	3.181	3.077	2.979
27-82	3.043	2.945	2.853	2.765
28-83	2.815	2.728	2.646	2.568
29-84	2.641	2.563	2.489	2.418
30-85	2.481	2.411	2.344	2.280
31-86	2.336	2.272	2.212	2.154
32-87	2.198	2.142	2.088	2.036
33-88	2.083	2.033	1.985	1.939
34-89	1.925	1.882	1.841	1.802
35-90	1.723	1.688	1.654	1.622
36-91	1.446	1.420	1.395	1.371
37-92	1.152	1.134	1.116	1.099
38-93	0.815	0.804	0.793	0.783
39-94	0.523	0.517	0.511	0.505
40-95	0.237	0.235	0.233	0.231
41-96	0.000	0.000	0.000	0.000

TABLE XXX.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *ſixty* Years.

Ages.	Value at 3 per Ct.	Value at 4 per Ct.	Value at 5 per Ct.	Value at 6 per Ct.
1-61	6.571	6.123	5.725	5.372
2-62	7.391	6.894	6.452	6.059
3-63	7.545	7.048	6.605	6.209
4-64	7.562	7.076	6.641	6.251
5-65	7.429	6.963	6.546	6.171
6-66	7.290	6.846	6.447	6.087
7-67	7.104	6.684	6.306	5.963
8-68	6.884	6.490	6.134	5.811
9-69	6.628	6.262	5.929	5.626
10-70	6.347	6.008	5.700	5.418
11-71	6.056	5.744	5.460	5.199
12-72	5.763	5.478	5.216	4.976
13-73	5.473	5.212	4.972	4.751
14-74	5.188	4.950	4.731	4.528
15-75	4.911	4.695	4.495	4.310
16-76	4.649	4.452	4.270	4.101
17-77	4.388	4.210	4.045	3.892
18-78	4.123	3.964	3.815	3.677
19-79	3.846	3.704	3.571	3.447
20-80	3.569	3.443	3.325	3.214
21-81	3.307	3.195	3.091	2.992
22-82	3.057	2.958	2.865	2.775

TABLE XXX. continued.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
23-83	2.828	2.740	2.657	2.579
24-84	2.653	2.574	2.499	2.429
25-85	2.492	2.421	2.354	2.290
26-86	2.346	2.282	2.221	2.163
27-87	2.208	2.151	2.096	2.044
28-88	2.091	2.041	1.992	1.946
29-89	1.933	1.889	1.848	1.808
30-90	1.729	1.694	1.660	1.628
31-91	1.451	1.425	1.400	1.376
32-92	1.155	1.137	1.119	1.102
33-93	0.817	0.806	0.795	0.785
34-94	0.524	0.518	0.512	0.506
35-95	0.238	0.235	0.233	0.231
36-96	0.000	0.000	0.000	0.000

TABLE XXXI.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *ſixty-five* Years.

Ages.	Value at 3 per Cent	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-66	5.633	5.295	4.996	4.728
2-67	6.266	5.896	5.569	5.276
3-68	6.330	5.965	5.641	5.352
4-69	6.277	5.924	5.611	5.332
5-70	6.102	5.768	5.472	5.209
6-71	5.925	5.610	5.331	5.084
7-72	5.714	5.418	5.157	4.929
8-73	5.480	5.204	4.963	4.752
9-74	5.225	4.969	4.747	4.556
10-75	4.962	4.725	4.522	4.350
11-76	4.707	4.487	4.301	4.148
12-77	4.449	4.368	4.195	3.943
13-78	4.185	4.022	3.871	3.729
14-79	3.904	3.759	3.624	3.497
15-80	3.621	3.492	3.372	3.259
16-81	3.348	3.235	3.128	3.028
17-82	3.087	2.987	2.893	2.804
18-83	2.849	2.760	2.677	2.598
19-84	2.668	2.589	2.513	2.442
20-85	2.503	2.431	2.364	2.299
21-86	2.354	2.290	2.229	2.171
22-87	2.216	2.158	2.104	2.051
23-88	2.099	2.048	1.999	1.953
24-89	1.939	1.895	1.854	1.814
25-90	1.734	1.699	1.665	1.633
26-91	1.455	1.429	1.404	1.379
27-92	1.158	1.140	1.122	1.105
28-93	0.819	0.808	0.797	0.786
29-94	0.525	0.519	0.513	0.507
30-95	0.238	0.236	0.234	0.231
31-96	0.000	0.000	0.000	0.000

TABLE XXXII.

Shewing the Value of an Annuity on the joint Continuance of Two Lives, according to the *Northampton* Table of Obſervations, p. 36.

Difference of Age *ſeventy* Years.

Ages.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.	Value at 6 per Cent.
1-71	4.611	4.380	4.169	3.976
2-72	5.061	4.814	4.588	4.380
3-73	5.051	4.811	4.591	4.389
4-74	4.953	4.726	4.516	4.323
5-75	4.768	4.557	4.362	4.181
6-76	4.599	4.403	4.221	4.053
7-77	4.402	4.222	4.055	3.899
8-78	4.180	4.016	3.864	3.722
9-79	3.921	3.775	3.638	3.510
10-80	3.647	3.517	3.395	3.281
11-81	3.380	3.264	3.156	3.054
12-82	3.122	3.020	2.924	2.833
13-83	2.884	2.794	2.709	2.628
14-84	2.703	2.622	2.545	2.472
15-85	2.535	2.462	2.393	2.327
16-86	2.380	2.315	2.253	2.194
17-87	2.235	2.177	2.121	2.069
18-88	2.112	2.061	2.012	1.965
19-89	1.948	1.904	1.862	1.822
20-90	1.739	1.704	1.670	1.638
21-91	1.459	1.432	1.407	1.382
22-92	1.160	1.142	1.124	1.107
23-93	0.820	0.809	0.798	0.788
24-94	0.526	0.520	0.514	0.508
25-95	0.238	0.236	0.234	0.232
26-96	0.000	0.000	0.000	0.000

Directions for uſing the preceding Tables of the Values of Two joint Lives.

IF the two lives have the ſame common age, or their difference of age is five years, or any multiple of five years, the value of their joint continuance is expreſſed in the Tables, and may be found by inſpection.

If their difference of age is any number of years between 1 and 5, 5 and 10, 10 and 15, &c. the required value may be eaſily found by the following rule.

" Find, in the preceding Tables, the va-
" lue of two joint lives, whoſe difference of
" age is that multiple of 5 which is greater
" *than*, but at the ſame time neareſt *to* the
" difference of age between the propoſed
" lives; and the oldeſt of which is of the
" ſame age with the oldeſt of the propoſed
" lives.——Find alſo, in the preceding Ta-
" bles, the value of two joint lives whoſe
" difference of age is five years leſs than
" the multiple of 5 juſt mentioned; and
" the oldeſt of which is, in like manner,
" of the ſame age with the oldeſt of the
" propoſed lives; and the 1ſt, 2d, 3d, or
" 4th arithmetical mean between the leaſt
" and the greateſt of theſe two values will
" be the value ſought, according as one of
" the propoſed lives is one year, 2 years,
" 3 years, or 4 years younger than the
" other."

EXAMI[illegible]

EXAMPLE.

Let the value be required of two joint lives aged 15 and 18, reckoning intereſt at 3 *per cent.*

That multiple of 5 which is greater than the difference between theſe ages, but comes neareſt to it, is 5.——The value of two joint lives, whoſe difference of age is 5 years, and the oldeſt of which is of the ſame age with the oldeſt of the two propoſed lives; that is, the value of two joint lives aged 18 and 13, is by Table 19th, 15.086. The value of two joint lives whoſe difference of age is 5 years leſs, and one of which is alſo 18; that is, the value of two joint lives aged 18 and 18, is, by Table 18th, 14.516.—— Theſe, then, being the values of two joint lives aged 18 and 13, and of two joint lives aged 18 and 18, it is obvious that the value of two joint lives, aged 18 and 15, muſt be the *third* of four arithmetical means between 14.516 and 15.086.

N. B. The 1ſt, 2d, 3d, or 4th arithmetical mean between the leaſt and greateſt of any two values, is the leaſt increaſed by 1, 2, 3, or 4 fifths of the difference between them.

In the preſent inſtance, the difference between the two values is .570; its fifth part

is .114; and 14.516 increaſed by thrice this fifth part, makes 14.858, the required value of two joint lives aged 18 and 15.

EXAMPLE II.

Let the value be required of two joint lives aged 31 and 45, reckoning intereſt at 3 *per cent.*

That multiple of 5 which is the next greater number to 14 (the difference of age between 45 and 31), is 15. The value of two joint lives, whoſe difference of age is this number, and the oldeſt of which is of the ſame age with the oldeſt of the propoſed lives; that is, the value of two joint lives aged 45 and 30, is, by Table 21ſt, 10.923.

The value of two joint lives, whoſe difference of age is 5 years leſs than 15, and the oldeſt of which is, in like manner, of the ſame age with the oldeſt of the propoſed lives; that is, the value of two joint lives aged 45 and 35, is, by Table 20th, 10.622.

Theſe then being the values of two joint lives aged 45 and 30, and of two joint lives aged 45 and 35, it follows that the value of two joint lives aged 45 and 31, muſt be the 4th of 4 arithmetical means between the leaſt and the greateſt of theſe two values. That is; it is 10.622 (the leaſt) increaſed by four-fifths of .301 (the difference),

rence), or by .240, which makes 10.862 the required value of two joint lives aged 45 and 31.

In the ſame manner may the values not ſpecified in the Tables be found univerſally for any of the four rates of intereſt. And that they are ſufficiently correct, will appear from the following compariſon.

Values of two joint Lives by the Rule juſt explained, reckoning intereſt at 3 per cent. *compared with the correct Values.*

Ages.	Value by Rule.	Correct Value.
18 and 14	14.972	14.978
18 and 15	14.858	14.864
18 and 16	14.744	14.744
18 and 17	14.630	14.626

Ages.	Value by Rule.	Correct Value.
45 and 31	10.862	10.869
45 and 32	10.802	10.811
45 and 33	10.742	10.751
45 and 34	10.682	10.688

Ages.	Value by Rule.	Correct Value.
66 and 27	7.092	7.095
66 and 28	7.076	7.080
66 and 29	7.060	7.063
66 and 30	7.044	7.046

In the higher rates of intereſt the agreement is greater.

I have been enabled to make this compariſon by the Tables in the office for

Equitable Aſſurances, where, in order to lay the foundation of accuracy in conducting the buſineſs of the office, it has been thought neceſſary to compute minutely to four places of decimals the values by the *Northampton* Obſervations, *at 3 per cent.* of two joint lives *for every poſſible difference of age.*

The values of any two joint lives being given, the values of the longeſt of any two ſingle lives are obtained by the following rule.

"From the ſum of the values of the "ſingle lives ſubtract the value of their "joint continuance. The remainder will "be the value of the longeſt of the two "lives."

In the former editions of this work, I gave a table of theſe values; but it is ſo eaſy to compute them by this rule, that it is by no means worth while to ſwell this volume with any ſuch table.

EXAMPLE. Let it be required to find the value of the longeſt of two lives aged 10 and 15, intereſt being at 4 *per cent.*

The value of a life aged 10, is, by Table 17th, 17.523. The value of a life aged 15, is 16.791. The ſum of theſe two values is 34.314. The value of the joint continuance of theſe two lives is (by Table 19th) 13.992, which ſubtracted from 34.314, leaves 20.322, the value ſought.

In

In the Firſt Volume, p. 173, I ſignified my intention to inſert, in this collection, the tables of the office juſt mentioned for *Equitable Aſſurances*. Some of theſe tables have been already inſerted; namely, Table 6th, and the columns ſhewing the values at 3 *per cent.* in all the Tables from the 17th to the laſt Table.——The values of ſingle and joint lives have been calculated in the office for this rate of intereſt, becauſe it is the intereſt by which it regulates all its demands. The values, in the preceding Tables, for the *other* rates of intereſt, have been calculated with much labour for this work, in order to ſet aſide all occaſion for having recourſe to Mr. *De Moivre*'s hypotheſis. See Vol. I. p. 308, &c.——The remaining Tables of this office are thoſe that follow.

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TABLE XXXIII.

Shewing the Value of an Annuity on a ſingle Life, for 1, 2, 3, 5, and 7 Years, reckoning the Probabilities of living at every Age as they are given in Table VI. and Intereſt at 3 *per cent*.

Ages.	One Year.	Two Years.	Three Years	Five Years.	Seven Years.
10	.962	1.887	2.778	4.459	6.015
15	.962	1.886	2.774	4.443	5.971
20	.957	1.873	2.748	4.385	5.880
25	.956	1.868	2.740	4.367	5.849
30	.954	1.864	2.733	4.349	5.816
35	.953	1.860	2.724	4.328	5.777
40	.951	1.853	2.710	4.294	5.716
45	.948	1.845	2.694	4.256	5.646
50	.943	1.832	2.669	4.195	5.538
55	.938	1.818	2.641	4.128	5.420
60	.932	1.798	2.604	4.041	5.266
65	.923	1.773	2.554	3.919	5.045

TABLE XXXIV.

Shewing the Value of an Aſſurance of 100*l.* on a ſingle Life, for 1, 5, or 7 Years, or the whole Duration of Life; reckoning the Probabilities of living as they are in the NORTHAMPTON Table of Obſervations (or Table VI.), and intereſt at 3 *per cent.*

N. B. With reſpect to the values in this Table, and alſo in thoſe that follow to Table XXXVI. it muſt be remembered, that the values in *annual* payments ſuppoſe, that the firſt payment is made at the time of purchaſing; and alſo that a purchaſer is allowed his option either to pay the value of the Aſſurance in the *annual* payments, or in the *ſingle* payments ſpecified in the Table; and that whichever of theſe he chuſes, he is excuſed the other.

Ages.	1 Year. Premium.	5 Years. Single Premium	5 Years. Annual Premium	7 Years. Single Premium	7 Years. Annual Premium	Whole durat. of life. Single Premium	Whole durat. of life. Annual Premium
8	1.336	4.632	1.004	6.052	.973	36.256	1.657
10	.890	4.069	.878	5.566	.890	36.903	1.704
15	.895	4.893	1.058	7.129	1.146	39.832	1.928
20	1.362	6.636	1.447	9.048	1.471	42.801	2.179
25	1.530	7.216	1.578	9.817	1.603	45.201	2.403
30	1.661	7.833	1.718	10.656	1.747	47.801	2.667
35	1.816	8.566	1.884	11.714	1.930	50.666	2.991
40	2.030	9.748	2.155	13.290	2.206	53.841	3.397
45	2.332	11.025	2.451	15.166	2.540	57.208	3.894
50	2.753	13.111	2.943	17.848	3.031	60.866	4.530
55	3.252	15.341	3.478	20.870	3.600	64.612	5.318
60	3.906	18.254	4.196	24.733	4.355	68.610	6.366
65	4.759	22.450	5.260	30.541	5.542	72.899	7.835

From theſe values of Aſſurances of 100*l.* the values of Aſſurances of any other ſum may be eaſily collected.

This Office makes aſſurances for any number of months, or years, of any ſums not exceeding 2000*l.* * on one life; and its tables contain the values for all the intermediate years omitted in this and the two following Tables.

It may be neceſſary here to add, for the information of thoſe who may not be converſant with decimal arithmetic, that in every value the number on the left hand of the point expreſſes ſo many pounds, and that allowing 2*s.* for every unit in the *firſt* figure on the right hand of the point, 2½*d.* for every unit in the *ſecond* figure, and one farthing for every unit in the *third*, will give very nearly the ſhillings and pence to be added to the pounds in each value.—Thus; 1.336 in the preceding Table is 1*l.* 6*s.* 9*d.*—4.632 is 4*l.* 12*s.* 8*d.*—1.004 is 1*l.* 0*s.* 1*d.*—6.052 is 6*l.* 1*s.* 1*d.*; and .973 is 19*s.* 6¼*d.* See the note in Vol. I. p. 14.

There is one remark more neceſſary to be here attended to; but which I cannot make without ſome reluctance. In giving an account of this Society, in Vol. I. p. 176, I have recommended, for reaſons there mentioned, that in tranſacting the buſineſs of the Society, an addition of 3 or 4 *per cent.* ſhould be made to all the calculated values.

* The Society has lately extended its aſſurances to the ſum of 5000*l.* *Ed.*

But

But the Society, having lately thought proper to increaſe its expences of management, and fearing the effect of too great and ſudden a reduction, has carried this addition as high as 15 *per cent.* * This, when added to the other advantages which the Society enjoys (and particularly that derived from eſtimating the improvement of the money it receives at 3 *per cent.*) would, without doubt, be a very *exorbitant*, were it intended to be a *permanent* charge. But this is not the caſe. Even this charge leaves a reduction in the payments of above a *quarter*; and ſhould the Society find that, notwithſtanding this reduction, it continues ſtill to proſper, as there is every reaſon to think it will, farther reductions may be expected; And, perhaps, in time it may find itſelf capable of reducing the payments for Aſſurances even BELOW thoſe in the preceding Table. Nothing renders this improbable, but the difficulty of keeping out bad lives, and preventing fraudulent aſſurances; for a compariſon of the *Northampton* Table of decrements with the Tables which will be given preſently for CHESTER, the pariſh of HOLY-CROSS, and for the kingdom of SWEDEN, will ſhew, that were the Society to take the *premiums* in the preceding Table without any addition, it would ſtill be governing itſelf by probabilities of living much below thoſe among mankind in general.

* This addition to the premiums of the Society has been diſcontinued ſince the 1ſt of January, 1786. *Ed.*

TABLE

TABLE XXXV.

Shewing the Value of an Assurance of 100*l.* on two joint Lives, according to the NORTHAMPTON Table of Observations, reckoning interest at 3 *per cent.*

Ages.		Single Premium.	Annual Premium.
10	10	49.498	2.855
	15	51.177	3.053
	20	52.958	3.279
	25	54.319	3.463
	30	55.873	3.688
	35	57.693	3.972
	40	59.832	4.339
	45	62.206	4.794
	50	64.919	5.390
	55	67.801	6.133
	60	71.012	7.135
	65	74.606	8.557
15	15	52.731	3.249
	20	54.388	3.473
	25	55.641	3.653
	30	57.083	3.874
	35	58.783	4.154
	40	60.799	4.517
	45	63.047	4.969
	50	65.634	5.563
	55	68.395	6.303
	60	71.485	7.302
	65	74.960	8.719
20	20	55.923	3.695
	25	57.065	3.871
	30	58.390	4.087
	35	59.968	4.363
	40	61.856	4.723
	45	63.979	5.173
	50	66.438	5.766
	55	69.077	6.506
	60	72.049	7.508
	65	75.406	8.930
25	25	58.106	4.040
	30	59.322	4.248
	35	60.736	4.515
	40	62.559	4.867
	45	64.571	5.308
	50	66.923	5.893
	55	69.461	6.625
	60	72.343	7.619
	65	75.621	9.035
30	30	60.418	4.446
	35	61.754	4.703
	40	63.392	5.044
	45	65.271	5.474

TABLE XXXV. continued.

Ages.		Single Premium.	Annual Premium.
30	50	67.495	6.048
	55	69.915	6.769
	60	72.685	7.751
	65	75.866	9.156
35	35	62.944	4.947
	40	64.428	5.275
	45	66.149	5.692
	50	68.217	6.252
	55	70.492	6.958
	60	73.125	7.925
	65	76.181	9.316
40	40	65.736	5.588
	45	67.274	5.988
	50	69.154	6.530
	55	71.250	7.218
	60	73.713	8.168
	65	76.612	9.541
45	45	68.611	6.367
	50	70.278	6.887
	55	72.164	7.551
	60	74.424	8.476
	65	77.134	9.825
50	50	71.705	7.381
	55	73.344	8.014
	60	75.357	8.907
	65	77.831	10.226
55	55	74.713	8.606
	60	76.443	9.451
	65	78.637	10.721
60	60	77.846	10.235
	65	79.699	11.434
65	65	81.152	12.541

TABLE XXXVI.

Shewing the Value of 100*l.* depending on the Contingency of one Life surviving another, according to the NORTHAMPTON Table of Observations, reckoning Interest at 3 *per cent.*

Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Possessor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
	10	24.749	1.427	5.723		35	24.176	1.759	7.570
	15	24.198	1.444	6.213		40	22.692	1.733	8.246
	20	23.498	1.455	6.738		45	21.058	1.703	9.059
	25	22.531	1.437	7.197	20	50	19.294	1.674	10.085
	30	21.468	1.417	7.746		55	17.410	1.640	11.356
	35	20.317	1.399	8.422		60	15.381	1.603	13.029
10	40	19.070	1.383	9.272		65	13.206	1.564	15.341
	45	17.696	1.364	10.314		70	10.892	1.523	18.634
	50	16.214	1.346	11.652					
	55	14.631	1.324	13.362		10	31.789	2.027	5.316
	60	12.925	1.299	15.671		15	31.093	2.042	5.729
	65	11.098	1.273	18.935		20	30.254	2.052	6.178
	70	9.153	1.246	23.651		25	29.053	2.020	6.557
						30	27.683	1.982	6.998
	10	26.979	1.609	5.505		35	26.198	1.946	7.540
	15	26.365	1.625	5.954	25	40	24.590	1.913	8.215
	20	25.602	1.635	6.435		45	22.819	1.876	9.027
	25	24.549	1.612	6.849		50	20.907	1.841	10.055
	30	23.391	1.588	7.340		55	18.866	1.799	11.329
	35	22.136	1.564	7.944		60	16.667	1.755	13.004
15	40	20.778	1.544	8.698		65	14.310	1.710	15.313
	45	19.281	1.520	9.617		70	11.803	1.662	18.595
	50	17.666	1.497	10.791					
	55	15.941	1.469	12.271		10	34.404	2.271	5.282
	60	14.083	1.439	14.264		15	33.694	2.287	5.689
	65	12.092	1.407	17.086		20	32.843	2.299	6.136
	70	9.973	1.373	21.219		25	31.640	2.266	6.526
						30	30.209	2.223	6.974
	10	29.461	1.824	5.345	30	35	28.589	2.177	7.510
	15	28.786	1.838	5.760		40	26.834	2.135	8.183
20	20	27.961	1.848	6.207		45	24.901	2.088	8.995
	25	26.811	1.819	6.582		50	22.815	2.044	10.025
	30	25.546	1.788	7.027		55	20.588	1.993	11.307

TABLE XXXVI. continued.

Poſſeſſor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Poſſeſſor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
	60	18.188	1.939	12.997		40	36.775	3.273	7.974
30	65	15.616	1.885	15.330		45	34.306	3.183	8.762
	70	12.880	1.829	18.642		50	31.432	3.080	9.727
					45	55	28.364	2.968	10.940
	10	37.375	2.573	5.236		60	25.057	2.854	12.552
	15	36.647	2.590	5.632		65	21.514	2.740	14.797
	20	35.794	2.604	6.073		70	17.744	2.629	18.012
	25	34.588	2.569	6.464					
	30	33.166	2.526	6.924		10	48.705	4.044	5.064
	35	31.472	2.474	7.466		15	47.968	4.066	5.415
35	40	29.540	2.419	8.128		20	47.144	4.091	5.809
	45	27.413	2.359	8.930		25	46.017	4.052	6.170
	50	25.116	2.302	9.952		30	44.680	4.004	6.608
	55	22.664	2.237	11.227		35	43.101	3.950	7.153
	60	20.022	2.170	12.917	50	40	41.208	3.891	7.838
	65	17.191	2.102	15.255		45	38.846	3.807	8.657
	70	14.179	2.034	18.590		50	35.853	3.691	9.634
						55	32.353	3.535	10.791
	10	40.763	2.956	5.178		60	28.581	3.378	12.338
	15	40.023	2.974	5.560		65	24.540	3.224	4.491
	20	39.164	2.991	5.986		70	20.239	3.075	17.570
	25	37.969	2.954	6.371					
	30	36.560	2.909	6.830		10	53.170	4.810	5.012
	35	34.888	2.857	7.384		15	52.454	4.834	5.349
40	40	32.868	2.794	8.048		20	51.668	4.867	5.727
	45	30.501	2.715	8.825		25	50.596	4.826	6.074
	50	27.946	2.639	9.821		30	49.329	4.776	6.497
	55	25.218	2.555	11.064		35	47.829	4.721	7.027
	60	22.278	2.468	12.714	55	40	46.034	4.664	7.702
	65	19.128	2.382	15.005		45	43.800	4.583	8.530
	70	15.776	2.296	18.274		50	40 993	4.479	9.569
						55	37.357	4.303	10.771
	10	44.511	3.430	5.124		60	33.002	4.080	12.272
	15	43.766	3.450	5.491		65	28.336	3.863	14.383
	20	42.921	3.471	5.903		70	23.370	3.656	17.409
45	25	41.753	3.433	6.278					
	30	40.369	3.386	6.730	60	10	58.087	5.836	4.960
	35	38.735	3.333	7.287		15	57.403	5.863	5.282

TABLE XXXVI. continued.

Poſſeſſor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.	Poſſeſſor.	Expectant.	Single Premium.	Annual Premium.	Equivalent Annuity.
	20	56.669	5.905	5.643		20	62.203	7.367	5.556
	25	55.675	5.863	5.975		25	61.311	7.325	5.871
	30	54.499	5.811	6.379		30	60.251	7.271	6.253
	35	53.103	5.755	6.887		35	58.990	7.213	6.734
	40	51.437	5.699	7.539		40	57.484	7.159	7.353
60	45	49.367	5.622	8.352	65	45	55.620	7.085	8.130
	50	46.777	5.529	9.402		50	53.293	7.002	9.149
	55	43.439	5.371	10.695		55	50.302	6.858	10.446
	60	38.923	5.117	12.274		60	46.279	6.640	12.156
	65	33.419	4.795	14.315		65	40.576	6.270	14.321
	70	27.563	4.490	17.283		70	33.466	5.787	17.150
65	10	63.510	7.285	4.906					
	15	62.870	7.313	5.213					

EXPLANATION.

THE annual premium in this Table is ſuppoſed to be payable during the joint continuance of the lives of the *poſſeſſor* and *expectant*; and the firſt payment is ſuppoſed to be made at the time of purchaſing the Aſſurance.

The *equivalent annuity* ſignifies that annuity to which either the *ſingle* premium ſpecified in the Table, or the *annual* premium, will entitle an *expectant* during his ſurvivorſhip, ſhould ſuch an annuity be preferred to a groſs ſum payable on ſurvivorſhip.—Thus; the payment of either *l.* 34.588 (34*l.* 11*s.* 10*d.*) in hand, or of *l.* 2.569 (2*l.* 11*s.* 5*d.*) annually, during the joint lives of a wife aged 25 and a huſband aged 35, the firſt payment to be made immediately, will, according to this Table, entitle the wife, ſhould ſhe ſurvive the huſband, either to 100*l.* payable to her when ſhe becomes a widow, or to an annuity payable during her life, after becoming a widow, of *l.* 6.464 (6*l.* 9*s.* 4*d.*) —If ſhe is 35 (or of the ſame age with her huſband) a ſingle payment of *l.* 31.472, or an annual payment of *l.* 2.474 will, by the Table, entitle her either to 100*l.* payable on her ſurvivorſhip, or to an annuity for her life of *l.* 7.466 after ſurvivorſhip.

Any payments greater or leſs will entitle to groſs ſums or annuities proportionably greater or leſs.

It

It is neceſſary to repeat here the obſervation made at the end of Table 34th, p. 104. that theſe are the exact premiums according to the *Northampton* Table of Obſervations, reckoning intereſt at 3 *per cent.* The *Equitable Society* adds to theſe premiums a charge of 15 *per cent.* *; and in this caſe, there is a reaſon which makes the addition leſs improper than in any other; I mean, the increaſe of value which the longer duration of the lives of *females* gives to all aſſurances depending on *their* ſurvivorſhip; and which the Society, for want of proper obſervations, have not yet had the means of calculating. Theſe means, however, will, I think, be furniſhed by ſome of the following Tables.

* See Note, p. 105.

TABLE

TABLE XXXVII.

Shewing the Values of *three equal* joint Lives, acording to the *Northampton* Table of Obſervations, reckoning Intereſt at 4 *per cent*.

Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.
1	5.309	25	9.796	49	6.482
2	8.251	26	9.685	50	6.317
3	9.632	27	9.572	51	6.161
4	10.661	28	9.457	52	6.011
5	11.170	29	9.340	53	5.859
6	11.707	30	9.221	54	5.705
7	12.058	31	9.099	55	5.550
8	12.266	32	8.975	56	5.393
9	12.298	33	8.848	57	5.235
10	12.200	34	8.718	58	5.076
11	12.043	35	8.585	59	4.916
12	11.865	36	8.448	60	4.755
13	11.678	37	8.309	61	4.593
14	11.481	38	8.165	62	4.432
15	11.274	39	8.017	63	4.263
16	11.056	40	7.865	64	4.093
17	10.845	41	7.714	65	3.914
18	10.656	42	7.567	66	3.733
19	10.490	43	7.423	67	3.550
20	10.342	44	7.276	68	3.366
21	10.222	45	7.126	69	3.181
22	10.118	46	6.972	70	2.995
23	10.012	47	6.813	71	2.810
24	9.905	48	6.650	72	2.627

TABLE XXXVII. continued.

Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.	Common Age.	Value at 4 per Ct.
73	2.448	81	1.245	89	0.614
74	2.277	82	1.092	90	0.563
75	2.119	83	0.949	91	0.452
76	1.985	84	0.860	92	0.337
77	1.855	85	0.782	93	0.185
78	1.720	86	0.716	94	0.085
79	1.563	87	0.662	95	0.015
80	1.400	88	0.646		

TABLE XXXVIII.

Shewing the Values of THREE joint Lives, whoſe Differences of Age are 10 and 20 Years, according to the *Northampton* Table of Obſervations, reckoning Intereſt at 4 *per cent.*

Differences of Age 10 and 20 Years.

Ages.			Value at 4 per Ct.	Ages.			Value at 4 per Ct.
1	11	21	8.627	23	33	43	8.586
2	12	22	9.914	24	34	44	8.451
3	13	23	10.344	25	35	45	8.313
4	14	24	10.598	26	36	46	8.171
5	15	25	10.655	27	37	47	8.027
6	16	26	10.708	28	38	48	7.878
7	17	27	10.700	29	39	49	7.725
8	18	28	10.654	30	40	50	7.571
9	19	29	10.562	31	41	51	7.420
10	20	30	10.438	32	42	52	7.272
11	21	31	10.305	33	43	53	7.123
12	22	32	10.170	34	44	54	6.971
13	23	33	10.031	35	45	55	6.816
14	24	34	9.887	36	46	56	6.658
15	25	35	9.738	37	47	57	6.497
16	26	36	9.584	38	48	58	6.332
17	27	37	9.429	39	49	59	6.164
18	28	38	9.278	40	50	60	5.994
19	29	39	9.131	41	51	61	5.827
20	30	40	8.986	42	52	62	5.662
21	31	41	8.850	43	53	63	5.494
22	32	42	8.718	44	54	64	5.322

TABLE XXXVIII. continued.

Ages.			Value at 4 per Ct.	Ages.			Value at 4 per Ct.
45	55	65	5.145	61	71	81	2.224
46	56	66	4.965	62	72	82	2.044
47	57	67	4.782	63	73	83	1.875
48	58	68	4.597	64	74	84	1.743
49	59	69	4.408	65	75	85	1.623
50	60	70	4.219	66	76	86	1.519
51	61	71	4.032	67	77	87	1.425
52	62	72	3.847	68	78	88	1.350
53	63	73	3.660	69	79	89	1.248
54	64	74	3.477	70	80	90	1.122
55	65	75	3.298	71	81	91	0.951
56	66	76	3.128	72	82	92	0.767
57	67	77	2.959	73	83	93	0.548
58	68	78	2.785	74	84	94	0.362
59	69	79	2.598	75	85	95	0.169
60	70	80	2.408				

REMARKS

REMARKS *on the two preceding Tables.*

THESE Tables contain the exact values of *three* joint lives having either the ſame common age, or whoſe differences of age are 10 and 20 years, according to the *Northampton* Table of Obſervations, or Table VI. intereſt being at 4 *per cent.*

In order to find the values nearly of three joint lives, having *other* differences of age, the following rules ſhould be obſerved.

If the age of the youngeſt of the three lives is between 10 and 50, and the difference of age between the youngeſt and oldeſt not more than *eight* years, take the *third* of the ſum of the three ages for a common age; and the value in the laſt Table but one, correſponding to that common age, will be the value ſought.

EXAMPLE.

Let the value be required of three joint lives whoſe ages are 15, 16, and 23.

The ſum of the ages is 54, the third part of which is 18, and the value (in Table 37th) correſponding to this age, is 10.656, the value required.

Within the limits I have mentioned this rule is tolerably correct. But theſe limits are ſo narrow as to render it of little uſe;

and, therefore, till ſome perſon will undertake to finiſh what has been begun in the two preceding Tables, it will be neceſſary to make uſe of the following *general* and very eaſy rule given by Mr. *Simpſon*, for finding the values of any *three* from the values given of any *two* joint lives.

"Let A be the youngeſt, and C the oldeſt of "the three propoſed lives. Take the value of "the *two* joint lives B and C, and find the "age of a *ſingle* life D of the ſame value. "Then find the value of the joint lives A "and D, which will be the anſwer."

EXAMPLE. Let the three given ages be 20, 30, and 40; and let the rate of intereſt be 4 *per cent.* The value of the two oldeſt joint lives B and C will (by Table XX.) be 10.490, anſwering in Table XVII. to a ſingle life D of 54 years, wanting $\frac{69}{221}$ (*a*) of a year. And the value of the joint lives A and D, which (by the rule in p. 75, and by Tables XXIV. and XXV.) (*b*) is 9.085, will be the value ſought.

(*a*) The value (in Table XVII.) which is *neareſt* to but *leſs* than 10.490, is 10.421; which is the value of a ſingle life aged 54. This value ſubtracted from 10.490 leaves 69, the *numerator* of this fraction. The *denominator* is the difference between 10.421 and 10.641, the laſt being the value of a life one year younger.

(*b*) The value deduced from the Tables (by the rule in p. 75) of two joint lives aged 20 and 54, is 9.038.——The value of two joint lives aged 20 and 55, is (by Table XXV.) 8.869; and of two joint lives aged 20 and 50, is (by Table XXIV.) 9.630. A fifth part of the difference between theſe values (that is, .153) multiplied by the fraction $\frac{69}{221}$, gives .047, which added to 9.038 makes 9.085, the value deduced from Tables XXIV. and XXV. of two joint lives, one aged 20 and the other wanting $\frac{69}{221}$ of a year of 54.——This ſhews the proper method of calculation in every caſe; but the difference will be little, if, for the ſake of more expedition, D is always taken for that age, whether greater or leſs, which anſwers moſt nearly to the value of the joint lives B and C, without regarding the fraction.

The

The following compariſon will ſhew how near this rule comes to correctneſs.

Values of *three joint* Lives.

Ages.	Correct Value at 4 *per cent.* by Table 38th.	Value by Rule.	Ages.	Correct Value at 4 *per cent.* by Table 37th.	Value by Rule.
10-20-30	10.438	10.563	10-10-10	12.200	12.244
15-25-35	9.738	9.840	15-15-15	11.274	11.376
20-30-40	8.986	9.085	20-20-20	10.342	10.504
25-35-45	8.313	8.395	25-25-25	9.796	9.937
30-40-50	7.571	7.651	30-30-30	9.221	9.351
35-45-55	6.816	6.886	35-35-35	8.585	8.701
40-50-60	5.994	6.046	40-40-40	7.865	7.984
45-55-65	5.145	5.169	45-45-45	7.126	7.249
50-60-70	4.219	4.238	50-50-50	6.317	6.432
55-65-75	3.298	3.292	55-55-55	5.550	5.636
			60-60-60	4.755	4.816
			65-65-65	3.914	3.942
			70-70-70	2.995	3.000
			75-75-75	2.119	2.110

My principal deſign in calculating the two preceding Tables has been, to enable myſelf to make this compariſon; and it may be inferred from it, that Mr. *Simpſon*'s rule gives the values of three joint lives *generally* within a *ninth* or *tenth*, and ſometimes within leſs than a 20th of a year's purchaſe.

It may be alſo obſerved, that when the oldeſt of the three ages does not exceed 75, and the youngeſt is not leſs than 10, the error falls always on the ſide of exceſs; and, conſequently, that if .05 (that is, a 20th of a year's purchaſe) is deducted from the value by the rule, the true value will be obtained, in *ſome* caſes, almoſt exactly; and, in *moſt* caſes, much more nearly.

The

The value of three joint lives being known, the value of the longeſt of any three lives is to be computed by the following rule.

"From the ſum of the values of all "the ſingle lives, ſubtract the ſum of the "values of all the joint lives combined "two and two. Then to the remainder "add the value of the three joint lives; "and this laſt ſum will be the value of the "longeſt of the three lives."——See Mr. *Simpſon*'s *Doctrine of Annuities*, &c. p. 23. —or Mr. *Dodſon*'s *Mathematical Repoſitory*, Vol. II. p. 244.

EXAMPLE. The ſum of the values of three ſingle lives whoſe ages are 10, 20, and 30, is, by Table XVII (reckoning intereſt at 4 *per cent.*) 48.338. The value of *two* joint lives whoſe ages are 10 and 20, is 13.355; of *two* joint lives whoſe ages are 10 and 30 is 12.586; of *two* joint lives whoſe ages are 20 and 30, is 11.873, by Tables XX. and XXII. And the ſum of theſe three values is 37.814. This ſum ſubtracted from 48.388, leaves 10.524, which remainder added to 10.485 (the value juſt found of the three joint lives) gives 20.009 the value of the *longeſt* of the three lives.

The value of three lives at the ſame ages by the Tables that follow ſhewing the values of ſingle and joint lives among mankind at large according to obſervations in *Sweden*, is 21.870

TABLE

TABLE XXXIX.

Shewing the Probability of the Duration of Human Life at all Ages among Males and Females, at *Warrington* in *Lancaſhire*; formed from a Regiſter of Mortality kept there by Mr. *Aikin*, for Nine years, from 1773 to 1781.——See the Introduction, p. 4, &c.

According to this Regiſter there were born at *Warrington* from 1773 to 1781.

	Males.	Females.	Total.
	1780	1777	3557
Died in the ſame time, including 14 males and 5 females who died at ages unknown - - -	1287	1432	2719

Marriages in the ſame time 778, or 86 annually.

	Males.	Females.
Died between birth and 1 month -	99	65
From 1 to 2 months — —	37	25
2 to 3 — —	26	19
3 to 6 — —	48	57
6 to 9 — —	62	67
9 to 12 — —	70	80
From birth to 1 year — —	342	313
From 1 to 2 years — —	182	210
2 to 3 — —	87	94
3 to 4 — —	53	51
4 to 5 — —	32	32
5 to 6 — —	22	21
6 to 7 — —	11	9
7 to 8 — —	7	9
8 to 9 — —	3	10
9 to 10 — —	4	7
10 to 14 — —	21	18
From birth to 14 years — —	764	774

Of

TABLE XXXIX. continued.

		Unknown.	Batchelors.	Husbands.	Widowers.	Total.
Of males turned of 14 died from	14 to 17	0	16	0	0	16
	17 to 20	0	21	1	0	22
	20 25	1	16	13	0	30
	25 30	5	14	15	1	35
	30 35	3	5	23	3	34
	35 40	5	3	28	2	38
	40 45	3	1	25	3	32
	45 50	2	0	21	3	26
	50 60	12	6	48	10	76
	60 70	21	6	39	25	91
	70 80	11	5	28	36	80
	80 90	4	0	10	11	25
Above	90 —	0	0	0	4	4
Total	—	67	93	251	98	509
		Died under 14 —				764
		Total —				1273

TABLE XXXIX. continued.

		Unknown.	Maids.	Wives.	Widows.	Total.
Of females turned of 14 died from	14 to 17	0	16	0	0	16
17	to 20	0	20	1	0	21
20	25	1	21	10	2	34
25	30	7	12	22	2	43
30	35	4	3	29	3	39
35	40	8	8	28	4	48
40	45	9	11	27	1	48
45	50	4	8	20	3	35
50	60	13	7	52	22	94
60	70	16	5	38	55	114
70	80	12	12	22	61	107
80	90	4	2	7	32	45
Above 90	—	1	0	1	7	9
Total	—	79	125	257	192	653
		Died under 14	—			774
		Total	—			1427

From

From theſe *data* the following Table has been formed.

Age.	Males. Living.	Decrements.	Females. Living.	Decrements.
0	1273	162	1427	109
3 months		48		57
6 months		62		67
9 months		70		80
1 year	931	182	1114	210
2 years	749	87	904	94
3	662	53	810	51
4	609	32	759	32
5	577	22	727	21
6	555	11	706	9
7	544	7	697	9
8	537	3	688	10
9	534	4	678	7
10	530	5	671	5
11	525	5	666	5
12	520	5	661	4
13	515	6	657	4
14	509	5	653	5
15	504	5	648	5
16	499	6	643	6
17	493	7	637	7
18	486	8	630	7
19	478	7	623	7
20	471	6	616	7
21	465	6	609	6
22	459	6	603	7
23	453	6	596	7
24	447	6	589	7
25	441	7	582	8
26	434	7	574	8
27	427	7	566	9

TABLE XXXIX. continued.

Ages.	Males. Living.	Decrements.	Females. Living.	Decrements.
28	420	7	557	9
29	413	7	548	9
30	406	6	539	8
31	400	7	531	8
32	393	7	523	7
33	386	7	516	8
34	379	7	508	8
35	372	7	500	9
36	365	8	491	9
37	357	8	482	10
38	349	8	472	10
39	341	7	462	10
40	334	7	452	10
41	327	7	442	10
42	320	6	432	10
43	314	6	422	9
44	308	6	413	9
45	302	6	404	8
46	296	5	396	7
47	291	5	389	6
48	286	5	383	7
49	281	5	376	7
50	276	6	369	8
51	270	6	361	8
52	264	7	353	9
53	257	7	344	9
54	250	8	335	10
55	242	8	325	10
56	234	8	315	10
57	226	8	305	10
58	218	9	295	10
59	209	9	285	10
60	200	9	275	11

TABLE XXXIX. continued.

Age.	Males. Living.	Decrements.	Females. Living.	Decrements.
61	191	9	264	11
62	182	9	253	11
63	173	9	242	11
64	164	9	231	12
65	155	10	219	12
66	145	9	207	12
67	136	9	195	12
68	127	9	183	11
69	118	9	172	11
70	109	9	161	11
71	100	9	150	11
72	91	9	139	11
73	82	9	128	11
74	73	8	117	11
75	65	8	106	11
76	57	8	95	11
77	49	7	84	10
78	42	7	74	10
79	35	6	64	10
80	29	25	54	45
Above 90	4	4	9	9
Totals —	27010	1273	36681	1427

It appears from this Table, and from the regiſter on which it is grounded, that though the probabilities of living among females are higher than among males, and a ſmaller number is born, yet more die. The reaſon muſt be, that more males emigrate, and that many of them die in the army, the navy,

navy, and the militia. To this alſo it is owing, that more *wives* die at WARRINGTON than huſbands.

It is proper to add, that in conſequence of this greater emigration, the preceding Tables gives the proportion of the expectations of life among *males* to thoſe among females lower than it really is. But at the ſame time it ſhould be remembered, that it does this only for the ages *before* which, and *during* which, the emigration happens. After theſe ages, (that is, probably after the age of 40 or 50) the correctneſs of the table cannot be affected by this cauſe.

See the remarks in the general introduction to theſe Tables, p. 4, &c.

TABLE

TABLE XL.

Shewing the Probability of the Duration of Human Life, at all Ages, among Males and Females; formed from a Regiſter kept by Dr. *Haygarth*, at CHESTER, for Ten Years, from 1772 to 1781.

According to this Regiſter there were born at CHESTER in ten years from 1772 to 1781.

	Males.	Females.
	2192	2115
There were buried at CHESTER during the ſame time, including 24 whoſe ages were unknown — —	1939	2151
Marriages 1500, or 150 annually.		
Died between birth and 1 month	115	80
from 1 to 2 months	67	51
2 to 3 —	38	30
Died from birth to 3 months	220	161
from 3 to 6 months	75	64
6 to 9 —	76	69
9 to 1 year —	67	74
Died from birth to 1 year —	438	368
from 1 to 2 years —	180	181
2 to 3 —	107	127
3 to 4 —	67	77
4 to 5 —	34	53
5 to 10 —	91	75
10 to 15 —	28	34
15 to 20 —	48	53
Died in all under 20 years of age	993	968

Of

TABLE XL. continued.

			Batchelors.	Husbands.	Widowers.	Total.
Of males turned of 20 died between - -	20	and 25	50	8	0	58
	25	and 30	30	31	1	62
	30	35	19	29	1	49
	35	40	16	38	5	59
	40	45	12	53	6	71
	45	50	9	61	7	77
	50	55	11	54	14	79
	55	60	10	49	13	72
	60	65	13	63	29	105
	65	70	7	40	17	64
	70	75	10	49	40	99
	75	80	3	29	27	59
	80	81	1	9	8	18
	81	82	2	1	6	9
	82	83	0	4	5	9
	83	84	0	1	2	3
	84	85	1	2	2	5
	85	86	0	4	1	5
	86	87	0	3	5	8
	87	88	0	1	2	3
	88	89	0	2	2	4
	89	90	0	0	2	2
	90	91	0	2	2	4
	91	92	0	0	1	1
	92	93	0	2	0	2
	93	94	0	0	1	1
	94	—	0	1	1	2
	97	—	0	0	1	1
	99	—	0	0	1	1
	106	—	1	0	1	2
Died in all of males above 20			195	536	203	934
Under 20			—	—	—	993
Total						1927

TABLE XL. continued.

		Maids.	Wives.	Widows.	Total.
Of females turned of 20 died between - -	20 and 25	38	13	2	53
	25 and 30	28	49	3	80
	30 35	21	40	4	65
	35 40	7	58	6	71
	40 45	11	54	9	74
	45 50	14	46	16	76
	50 55	16	34	21	71
	55 60	13	32	24	69
	60 65	26	53	37	116
	65 70	9	28	46	83
	70 75	19	37	86	142
	75 80	18	20	70	108
	80 81	3	3	29	35
	81 82	1	0	12	13
	82 83	1	2	15	18
	83 84	1	0	10	11
	84 85	4	6	15	25
	85 86	2	0	8	10
	86 87	1	0	8	9
	87 88	1	0	6	7
	88 89	0	2	6	8
	90	0	0	6	6
	91	0	0	3	3
	92	1	0	4	5
	94	0	0	2	2
	95	0	0	1	1
	96	0	0	1	1
	97	0	1	0	1
	98	0	1	3	4
	99	0	0	1	1
	101	0	0	1	1
	102	0	0	1	1
	103	0	0	1	1
Died in all of females *above* 20		235	479	456	1171
Under 20		—	—		968
	Total		—		2139

Of 22 females above the age of 80 who died at *Cheſter* in 1772, the regiſter ſpecifies nō more that that 4 of them were maids, and 14 of them widows who died between 80 and 90; and that the remaining 4 were widows who died above 90.——Of the 4 who had never been married, one has been ſuppoſed to die at each of the ages 81, 83, 84, and 85.——Of the 18 widows, 2 have been ſuppoſed to die at each of the ages between 80 and 88; two at 91; one at 92; and one at 93.——It was proper to make ſome diſtribution of this kind; but it is of little conſequence whether it is right or wrong. In every other inſtance the numbers dying at every age have been taken juſt as the regiſter has given them; and the following Table has been formed from them.

TABLE XL. continued.

Age.	Males. Living.	Decrements.	Females. Living.	Decrements.
0	1927	220	2139	161
3 months	——	75	——	64
6 months	——	76	——	69
9 months	——	67	——	74
1 year	1489	180	1771	181
2 years	1309	107	1580	127
3	1202	67	1463	77
4	1135	34	1386	53
5	1101	30	1333	30
6	1071	24	1303	10
7	1047	18	1285	11
8	1029	11	1274	9
9	1018	8	1265	7
10	1010	6	1258	6
11	1004	5	1252	6
12	999	5	1246	7
13	994	6	1239	7
14	988	6	1232	8
15	982	7	1224	9
16	975	9	1215	10
17	966	10	1205	11
18	956	11	1194	12
19	945	11	1182	11
20	934	11	1171	10
21	923	11	1161	10
22	912	12	1151	10
23	900	12	1141	11
24	888	12	1130	12
25	876	13	1118	16
26	863	13	1102	16
27	850	13	1086	16
28	837	12	1070	16
29	825	11	1054	16

TABLE XL. continued.

Age.	Males. Living	Decrements.	Females. Living.	Decrements.
30	814	10	1038	13
31	804	9	1025	13
32	795	10	1012	13
33	785	10	999	13
34	775	10	986	13
35	765	11	973	14
36	754	11	959	14
37	743	12	945	14
38	731	12	931	14
39	719	13	917	15
40	706	13	902	15
41	693	14	887	15
42	679	14	872	15
43	665	15	857	14
44	650	15	843	15
45	635	15	828	15
46	620	15	813	15
47	605	15	798	15
48	590	16	783	16
49	574	16	767	15
50	558	16	752	15
51	542	16	737	14
52	526	16	723	14
53	510	16	709	14
54	494	15	695	14
55	479	14	681	13
56	465	14	668	13
57	451	14	655	13
58	437	14	642	15
59	423	16	627	15
60	407	19	612	20
61	388	22	592	25
62	366	22	567	25
63	344	22	542	25

TABLE XL. continued

Age.	Males. Living.	Decrements.	Females. Living.	Decrements.
64	322	20	517	21
65	302	16	496	17
66	286	13	479	15
67	273	11	464	15
68	262	11	449	16
69	251	13	433	20
70	238	16	413	25
71	222	22	388	30
72	200	22	358	30
73	178	21	328	30
74	157	18	298	27
75	139	15	271	23
76	124	12	248	22
77	112	11	226	21
78	101	11	205	21
79	90	10	184	21
80	80	10	163	21
81	70	10	142	21
82	60	9	121	21
83	51	8	100	21
84	43	7	79	18
85	36	6	61	12
86	30	5	49	8
87	25	4	41	6
88	21	4	35	4
89	17	3	31	4
90	14	3	27	4
91	11	3	23	4
92	8	3	19	4
93	5	2	15	4
94	3	2	11	4
95	1	1	7	3
96			4	3
97			1	1

In this and the laſt Table there are ſeveral irregularities in the decreaſe of the probabilities of the duration of life, which would not have taken place, had the obſervations been made on a larger body of people, or for a longer period of years; but they do not much affect the correctneſs of the expectations and values of lives deducible from theſe Tables, except at the extremity of life after the age of 80 or 85. According to the *Cheſter* regiſter, the whole number of males that died at every age for ten years between 80 and 85, was 44——22 died between 85 and 90, and 14 above 90. This regiſter alſo makes 102 the number of females that died between 80 and 85, and 34 and 27 the numbers that died between 85 and 90, and *above* 90. The preceding Table, from the age of 80 to 97, is formed juſt as it would have been formed had the regiſter given only this information without particularizing the numbers dying in every ſingle year of life after 80. It will be eaſily ſeen that this was neceſſary. The deaths at the extreme ages beyond 96 or 97, bear ſo ſmall a proportion to the reſt, that there is no occaſion for including them in a Table of Obſervations; nor is it poſſible to do it properly.

It ſhould be further conſidered, that the remark at the end of the Table for *Warrington* is applicable to this Table.

COMPARISON of the Duration of the Lives of *Males* and *Females*, according to the preceding Table.

Ages.	Expectations of Males.	Expectations of Females.
Birth	28.13	33.27
5	43.20	47.44
10	41.92	45.17
15	38.05	41.36
20	34.86	38.10
25	32.00	34.78
30	29.25	32.27
35	25.97	29.26
40	22.92	26.37
45	20.20	23.50
50	17.64	20.62
55	15.14	17.52
60	12.36	14.20
65	10.79	11.94
70	8.05	8.81
75	7.00	7.14
80	5.43	5.20
85	4.25	4.81
90	2.50	3.46

ABSTRACT

ABSTRACT *of the Rev. Mr.* GORSUCH'S *Obſervations and Regiſter in the Pariſh of* HOLY CROSS, *near* SHREWSBURY.

In 1755 the number of inhabitants in this pariſh was 1049.

In 1760 the families were 235—the inhabitants 1048, of whom two were males, and 13 females above 80.

In 1765 the families were 249—the inhabitants 1096.

	In 1770.	1775.	1780.
Families - -	240	——	246
Inhabitants - - -	1046	1057	1113
Males under 10 - -	126	——	155
Females under 10 -	122	——	135
Males from 70 to 80 -	20	20	11
Females from 70 to 80 -	24	21	19
Males above 80 - -	6	9	4
Females above 80 - -	11	7	5

The increaſe in 1765 was occaſioned by the removal of four numerous families into four great houſes in the Pariſh, which for many years before had been almoſt uninhabited.

In 1767 ſeveral houſes were pulled down to open a way to a new ſtone bridge over the *Severn*, and 38 perſons went out of the pariſh.

In

In 1774 a fire deſtroyed 48 houſes, moſtly thatched; but the ſufferers provided themſelves with lodgings in the pariſh, and only 24 left it.——The vacant ground was covered with little tenements fit for poor people, and ſo commodious as to draw into the pariſh a greater number of perſons than had reſided there before. ——See a further account of this pariſh in Vol. I. p. 261.

Births for 30 years, from 1750 to 1780	Males - 565	Females 533	1098
Burials - -	Males - 458	Females 508	966

The births have exceeded the burials in the proportion of 15 to 13; and this ought to have increaſed the inhabitants in 30 years to at leaſt 1200; but it appears that it has occaſioned little or no increaſe; and, conſequently, that the exceſs of the births has been but juſt ſufficient to ſupply the loſs produced by emigrations to the navy and army, and ſettlements in towns.

From 1750 to 1760.		Males and Females.	From 1760 to 1780. Males.	Females.	Total.	
Died under	1 month	17	22	25		64
	1 year	27	42	44		113
	2 years	15	23	23		61
From	2 to 5	28	33	38		99
	5 10	23	16	21		60
	10 15	6	4	4	14	35
	15 20	7	6	8	21	
	20 25	18	4	7	29	57
	25 30	11	10	7	28	
	30 35	9	4	8	21	57
	35 40	11	16	9	36	
	40 45	13	16	11	40	67
	45 50	8	9	10	27	
	50 55	10	16	17	43	80
	55 60	13	12	12	37	
	60 65	13	22	20	55	94
	65 70	15	11	13	39	
	70 75	10	17	29	56	95
	75 80	10	15	14	39	
	80 85	15	22	20	57	71
	85 90	8	1	5	14	
	90 95	1	0	9	10	13
	96	1	0	1	2	
	101	0	0	1	1	
		289	321	356		966

It is obvious, that these observations do not give sufficient *data* for forming distinct tables of the probabilities of living among males

males and females: And it is alſo obvious, that the numbers dying in every period of five years after 10, are much more irregular than they would have been had theſe obſervations been made for a greater number of years, or on a larger body of inhabitants. In conſtructing, therefore, the following Table, the decrements of life have been taken as the regiſter gives them for both ſexes in every period of ten years after the age of ten. And in this way the regiſter exhibits with remarkable regularity and conſiſtency the progreſs of human mortality from birth to old age, repreſenting human life in conformity to other obſervations, as particularly weak in the firſt month, (though much leſs ſo than in towns) and from that age as growing gradually ſtronger, till at 10 it acquires its greateſt ſtrength, which it afterwards loſes, but more ſlowly till 50, and after 50 more rapidly, till at 70 or 75 it is brought back to all the weakneſs of the firſt month.

TABLE

TABLE XLI.

Shewing the Probabilities of the Duration of Human Life at all Ages, as deduced from the Rev. Mr. GORSUCH's Obſervations, during a Period of 30 Years, in the Pariſh of HOLY CROSS, near SHREWSBURY. See Vol. I. p. 261.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	966	64	21	529	5	44	395	7
Under 1 month		113	22	524	5	45	388	7
			23	519	6	46	381	7
1 year	789	61	24	513	6	47	374	7
2	728	44	25	507	6	48	367	7
3	684	30	26	501	6	49	360	7
4	654	25	27	495	6	50	353	7
5	629	20	28	489	6	51	346	7
6	609	16	29	483	6	52	339	7
7	593	12	30	477	5	53	332	8
8	581	7	31	472	5	54	324	8
9	574	5	32	467	5	55	316	8
10	569	4	33	462	6	56	308	8
11	565	3	34	456	6	57	300	9
12	562	3	35	450	6	58	291	9
13	559	3	36	444	6	59	282	9
14	556	3	37	438	6	60	273	9
15	553	3	38	432	6	61	264	9
16	550	4	39	426	6	62	255	9
17	546	4	40	420	6	63	246	9
18	542	4	41	414	6	64	237	9
19	538	4	42	408	6	65	228	9
20	534	5	43	402	7	66	219	10

TABLE XLI. continued.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
67	209	10	77	111	9	87	28	6
68	199	10	78	102	9	88	22	5
69	189	10	79	93	9	89	17	4
70	179	10	80	84	9	90	13	3
71	169	10	81	75	8	91	10	2
72	159	10	82	67	8	92	8	2
73	149	10	83	59	8	93	6	2
74	139	10	84	51	8	94	4	2
75	129	9	85	43	8	95	2	1
76	120	9	86	35	7	96	1	1

EXPECTATIONS of Life by the preceding Table.

Age.	Expectation.
Birth	33.93
5	46.30
10	46.00
15	42.25
20	38.66
25	35.58
30	32.66
35	29.43
40	26.40
45	23.35
50	20.40
55	17.47
60	14.86
65	12.30
70	10.00
75	7.87
80	5.75

The proportion of the living under ten years of age to the living at ten and upwards, is, by this Table, as 6807 to 26452, or as 10 to 39; but the real proportion appears from the ſurvey to be greater: And it is evident, that the exceſs of the births above the burials, and the emigrations from the pariſh after ten, muſt make it conſiderably greater; and it ſhould not be forgotten, that theſe alſo are circumſtances which muſt render the probabilities and expectations of life, as given by the Table, leſs than they *really* are.

TABLE

TABLE XLII.

Shewing the Probabilities of the Duration of Human Life among Males and Females, deduced from Obſervations of the Proportions of the Living to the Numbers who have died at all Ages for 21 Years, from 1755 to 1776, in the Kingdom of SWEDEN.

PRELIMINARY OBSERVATIONS.

According to the medium of ſeven different enumerations in 1757, 1760, 1763, 1766, 1769, 1772, and 1775, there were living in the kingdom of SWEDEN:

Ages.	Males.	Females.
Under 1 year	33882	33640
Between 1 and 3 years	62155	63005
3 5	62696	63551
5 10	121871	122460
10 15	117879	118419
15 20	103093	105845
20 25	91907	102306
25 30	82919	93315
30 35	78615	87129
35 40	70390	77077
40 45	63961	70405
45 50	52083	59580
50 55	44908	52689
55 60	36253	44211
60 65	30772	39416
65 70	21170	29610
70 75	14610	21776
75 80	8224	12515
80 85	4036	6418
85 90	1522	2492
Above 90	486	869
Total —	1,103,432	1,206,728
And females —	1,206,728	
Total of males and females	2,310,160	

Fencible men between 15 and 55	587,876	or a *quarter* nearly of the inhabitants.
Males and females under the age of 25	1,201,909	or a little more than *half* the inhabitants

Of theſe numbers there died annually in SWEDEN during twenty-one years from 1755 to 1776,

Ages.	Males.		Females.	
Under 1 Yr.	9664	or 1 of 3.5*	8355	or 1 of 4.0*
Between 1 and 3 Yrs.	3592	or 1 of 17.3	3531	or 1 of 17.8
3 and 5	1816	or 1 of 34.5	1774	or 1 of 35.8
5 10	1789	1 of 68.1	1672	1 of 73.2
10 15	898	1 of 131.2	802	1 of 147.6
15 20	741	1 of 139.1	714	1 of 148.2
20 25	874	1 of 105.1	776	1 of 131.8
25 30	879	1 of 94.3	872	1 of 106.9
30 35	955	1 of 82.3	1058	1 of 82.3
35 40	907	1 of 77.6	901	1 of 85.5
40 45	1119	1 of 57.1	1129	1 of 62.3
45 50	1077	1 of 48.3	958	1 of 62.2
50 55	1233	1 of 36.4	1127	1 of 46.7
55 60	1180	1 of 30.7	1163	1 of 38.0
60 65	1383	1 of 22.2	1597	1 of 24.6
65 70	1328	1 of 15.9	1510	1 of 19.6
70 75	1360	1 of 10.7	1935	1 of 11.2
75 80	1023	1 of 8.0	1527	1 of 8.2
80 85	784	1 of 5.1	1230	1 of 5.2
85 90	383	1 of 4.0	609	1 of 4.1
Above 90	195	1 of 2.5	339	1 of 2.6
Of all ages	33180	1 of 33.25	33579	1 of 35.94

* It ſhould be conſidered, that this is a *higher* proportion than that of the number that dies under one year of age to the number born in a year. The latter number is equal to the former increaſed by the number living at one time under one year. See the note, p. 158.

The enumerations and deaths for the firſt 9 years from 1755 to 1763 included the whole kingdom of *Sweden*, conſiſting of 26 principalities or provinces.——In 1764 there was a ſuſpenſion of all the obſervations. In 1765 they were taken up again; but in this and the following years, the enumeration of one of the provinces was omitted, together with the regiſtration of the deaths in that province.—In the three years from 1767 to 1770 three provinces were omitted, in the enumerations and regiſters.——In the three years from 1770 to 1773, there was alſo an omiſſion of three provinces, together with the city of STOCKHOLM. And in the remaining three years (to 1776) four out of the 15 dioceſſes in *Sweden* were omitted. But theſe omiſſions will produce no incorrectneſs in the tables of the decrements and values of lives formed from the preceding *data*.

I have formed tables from the enumerations and deaths in the firſt nine years, comprehending all *Sweden*; but there is no other difference between them and the following Tables, except that the latter give the probabilities of the duration of life a little lower than the former; and the reaſon of this is, that the mortality of the years 1771, 1772, and 1773, exceeded greatly the mortality of the other years (*a*).

(*a*) It is alſo owing to this that the proportions of annual deaths to the living at all ages, as here given, are ſomewhat greater than thoſe in the Second Eſſay at the end of this work.

In

In the healthieſt of the ſeven ternaries of years into which theſe obſervations have been divided (that is, in the three years (*b*) from 1765 to 1767) only one in $36\frac{1}{2}$ of males, and 1 in $39\frac{2}{3}$ of females, died. The average proportion for the whole period of 21 years is 1 in $33\frac{1}{4}$ of males; and 1 in $35,\frac{94}{100}$ of females. But, in the ſickly years juſt mentioned, there died 1 in 27 of males, and 1 in 29 of females.——The number of the living in the following Tables, at the end of one year of age, is the *difference* between the number born in *Sweden* in a year,

(*b*) The whole number of males living in theſe years was 1,182,848; of females 1,290,068. I have ſaid that one of the 26 provinces of SWEDEN was omitted in the obſervations for theſe three years. The addition of this province will make the inhabitants of SWEDEN in 1766 above two millions and a half. In 1757 they were 2,323,195. They increaſed, therefore, at the rate of near 200,000 in *nine* years. But it appears that this increaſe had not been of long continuance; for had it been ſo, a table formed from the decrements as given by the regiſters, and by taking the medium of annual deaths from 1755 to 1763 for the *radix*, would have given the probabilities of living much too ſmall (and much leſs than thoſe in the following Table) through the whole duration of life; whereas it does this only in the firſt ſtages of life. From 45 to 60 it gives them nearly equal; and after 60 it gives them greater, which is a plain proof that about the beginning of this century *Sweden* was decreaſing.——To the ſame purpoſe it appears from the enumerations, that while the numbers living in the *firſt* ſtages of life were increaſing faſt, the numbers in the *laſt* ſtages were decreaſing.

and the number of deaths under one year of age (excluſive in both caſes of ſtill-borns) accommodated to 10,000 as a radix.

The decrements among *males* in the following Table, increaſe regularly through every period of life from 10 to 75. But among *females* this increaſe is interrupted for a few years after 35, and again for a few years after 45.——This cannot be an accidental irregularity, the numbers being too great, and the period for which the obſervations have been made, too long, to admit of ſuch an irregularity.——Probably, therefore, it muſt be accounted for in the following manner.——From the age of 30 to 35, the number of married, and conſequently of child-bearing women, is greater than at any other ages; and this raiſes the decrements in that diviſion of life. After 35, this number is diminiſhed, and the decrements fall. Between 40 and 45 the critical periods come on, and the decrements are raiſed again; but after 45 the number of deaths ariſing from hence becoming leſs, the decrements become alſo leſs, but continue afterwards to increaſe with increaſing years, till they become greateſt at 74 or 75.——It is, however, remarkable that notwithſtanding the peculiar dangers to which the lives of females are ſubject from the cauſes juſt mentioned, there

there are no ages at which a *ſmaller* proportion of them does not die than of males, except the ages in which the number of deliveries is greateſt; and that even *then* the probabilities of living among them are nearly equal to thoſe among males.

TABLE XLII. continued.

	Males.			Females.		
	Born 10,282—282 born dead			10,277-217born dead		
Ages.	Living.	Decr.	Expectat.	Living.	Decr.	Expect.
Born alive	10,000	2300	33.20	10,000	2090	35.70
1 year	7,700	500	42.45	7,910	518	44.00
2	7,200	337	43.83	7,392	350	46.05
3	6,863	240	44.96	7,042	250	47.31
4	6,623	150	45.57	6,792	135	48.04
5	6,473	125	45.62	6,657	120	48.00
6	6,348	105	45.50	6,537	105	47.87
7	6,243	90	45.26	6,432	85	47.64
8	6,153	75	44.91	6,347	70	47.28
9	6,078	65	44.46	6,277	60	46.80
10	6,013	55	43.94	6,217	52	46.25
11	5,958	45	43.26	6,165	46	45.55
12	5,913	45	42.58	6,119	40	44.85
13	5,868	40	41.91	6,079	35	44.15
14	5,828	40	41.24	6,044	35	43.46
15	5,788	39	40.56	6,009	35	42.76
16	5,749	39	39.83	5,974	40	42.04
17	5,710	39	39.11	5,934	40	41.31
18	5,671	44	38.39	5,894	42	40.59
19	5,627	44	37.67	5,852	43	39.87
20	5,583	50	36.95	5,809	43	39.15
21	5,533	50	36.28	5,766	43	38.43
22	5,483	50	35.62	5,723	43	37.72
23	5,433	55	34.96	5,680	44	37.01
24	5,378	55	34.30	5,636	45	36.29
25	5,323	55	33.63	5,591	45	35.58
26	5,268	55	32.98	5,546	50	34.90
27	5,213	55	32.32	5,496	52	34.21
28	5,158	55	31.66	5,444	55	33.53
29	5,103	56	31.00	5,389	55	32.85
30	5,049	59	30.34	5,334	60	32.17

TABLE XLII. continued.

	Males.			Females.		
Ages.	Living.	Decr.	Expectat.	Living.	Decr.	Expectat.
31	4,988	60	29.69	5,274	60	31.54
32	4,928	60	29.04	5,214	65	30.91
33	4,868	60	28.39	5,149	65	30.28
34	4,808	60	27.74	5,084	65	29.66
35	4,748	60	27.09	5,019	60	29.03
36	4,688	60	26.43	4,959	56	28.26
37	4,628	60	25.76	4,903	56	27.50
38	4,568	60	25.09	4,847	56	26.74
39	4,508	60	24.42	4,791	58	25.97
40	4,448	65	23.75	4,733	65	25.21
41	4,383	72	23.15	4,668	75	24.68
42	4,311	80	22.54	4,593	76	24.75
43	4,231	80	21.93	4,517	76	23.62
44	4,151	80	21.32	4,441	75	23.10
45	4,071	80	20.71	4,366	72	22.57
46	3,991	80	20.12	4,294	67	21.91
47	3,911	80	19.52	4,227	65	21.24
48	3,831	80	18.92	4,162	65	20.58
49	3,751	85	18.32	4,097	70	19.92
50	3,666	95	17.72	4,027	75	19.26
51	3,571	95	17.17	3,952	80	18.64
52	3,476	95	16.63	3,872	85	18.01
53	3,381	95	16.08	3,787	85	17.39
54	3,286	95	15.53	3,702	85	16.77
55	3,191	95	14.98	3,617	85	16.15
56	3,096	95	14.43	3,532	85	15.53
57	3,001	100	13.87	3,447	90	14.92
58	2,901	100	13.33	3,357	90	14.31
59	2,801	100	12.79	3,267	100	13.69
60	2,701	105	12.24	3,167	110	13.08
61	2,596	110	11.72	3,057	118	12.56
62	2,486	115	11.21	2,939	120	12.04
63	2,371	115	10.73	2,819	120	11.52
64	2,256	115	10.26	2,699	120	11.01

TABLE XLII. continued.

	Males.			Females.		
Ages.	Living	Decr	Expectat.	Living.	Decr.	Expect.
65	2,141	115	9.78	2,579	120	10.49
66	2.026	115	9.30	2,459	120	9.97
67	1,911	120	8.84	2,339	120	9.46
68	1,791	125	8.40	2,219	120	8.94
69	1,666	125	7.99	2,099	120	8.42
70	1,541	125	7.60	1,979	130	7.91
71	1,416	125	7.22	1,849	140	7.53
72	1,291	120	6.87	1,709	150	7.16
73	1,171	120	6.53	1,559	160	6.78
74	1,051	110	6.22	1,399	150	6.40
75	941	105	5.89	1,249	140	6.03
76	836	100	5.56	1,109	130	5.73
77	736	90	5.25	979	120	5.43
78	646	85	4.92	859	110	5.11
79	561	80	4.59	749	100	4.79
80	481	75	4.27	649	95	4.47
81	406	70	3.96	554	90	4.13
82	336	65	3.69	464	85	3.84
83	271	60	3.45	379	80	3.59
84	211	50	3.30	299	75	3.42
85	161	40	3.16	224	55	3.40
86	121	30	3.04	169	40	3.34
87	91	22	2.88	129	30	3.22
88	69	17	2.64	99	23	3.05
89	52	14	2.34	76	18	2.82
90	38	12	2.02	58	15	2.55
91	26	9		43	12	
92	17	7		31	10	
93	10	6		21	8	
94	4	3		13	6	
95	1	1		7	4	
96	0	0		3	2	
97	0	0		1	1	

TABLE XLIII.

Shewing the Probabilities of the Duration of Human Life among Males and Females, taken collectively, deduced from the preceding Table.

Born - 10,249 — 249 born dead				Age.	Living.	Decr.	Expect.
Age.	Living.	Decr.	Expect.				
				35	4884	59	28.03
Born alive	10000	2195	34.42	36	4825	58	27.31
1 year	7805	509	42.95	37	4767	58	26.68
2 years	7296	344	44.92	38	4709	58	26.01
3	6952	245	46.11	39	4651	60	25.33
4	6707	143	46.78	40	4591	65	24.66
5	6564	122	46.79	41	4526	73	24.05
6	6442	105	46.66	42	4453	78	23.44
7	6337	87	46.43	43	4375	78	22.83
8	6250	73	46.07	44	4297	78	22.22
9	6177	62	45.61	45	4219	76	21.61
10	6115	54	45.07	46	4143	74	20.98
11	6061	45	44.38	47	4069	72	20.35
12	6016	42	43.70	48	3997	73	19.72
13	5974	38	43.01	49	3924	78	19.09
14	5936	37	42.33	50	3846	85	18.46
15	5899	37	41.64	51	3761	87	17.87
16	5862	40	40.92	52	3674	90	17.29
17	5822	40	40.19	53	3584	90	16.70
18	5782	42	39.47	54	3494	91	16.12
19	5740	43	38.74	55	3403	91	15.53
20	5697	47	38.02	56	33 2	92	14.95
21	5650	47	37.33	57	3220	95	14.37
22	5603	48	36.64	58	3125	95	13.79
23	5555	48	35.96	59	3030	100	13.21
24	5507	50	35.27	60	2930	108	12.63
25	5457	50	34.58	61	2822	114	12.12
26	5407	52	33.91	62	2708	118	11.62
27	5355	54	33.23	63	2590	118	11.11
28	5301	55	32.56	64	2472	118	10. 1
29	5246	55	31.88	65	2354	118	10.10
30	5191	59	31.21	66	2236	118	9.62
31	5132	60	30.57	67	2118	121	9.15
32	5072	62	29.94	68	1997	124	8.67
33	5010	63	29.30	69	1873	124	8.20
34	4947	63	28.67	70	1749	127	7.72

TABLE XLIII. continued.

Age.	Living.	Decr.	Expectat.	Age.	Living.	Decr.	Expect.
71	1622	133	7.32	86	144	35	3.09
72	1489	135	6.89	87	109	27	2.92
73	1354	140	6.53	88	82	20	2.71
74	1214	130	6.23	89	62	15	2.43
75	1084	121	5.91	90	47	14	2.05
76	963	115	5.59	91	33	12	1.71
77	848	105	5.28	92	21	10	1.40
78	743	95	4.96	93	11	6	
79	648	90	4.61	94	5	3	
80	558	90	4.28	95	2	1	
81	468	84	4.01	96	1	1	
82	384	75	3.80				
83	309	65	3.57				
84	244	55	3.39				
85	189	45	3.23				

In forming this Table from the decrements of life among males and females in Table XLII. it is neceſſary to conſider that the proper decrements for a body of males and females taken collectively, are not the means between thoſe for males and females in that Table; but the numbers dying in every period of life out of a given number living at the beginning of that period, ſuppoſed to conſiſt of *equal* numbers of males and females.

For example. Table XLII. ſhews that of 2701 males living at 60 years of age, 560 will die in five years; and that of 3167 *females* living at the ſame age, 588 will die in the ſame time. From hence it may be eaſily deduced, that of 2930 perſons (the number

number in this Table living at 60) conſiſting one half of males and one half of females, 576 will die in the ſame time. The number, therefore, living at 60 will at 65 be reduced to 2354; which number muſt again be ſuppoſed to conſiſt one half of males and the other half of females, and the proper decrement for the next five years, deduced in the ſame manner from Table XLII. And it is in this method the whole of this Table has been conſtructed, which, therefore muſt exhibit more accurately than any other, the probabilities of living among the general maſs of mankind, conſiſting of males and females taken collectively.

TABLE XLIV.

Shewing the Probabilities of the Duration of Human Life among Males and Females in STOCKHOLM, formed from the Proportions of the Living to the Numbers who have died in STOCKHOLM at all Ages for Nine Years from 1755 to 1763.

	Males.	Females.
There were born alive in STOCKHOLM annually from 1755 to 1763 — — —	1335	1207
Still-born — — —	43 1/3	31

According to the medium of three different enumerations in 1757, 1760, and 1763, there were living in STOCKHOLM,

	Males.	Females.
Under 1 year	666	727
From 1 to 3 years	1239	1376
3 to 5	1185	1281
5 10	2662	2769
10 15	2971	2791
15 20	2780	2662
20 25	3293	4255
25 30	3371	4325
30 35	3533	4156
35 40	2763	3101
40 45	2528	2837
45 50	1668	1911
50 55	1402	1892
55 60	874	1340
60 65	705	1247
65 70	404	806
70 75	285	626
75 80	131	314
80 85	57	148
85 90	15	51
Above 90	8	27
Under 15	8723	8944
Between 15 and 55	21338	25139
Above 55	2479	4559
Of all ages	32540	38642

Of theſe numbers there died annually at STOCKHOLM during nine years from 1755 to 1763.

Ages.		MALES.	FEMALES.
Under 1 year, excluſive of ſtillborn - - -		565 or 1 of 1.17	505 or 1 of 1.44
Between 1 and 3 yrs.		179 or 1 of 6.93	187 or 1 of 7.37
3	5	89 1 of 13.27	81 1 of 15. 8
5	10	77 1 of 34. 5	71 1 of 38. 8
10	15	38 1 of 78. 8	24 1 of 114.7
15	20	37 1 of 59. 1	27 1 of 99. 8
20	25	74 1 of 44. 3	54 1 of 79. 3
25	30	101 1 of 33. 2	75 1 of 57. 9
30	35	119 1 of 29. 6	96 1 of 43. 3
35	40	104 1 of 26.56	79 1 of 39. 1
40	45	110 1 of 23. 0	92 1 of 31. 0
45	50	86 1 of 19. 4	69 1 of 27. 7
50	55	85 1 of 16. 4	75 1 of 25. 3
55	60	62 1 of 14. 1	56 1 of 24. 0
60	65	69 1 of 10.74	77 1 of 16.06
65	70	43 1 of 9.47	60 1 of 13.35
70	75	37 1 of 7.63	77 1 of 8.09
75	80	29 1 of 4.56	61 1 of 5.15
80	85	16 1 of 3.51	47 1 of 3.42
85	90	7 1 of 2.00	21 1 of 2.37
Above	90	3 1 of 2.66	12 1 of 2.31
Of all ages -		1930 1 of 10.86	1846 1 of 20.93

TABLE

From theſe *data* the following Table has been formed.

	Males.		Females.	
Born -	10324—324 born dead		born 10235—225 born dead	
Ages.	Living.	Decr.	Living.	Decrements.
Born alive	10000	4232	10000	3885
1 year	5768*	800	6115*	900
2 years	4968	541	5215	530
3	4427	380	4685	350
4	4047	235	4335	200
5	3812	150	4135	155
6	3662	110	3980	115
7	3552	90	3865	90
8	3462	85	3775	75
9	3377	75	3700	60
10	3302	55	3640	45
11	3247	40	3595	30
12	3207	35	3565	25
13	3172	35	3540	25
14	3137	37	3515	30
15	3100	40	3485	30
16	3060	45	3455	30
17	3015	50	3425	35
18	2965	55	3390	35
19	2910	60	3355	40
20	2850	60	3315	40
21	2790	60	3275	40

* The annual average of males born alive at *Stockholm* for 9 years from 1755 to 1763, was 1335. Of theſe 565 died annually under one year of age. The number, therefore, that lived to one year of age was 770; and 770 is the ſame part of 1335 that 5768 is of 10000.

In the ſame manner the number of females who lived to one year of age has been determined; after which, the totals living between 1 and 3, and between 3 and 5, and between 5 and 10, &c. &c. are always made to be in the ſame ratio to the number dying at thoſe ages that they were found to be by obſervation.

In this method alſo the laſt Table, ſhewing the probabilities of life in the kingdom of Sweden at large, has been formed.

TABLE XVII. continued.

	Males.		Females.	
Age.	Living.	Decrements.	Living.	Decrements.
22	2730	60	3235	40
23	2670	60	3195	40
24	2610	65	3155	43
25	2545	70	3112	45
26	2475	70	3065	47
27	2405	70	3020	50
28	2335	70	2970	55
29	2265	70	2915	60
30	2195	70	2855	60
31	2125	70	2795	60
32	2055	70	2735	63
33	1985	65	2672	65
34	1920	65	2607	65
35	1855	65	2542	62
36	1790	65	2480	60
37	1725	65	2420	60
38	1660	60	2360	60
39	1600	60	2300	65
40	1540	60	2235	66
41	1480	60	2169	66
42	1420	60	2103	67
43	1360	60	2036	67
44	1300	60	1969	67
45	1240	60	1902	65
46	1190	57	1837	65
47	1133	55	1772	65
48	1078	55	1707	63
49	1023	55	1644	60
50	968	53	1584	60
51	915	50	1524	60
52	865	50	1464	55
53	815	50	1409	55
54	765	50	1354	53
55	715	45	1301	50
56	670	45	1251	50
57	625	45	1201	50
58	580	40	1151	50
59	540	40	1101	50
60	500	40	1051	55
61	460	40	996	60
62	420	38	936	60

TABLE XLIV. continued.

Age.	Males. Living.	Decrements.	Females. Living.	Decrements.
63	382	35	876	55
64	347	32	821	53
65	315	30	768	49
66	285	28	719	49
67	257	25	670	49
68	232	22	621	49
69	210	22	572	49
70	198	20	523	49
71	168	20	474	49
72	148	18	425	49
73	130	17	376	49
74	113	17	327	49
75	96	16	278	45
76	80	15	233	40
77	65	15	193	35
78	50	11	158	30
79	39	9	128	25
80	30	7	103	23
81	23	5	80	20
82	18	4	60	17
83	14	4	43	12
84	10	3	31	10
85	7	2	21	7
86	5	2	14	5
87	3	2	9	4
88	1	1	5	2
89	0	0	3	2
90	0	0	1	1
Total	147593	10000	185924	10000

COMPARISON of the Duration of the Lives of Males and Females, according to the preceding Table.

Ages.	Expectations of Males.	Expectations of Females.
Birth	14.25	18.10
5	31.05	37.12
10	30.00	36.89
15	26.74	33.43
20	23.85	30.01
25	21.40	26.80
30	19.42	23.98
35	17.58	21.62
40	15.61	19.25
45	13.78	17.17
50	11.95	15.12
55	10.30	12.89
60	8.69	10.45
65	7.39	8.39
70	5.81	6.16
75	4.09	4.39

From this comparison, and from Tables XL. and XLII. p. 136 and 150, it appears, that the difference between the duration of the lives of *males* and *females* is *leaſt* in the kingdom of SWEDEN at large, *greater* at CHESTER, and *greateſt* at STOCKHOLM, which ſeems to indicate that this is a difference not entirely natural.

TABLE XLV.

Shewing the Values of Annuities on Single Lives among Males and Females, according to the Probabilities of the Duration of Life in the Kingdom of SWEDEN. See Table XLII. page 150.

	MALES.		FEMALES.		Lives in general.	
Ages.	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.	4 per Ct.	5 per Ct.
1	16.503	14.051	16.820	14.271	16.661	14.161
2	17.355	14.778	17.719	15.034	17.537	14.906
3	17.935	15.279	18.344	15.571	18.139	15.425
4	18.328	15.624	18.780	15.951	18.554	15.787
5	18.503	15.786	18.927	16.088	18.715	15.937
6	18.622	15.901	19.045	16.203	18.833	16.052
7	18.693	15.977	19.131	16.291	18.912	16.134
8	18.725	16.021	19.162	16.335	18.943	16.178
9	18.715	16.030	19.151	16.343	18.933	16.186
10	18.674	16.014	19.109	16.325	18.891	16.169
11	18.600	15.970	19.041	16.286	18.820	16.128
12	18.491	15.896	18.952	16.229	18.721	16.062
13	18.378	15.819	18.840	16.153	18.609	15.986
14	18.246	15.724	18.707	16.059	18.476	15.891
15	18.105	15.624	18.568	15.960	18.336	15.792
16	17.958	15.517	18.424	15.856	18.191	15.686
17	17.803	15.404	18.290	15.761	18.046	15.582
18	17.643	15.285	18.151	15.662	17.897	15.473
19	17.492	15.175	18.013	15.563	17.752	15.369
20	17.335	15.059	17.872	15.462	17.603	15.260
21	17.192	14.955	17.725	15.356	17.458	15.155
22	17.042	14.846	17.573	15.245	17.307	15.045
23	16.887	14.732	17.414	15.129	17.150	14.930
24	16.742	14.627	17.252	15.009	16.997	14.818
25	16.592	14.517	17.087	14.886	16.839	14.701
26	16.436	14.402	16.915	14.757	16.675	14.579
27	16.274	14.282	16.751	14.636	16.512	14.459
28	16.105	14.155	16.588	14.515	16.346	14.335
29	15.930	14.024	16.427	14.396	16.178	14.210
30	15.751	13.889	16.261	14.272	16.006	14.080
31	15.575	13.756	16.104	14.156	15.839	13.956
32	15.395	13.619	15.941	14.035	15.668	13.827
33	15.208	13.477	15.787	13.923	15.497	13.700
34	15.014	13.327	15.629	13.806	15.321	13.566
35	14.812	13.170	15.465	13 684	15.138	13.427

TABLE XLV. continued.

Ages.	Males. 4 per Ct.	Males. 5 per Ct.	Females. 4 per Ct.	Females. 5 per Ct.	Lives in general. 4 per Ct.	Lives in general. 5 per Ct.
36	14.601	13.006	15.278	13.542	14.939	13.274
37	14.382	12.833	15.070	13.382	14.726	13.107
38	14.154	12.652	14.854	13.213	14.504	12.932
39	13.916	12.462	14.629	13.036	14.272	12.749
40	13.668	12.261	14.401	12.856	14.034	12.558
41	13.426	12.065	14.185	12.687	13.805	12.376
42	13.196	11.880	13.994	12.538	13.595	12.209
43	12.984	11.710	13.798	12.387	13.391	12.048
44	12.763	11.532	13.596	12.229	13.179	11.880
45	12.535	11.347	13.383	12.061	12.959	11.704
46	12.297	11.153	13.151	11.876	12.724	11.514
47	12.051	10.951	12.894	11.668	12.472	11.309
48	11.795	10.738	12.620	11.443	12.217	11.090
49	11.528	10.516	12.333	11.205	11.930	10.860
50	11.267	10.298	12.049	10.970	11.658	10.634
51	11.030	10.100	11.769	10.737	11.399	10.418
52	10.785	9.895	11.492	10.507	11.138	10.201
53	10.531	9.682	11.220	10.280	10.875	9.981
54	10.269	9.460	10.937	10.042	10.603	9.751
55	9.998	9.229	10.642	9.792	10.320	9.510
56	9.717	8.988	10.334	9.529	10.025	9.258
57	9.425	8.736	10.012	9.253	9.718	8.994
58	9.140	8.489	9.692	8.976	9.416	8.732
59	8.845	8.232	9.358	8.684	9.101	8.458
60	8.540	7.963	9.039	8.406	8.789	8.184
61	8.241	7.700	8.739	8.144	8.480	7.922
62	7.950	7.442	8.453	7.895	8.201	7.668
63	7.669	7.193	8.166	7.643	7.917	7.418
64	7.382	6.938	7.870	7.382	7.626	7.160
65	7.090	6.676	7.566	7.111	7.328	6.893
66	6.792	6.408	7.252	6.831	7.022	6.619
67	6.489	6.134	6.930	6.541	6.709	6.337
68	6.201	5.872	6.596	6.239	6.398	6.055
69	5.933	5.628	6.253	5 926	6.093	5.777
70	5.670	5.389	5.897	5.599	5.783	5.494
71	5.418	5.158	5.564	5.293	5.491	5.225
72	5.180	4.940	5.261	5.013	5.220	4.976
73	4.940	4.719	4.998	4.770	4.969	4.744
74	4.724	4.521	4.792	4.581	4.758	4.551
75	4.487	4.302	4.582	4.388	4.534	4.345
76	4.253	4.084	4.367	4.189	4.310	4.136
77	4.024	3.871	4.145	3.983	4.084	3.927

TABLE XLV. continued.

Ages.	Males. 4 per Ct.	Males. 5 per Ct.	Females. 4 per Ct.	Females. 5 per Ct.	Lives in general. 4 per Ct.	Lives in general. 5 per Ct.
78	3.768	3.631	3.913	3 767	3.840	3.699
79	3.512	3.390	3.668	3.536	3.590	3.463
80	3.260	3.152	3.402	3.285	3.331	3.218
81	3.017	2.921	3.145	3.041	3.081	2.981
82	2.792	2.706	2.905	2.812	2.848	2.759
83	2.600	2.523	2.699	2.615	2.649	2.569
84	2.473	2.403	2.559	2.480	2.516	2.441
85	2.371	2.306	2.552	2.476	2.461	2.391
86	2.281	2.222	2.518	2.446	2.399	2.334
87	2.154	2.103	2.431	2.365	2.292	2.238
88	1.955	1.912	2.294	2.236	2.124	2.074
89	1.698	1.664	2.108	2.059	1.903	1.861
90	1.417	1.392	1.873	1.833	1.645	1.612
91	1.154	1.136	1.628	1.596	1.391	1.366
92	0.835	0.824	1.349	1.325	1.092	1.074
93	0.477	0.471	1.071	1.054	0.774	0.762
94	0.240	0.238	0.799	0.788	0.519	0.513
95	0.000	0.000	0.544	0.537		
96	0.000	0.000	0.320	0.317		

TABLE XLVI.

Shewing the Values of Annuities on two *joint* Lives, according to the Probabilities (in Table XLIII.) of the Duration of Human Life among Males and Females collectively, reckoning Intereſt at 4 *per cent.*

INTEREST 4 *per cent.*

Differences of Age, 0, 6, 12, and 18 Years.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
1- 1	12.252	1- 7	13.989	1-13	13.894	1-19	13.309
2- 2	13.583	2- 8	14.780	2-14	14.557	2-20	14.008
3- 3	14.558	3- 9	15.323	3-15	14.988	3-21	14.417
4- 4	15.267	4-10	15.685	4-16	15.259	4-22	14.671
5- 5	15.577	5-11	15.817	5-17	15.326	5-23	14.725
6- 6	15.820	6-12	15.887	6-18	15.354	6-24	14.740
7- 7	16.003	7-13	15.914	7-19	15.351	7-25	14.727
8- 8	16.109	8-14	15.888	8-20	15.310	8-26	14.673
9- 9	16.152	9-15	15.824	9-21	15.244	9-27	14.590
10-10	16.141	10-16	15.729	10-22	15.149	10-28	14.484
11-11	16.087	11-17	15.617	11-23	15.033	11-29	14.357
12-12	15.982	12-18	15.477	12-24	14.889	12-30	14.202
13-13	15.855	13-19	15.327	13-25	14.736	13-31	14.045
14-14	15.701	14-20	15.164	14-26	14.566	14-32	13.874
15-15	15.535	15-21	15.001	15-27	14.392	15-33	13.700
16-16	15.361	16-22	14.832	16-28	14.216	16-34	13.520
17-17	15.196	17-23	14.665	17-29	14.042	17-35	13.340
18-18	15.023	18-24	14.491	18-30	13.860	18-36	13.141
19-19	14.854	19-25	14.320	19-31	13.687	19-37	12.934
20-20	14.682	20-26	14.144	20-32	13.512	20-38	12.720
21-21	14.525	21-27	13.976	21-33	13.345	21-39	12.505
22-22	14.360	22-28	13.807	22-34	13.173	22-40	12.286
23-23	14.194	23-29	13.635	23-35	12.997	23 41	12.073
24-24	14.020	24-30	13.455	24-36	12.801	24-42	11.873
25-25	13.849	25-31	13.284	25-37	12 599	25-43	11.683
26-26	13.671	26-32	13.108	26-38	12.387	26-44	11.485
27-27	13.495	27-33	12.935	27-39	12.170	27-45	11.284
28-28	13.323	28-34	12.763	28-40	11.953	28-46	11.072
29-29	13.148	29-35	12.586	29-41	11.742	29-47	10.847
30-30	12.965	30-36	12.390	30-42	11.543	30-48	10.606
31-31	12.795	31-37	12.192	31-43	11.359	31-49	10.365
32-32	12.624	32-38	11.988	32 44	11.170	32-50	10.128

TABLE XLVI continued.

INTEREST 4 *per cent.*

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
33-33	12.456	33-39	11.779	33-45	10.978	33-51	9.905
34-34	12.286	34-40	11.568	34-46	10.775	34-52	9.679
35-35	12.109	35-41	11.361	35-47	10.557	35-53	9.452
36-36	11.904	36-42	11.156	36-48	10.314	36-54	9.207
37-37	11.683	37-43	10.953	37 49	10.059	37-55	8.951
38-38	11.452	38-44	10.741	38-50	9.805	38-56	8.683
39-39	11.209	39-45	10.519	39-51	9.558	39-57	8.404
40-40	10.964	40-46	10.286	40-52	9.308	40-58	8.124
41-41	10.732	41-47	10.049	41-53	9.066	41-59	7.839
42-42	10.531	42-48	9.813	42-54	8.830	42-60	7.569
43-43	10.346	43 49	9.581	43 55	8.597	43-61	7.318
44-44	10.154	44-50	9.351	44-56	8.354	44-62	7.075
45-45	9.954	45-51	9.129	45-57	8.101	45-63	6.836
46-46	9.736	46-52	8.897	46-58	7.841	46-64	6.586
47-47	9.497	47-53	8.658	47-59	7.563	47-65	6.323
48-48	9.236	48-54	8.402	48-60	7.281	48-66	6.048
49-49	8.966	49-55	8.139	49-61	7.008	49-67	5.764
50-50	8.707	50-56	7.874	50-62	6.749	50-68	5.487
51-51	8.469	51-57	7.613	51-63	6.505	51-69	5.221
52-52	8.230	52-58	7.351	52-64	6.256	52-70	4.953
53-53	7.994	53-59	7.083	53 65	6.004	53-71	4.694
54-54	7.748	54 60	6.814	54 66	5.743	54-72	4.455
55-55	7.495	55-61	6.555	55-67	5.474	55-73	4.231
56-56	7.229	56-62	6.299	56-68	5.204	56-74	4.043
57-57	6.954	57-63	6 045	57-69	4.936	57-75	3.844
58-58	6.678	58-64	5.788	58-70	4.664	58-76	3.637
59-59	6.388	59-65	5.519	59-71	4.395	59-77	3.430
60-60	6.104	60.66	5.249	60-72	4.149	60-78	3.210
61-61	5.844	61-67	4.984	61-73	3.927	61-79	2.974
62-62	5.600	62.68	4.729	62-74	3.747	62-80	2.744
63-63	5.367	63-69	4 482	63-75	3.563	63-81	2.557
64-64	5.128	64-70	4.231	64-76	3.370	64-82	2.396
65-65	4.881	65-71	3.982	65 77	3.180	65-83	2.252
66-66	4.626	66-72	3.750	66-78	2.974	66-84	2.123
67-67	4.362	67-73	3 527	67-79	2.743	67-85	2.010
68-68	4.130	68-74	3.340	68-80	2.514	68-86	1.910
69-69	3.851	69-75	3.147	69-81	2.324	69-87	1.798
70-70	3.593	70-76	2.946	70-82	2.155	70-88	1.661
71-71	3.345	71-77	2.752	71-83	2.004	71-89	1.464
72-72	3.128	72-78	2.558	72-84	1.875	72-90	1.189
73-73	2.935	73-79	2,355	73-85	1.768	73-91	0,937

TABLE XLVI. continued.

INTEREST 4 *per cent.*

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
74-74	2.797	74-80	2.172	74-86	1.692	74-92	0.708
75-75	2 648	75-81	2.017	75-87	1.605	75-93	0.575
76-76	2.490	76-82	1.877	76-88	1.497	76-54	0.481
77-77	2.340	77-83	1.756	77-89	1.339	77-95	0.421
78-78	2. 70	78-84	1.639	78-90	1.097		
79-79	1.967	79-85	1.524	79-91	0.863		
80-80	1.758	80-86	1.416	80-92	0.638		
81-81	1.600	81-87	1.320	81-93	0.511		
82-82	1.472	82-88	1.225	82-94	0.427		
83-83	1.364	83-89	1.094	83-95	0.379		
84-84	1.276	84-90	0.902				
85-85	1.212	85-91	0.725				
86-86	1.172	86-92	0.556				
87-87	1.127	87-93	0.459				
88-88	1.071	88-94	0.396				
89-89	0.949	89-95	0.364				
90-90	0.718						
91-91	0.516						
92-92	0.326						
93-93	0.236						
94-94	0.190						
95-95	0.024						

TABLE XLVII.

Shewing the Values of two *joint* Lives, according to the Probabilities (in Table XLIII.) of the Duration of Human Life among Males and Females collectively.

INTEREST 4 *per cent.*

Differences of Age 24, 30, 36, and 42 Years.

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
1-25	12.832	1-31	12.196	1-37	11.465	1-43	10.546
2-26	13.409	2-32	12.730	2-38	11.913	2-44	10.946
3-27	13.778	3-33	13.066	3-39	12.164	3 45	11.168
4-28	14.003	4 34	13.264	4 40	12.284	4-46	11.260
5-29	14.037	5-35	13.277	5-41	12.242	5-47	11.183
6-30	14.033	6-36	13.242	6-42	12.185	6-48	11.064
7-31	14.006	7-37	13.170	7-43	12.112	7 49	10.915
8-32	13.944	8-38	13.059	8-44	12.004	8-50	10.743
9-33	13.855	9-39	12.913	9-45	11.865	9-51	10.560
10-34	13.741	10-40	12.743	10-46	11.694	10 52	10.357
11-35	13.604	11-41	12.563	11-47	11.493	11-53	10.140
12-36	13.428	12-42	12.379	12-48	11.259	12-54	9.898
13-37	13.234	13-43	12.196	13-49	11.011	13 55	9.644
14-38	13.023	14-44	11.997	14-50	10.759	14 56	9.371
15-39	12.798	15-45	11.787	15-51	10.514	15-57	9.087
16-40	12.570	16-46	11.562	16-52	10.264	16-58	8.799
17-41	12.351	17 47	11.328	17-53	10.018	17 59	8.503
18-42	2.146	18-48	11.076	18-54	9.761	18-60	8.208
19 43	11.951	19-49	10.819	19-55	9.500	19-61	7.928
20-44	11.751	20 50	10.567	20-56	9.228	20-62	7.658
21-45	11.550	21-51	10.332	21-57	8.953	21 63	7.396
22-46	11.335	22-52	10.092	22-58	8.675	22-64	7 127
23-47	11.107	23 53	9.852	23-59	8.385	23-65	6.851
24-48	10.862	24 54	9.602	24-60	8.097	24 66	6.566
25-49	10.612	25-55	9.347	25-61	7.823	25-67	6.275
26-50	10.364	26 56	9.080	26-62	7.557	26-68	5.986
27-51	10.130	27-57	8.807	27 63	7.297	27 69	5.702
28-52	9.894	28-58	8.534	28-64	7.032	28-70	5.415
29 53	9.659	29-59	8.250	29-65	6.761	29-71	5.136
30-54	9.413	30-60	7.967	30-66	6.481	30-72	4.881
31-55	9.167	31-61	7.702	31 67	6.197	31 73	4.646
32-56	8.912	32-62	7.446	32-68	5 917	32-74	4.453
33-57	8.651	33-63	7.196	33-69	5.642	33-75	4.251
34-58	8.389	34 64	6.942	34 70	5.364	34-76	4.040
35-59	8.114	35-65	6.679	35-71	5.093	35-77	3.833
36-60	7.833	36-66	6.402	36-72	4.840	36-78	3.605

TABLE XLVII. continued.

INTEREST 4 *per Cent.*

Ages.	Values.	Ages.	Values.	Ages.	Values.	Ages.	Values.
37-61	7.561	37 67	6.115	37-73	4.603	37.79	3.352
38-62	7.296	38-68	5.828	38-74	4.405	38.80	3.098
39-63	7.033	39 69	5.543	39-75	4.195	39.81	2.889
40-64	6.763	40 70	5.254	40-76	3.975	40.82	2.710
41-65	6.492	41-71	4.977	41-77	3.762	41.83	2.553
42-66	6.225	42-72	4.730	42-78	3.539	42.84	2.418
43-67	5.957	43-73	4.507	43-79	3.295	43.85	2.305
44-68	5.689	44-74	4.322	44 80	3.052	44.86	2.203
45-69	5.426	45-75	4.128	45-81	2.854	45.87	2.083
46-70	5.153	46-76	3.921	46-82	2.684	46.88	1.933
47-71	4.884	47-77	3.715	47-83	2.533	47.89	1.708
48-72	4.633	48-78	3.489	48 84	2.396	48.90	1.385
49-73	4.398	49-79	3.238	49 85	2.277	49.91	1.090
50-74	4.205	50 80	2.990	50 86	2.171	50.92	0.818
51-75	4.008	51-81	2.792	51 87	2.050	51.93	0.662
52-76	3.803	52-82	2.623	52-88	1.901	52 94	0.551
53-77	3.605	53-83	2.475	53 89	1 681	53.95	0.468
54-78	3.389	54-84	2.344	54-90	.366		
55-79	3.150	55-85	2.232	55 91	1.078		
56-80	2.909	56-86	2.130	56-92	0.810		
57-81	2.710	57-87	2.010	57 93	0.655		
58-82	2.539	58-88	1.864	58-94	0.546		
59 83	2.385	59-89	1.644	59-95	0.464		
60-84	2.248	60-90	1.333				
61-85	2.135	6 -91	1.050				
62-86	2.037	62-92	0.789				
63 87	1.926	63-93	0.639				
64-88	1.790	64-94	0.533				
65-89	1.585	65-95	0.456				
66-90	1.290						
67-91	1.017						
68-92	0.764						
69-93	0.617						
70 94	0.514						
71-95	0.441						

THE

THE directions given in p. 96, for uſing the tables of the values of joint lives deduced from the *Northampton* Obſervations, are applicable to the two laſt Tables, and may be eaſily adapted to them, by taking the differences of age in thoſe directions at *ſix* years and its multiples, inſtead of *five* years and its multiples.

REMARKS.

The values of *joint* lives in theſe Tables have been computed for only one rate of intereſt; and of *ſingle* lives in Table XLV. for only two rates of intereſt. The following rules will ſhew, that it would be a needleſs labour to compute theſe values (in ſtrict conformity to the obſervations) for any other rates of intereſt.

ACCOUNT of a method of deducing, from the correct *values (according to any obſervations) of any ſingle or joint lives at* one *rate of intereſt, the ſame values at* other *rates of intereſt.*

PRELIMINARY PROBLEMS.

PROBLEM I. The *expectation* given of a ſingle life by any table of obſervations, to find its value, ſuppoſing the decrements of life equal, at any given rate of intereſt.

SOLU-

SOLUTION, Find in Table II. the value of an annuity certain for a number of years equal to twice the expectation. Multiply this value by the PERPETUITY increaſed by unity, and divide the product by twice the expectation. The quotient ſubtracted from the perpetuity will be the value required.

EXAMPLE.

The *expectation* of a male life aged ten, by the SWEDEN obſervations (See Table XLV.) is 43.94. Twice this expectation is 87.88. The value of an annuity certain for 87.88 years is, by Table II. (reckoning intereſt at 4 *per cent.*) 24.200. The product of 24.200 into 26 (the perpetuity increaſed by unity) is 629.2, which, divided by 87.88, gives 7.159. And this quotient ſubtracted from 25 (the perpetuity) gives 17.84 years purchaſe, the value of a life aged ten, deduced from the expectation of life at that age, according to the SWEDEN obſervations.

This is the rule by which Mr. *De Moivre* has calculated the table commonly uſed of the values of lives according to his hypotheſis; and from this Table (the firſt of the two Tables at the end of this volume) the value required in this problem may be deduced more compendiouſly in the following

following manner, provided the expectation does not exceed 38.——"Take the dif-"ference between twice the expectation "and 86; and the value in the Table cor-"reſponding to that difference, if not leſs "than 10, will be the value ſought."—— Thus; twice the *expectation* of a female life aged 30 (that is, its *complement*) is, by Table XLII. 64.34. The difference between it and 86, is 21.66. And ſince the value correſponding to age 21 in Mr. *De Moivre*'s valuation of lives (or in Table I. at the end of this volume) is (reckoning intereſt at 4 *per cent.*) 15.781; and the value correſponding to age 22 is 15.669; it is obvious, that the value correſponding to age 21.66 muſt be the *greateſt* of theſe two values leſſened by $\frac{66}{100}$ of the difference between it and the *leaſt*. This difference is .112; and $\frac{66}{100}$ of it (or .112 multiplied by .66) is .074, which ſubtracted from 15.781, gives 15.707 the value ſought of a life whoſe expectation is 32.17 (or whoſe complement is 64.34) on the ſuppoſition of an equal decrement of life.

PROBLEM II.

Having the expectations given of any two lives by any table of obſervations, to deduce from thence the value of the joint lives at any

any rate of intereſt ſuppoſing an equal decrement of life.

SOLUTION. Find the difference between twice the expectation of the youngeſt life, and twice the expectation of the oldeſt life increaſed by unity and twice the perpetuity. Multiply this difference by the value of an annuity certain for a time equal to *twice* the expectation of the *oldeſt* life; and by twice the ſame expectation divide the product, reſerving the quotient.

From twice the perpetuity ſubtract the reſerved quotient, and multiply the remainder by the perpetuity increaſed by unity (*a*). This laſt product divided by twice the expectation of the *youngeſt* life, and then ſubtracted from the perpetuity, will be the required value.

EXAMPLE.

Let the joint lives propoſed be a female life aged 10, and a male life aged 15, and let the table of obſervations be the *Sweden*

(*a*) When twice the expectation of the youngeſt life is *greater* than twice the expectation of the oldeſt life increaſed by unity and twice the perpetuity, the reſerved quotient inſtead of being *ſubtracted* from twice the perpetuity, muſt be added to it, and the *ſum*, not the *difference*, multiplied by the perpetuity increaſed by unity.

Table

Table for lives in general, and the rate of intereſt 4 *per cent.* Twice the expectations of the two lives are 90.14 and 83.28. (See Table XLIII.)

Twice the expectation of the *oldeſt* life, increaſed by unity and twice the perpetuity, is 134.28, which leſſened by 90.14 (twice the expectation of the *youngeſt* life) leaves 44.14 for the *reſerved* remainder.——This remainder multiplied by 24.045 (the value of an annuity certain (*a*) for 83.28 years) and the product divided by 83.28 (twice the expectation of the *oldeſt* life) gives 12.744 the quotient to be *reſerved*; which ſubtracted from double the perpetuity, and the remainder (or 37.255) multiplied by the perpetuity increaſed by unity (or by 26) gives 968.630, which divided by 90.14 (twice the expectation of the *youngeſt* life) and the quotient ſubtracted from the perpetuity, we have 14.254 for the required value.

(*a*) This value, when the number of years is a whole number with a fraction added (as will be commonly the caſe) may be beſt computed in the following manner.

In this example the number of years is 83.28.

The value of an annuity certain for 83 years is (by Table II. p. 21. 24.035.——The ſame value for 84 years is 24.072.——The difference between theſe two values is .037; which difference multiplied by .28 (the fractional part of the number of years) and the product (.0103) added to the *leaſt* of the two values, will give 24.045 the value for 83.28 years.

This

This calculation may be made more eaſily by logarithms in the following manner.

Twice the expectation of the eldeſt is	83.28
ADD twice the perpetuity increaſed by unity - -	51.00
Sum —	134.28
SUBTRACT twice the expectation of the youngeſt -	90.14
Remainder —	44.14
Log. of 44.14 is	1.644,832
Log. of 24.045 is (See the note in the laſt page)	1.381,024
Sum —	3.025,856
Subtract	1.920,540 Log. of 83.28
Remainder -	1.105,316 the number of which is 12.744. or the remainder to be reſerved.

Twice the perpetuity is	50	
Subtract	12.744	
Remains —	37.255 Logar. of which is	1.571,184
Add Log. of the perpetuity increaſed by unity		1.414,973
Sum —		2.986,157
Subtract Log. of twice the expectation of the youngeſt — — —		1.954,917
Remainder —		1.031,240

The

The number of this laſt remainder is 10.745, which ſubtracted from 25 (the perpetuity) leaves 14.254, the value ſought.—See the algebraical canon in Note (L) at the end of this volume.

GENERAL RULE.

Call the *correct* value (ſuppoſed to be computed for any rate of intereſt) the FIRST value.

Call the value deduced (by the preceding problems) from the *expectations* at the ſame rate of intereſt, the SECOND value.

Call the value deduced from the *expectations* for any other rate of intereſt the THIRD value.

Then, the difference between the *firſt* and *ſecond* values added *to* or ſubtracted *from* the *third* value, juſt as the *firſt* is *greater* or *leſs* than the *ſecond*, will be the value at the rate of intereſt for which the THIRD value has been deduced from the expectations.

The following examples will make this perfectly plain.

EXAMPLE I.

In the laſt Tables the correct values are given of two joint lives among mankind at large, without diſtinguiſhing between males and females, according to the SWEDEN obſervations, reckoning intereſt at 4 *per cent.*

Let

Let it be required to find from theſe values the values at 3 *per cent.*; and let the ages of the joint lives be ſuppoſed 10 and 10.

The correct value by Table XLVI. (reckoning intereſt at 4 *per cent.*) is 16.141. The expectation of a life aged 10 is (by Table XLVII.) 45.07.——The value deduced from this expectation at 4 *per cent.* by Prob. II. is 14.539.——The value deduced by the ſame problem from the ſame expectation at 3 *per cent.* is 16.808.—— The difference between the *firſt* and *ſecond* values, is 1.602, which, added to the *third* value (the firſt being *greater* than the ſecond) makes 18.410 the value required.

EXAMPLE II.

Let the values be required of two joint lives aged 50 and 60, at an intereſt of 3 *per cent.* from the correct value given at an intereſt of 4 *per cent.* according to the *Northampton* obſervations.

Firſt or correct value at 4 *per cent* by Table XX. is 6.989. The expectation of 50 is 17.99; of 60, is 13.21, by Table VII. p. 38. The *ſecond* value, or the value deduced from theſe expectations at 4 *per cent.* is, by Prob. II. 7.182. The *third* value, or the value deduced from the ſame expectations at 3 *per cent.* is 7.704.——The

 difference

difference between the firſt and ſecond is .193, which (ſince the *ſecond* is greater than the *firſt*) muſt be ſubtracted from the *third*, and the remainder (or 7.511) will be the value required.——The exact value at 3 *per cent.* is, by Table XX. 7.460.

If the value is required at 5 *per cent.* the *third* value will be 6.732; and the difference ſubtracted from 6.732, will leave 6.539 the value at 5 *per cent.*

The *exact* value at this rate of intereſt, is (by Table XX.) 6.568.

EXAMPLE III.

Let the value be required of a ſingle male life aged 10, at 3 *per cent.* intereſt, from the correct value at 4 *per cent.* according to the *Sweden* obſervations.

Firſt, or correct value at 4 *per cent.* (by Table XLV.) is 18.674. The expectation of a male life aged 10, is (by Table XLII.) 43.94.

The *ſecond* value (or the value deduced from this expectation by Prob. I.) is 17.838.

The *third* value (or the value deduced from the ſame expectation at 3 *per cent.*) is 21.277.

The difference between the firſt and ſecond is .836; which (ſince the firſt is greater than the ſecond) muſt be added to the

tne *third*; and the *ſum*, (that is, 22.113) will be the value required.

The *third* value at 5 *per cent.* is 15.286; and the difference added to 15.286 makes 16.122 the value of a male life aged 10 at 5 *per cent.* according to the SWEDEN obſervations.——The exact value at 5 *per cent.* is (by Table 45th) 16.014.

Again. The difference between 16.014 (the correct value at 5 *per cent.*) and 15.286 (the value at the ſame intereſt deduced from the expectation) is .728; which, *added* (becauſe the firſt value is greater than the ſecond) to 13.335, (the value deduced at 6 *per cent.* from the expectation) gives 14.063, the value of the ſame life, reckoning intereſt at 6 *per cent.*

Theſe deductions, in the caſe of ſingle lives particularly, are ſo eaſy, and give the true values ſo nearly, that it will be ſcarcely ever neceſſary to calculate the *exact* values (according to any given obſervations) for more than *one* rate of intereſt.

If, for inſtance, the correct values are computed at 4 *per cent.* according to any obſervations, the values at 3, $3\frac{1}{2}$, $4\frac{1}{2}$, 5, 6, 7, or 8 *per cent.* may be deduced from them by the preceding rules, as occaſion may require, without much labour or any danger of conſiderable errors.——The following compariſons will ſhew in ſome meaſure how far theſe deductions may be depended on.

Value of Single Male Lives by the SWEDEN Table of Obſervations, p. 162.

Ages.	Values at 5 per cent. deduced from the correct values by Table XLV. at 4 per cent.	Correct values by Table XLV. at 5 *per cent.*
5	15.879	15.786
10	16.122	16.014
15	15.707	15.624
30	13.909	13.889
60	7.969	7.963
70	5.417	5.389

Values of two joint Lives by the NORTHAMPTON Table of Obſervations.

Ages.	Values at 5 *per cent.* deduced from the correct values at 4 *percent.* by Table 18th &c.	Correct values at 5 *per cent.* by Tables 18, 19, &c.	Values at 3 *per cent.* deduced from the correct values at 4 *per cent.* by Table 18th, &c.	Correct values at 3 *per cent.* by Table 18, 19, &c.
5- 5	11.989	11.984	15.618	15.638
15-15	11.986	11.960	15.184	15.229
25-25	10.775	10.764	13.389	13.383
40-40	9.006	9.016	10.756	10.764
60-60	5.842	5.888	6.692	6.606
15-40	10.214	10.205	12.368	12.459
30-60	7.285	7.292	8.396	8.378
50-60	6.555	6.568	7.471	7.461

Values of Single Lives by the NORTHAMPTON Table of Obſervations.

Ages.	Values at 5 *per cent.* deduced from the correct values at 4 *per cent.* by Table 17.	Correct values at 5 *per cent.* by Table 17.	Values at 3 *per cent.* deduced from the correct values at 5 *per cent.* by Table 17.	Correct values at 3 *per cent.* by Table 17.
5	14.825	14.827	20.435	20.473
10	15.162	15.139	20.652	20.663
68	6.546	6.536	7.353	7.367

Ages.	Values at 4 *per cent.* deduced from the correct values at 5 *per cent.* by Table 17.	Correct values at 4 *per cent.* by Table 17.	Values at 5 *per cent.* deduced from the correct values at 3 *per cent.* by Table 17.	Correct values at 5 *per cent.*
5	17.239	17.248	14.850	14.827
10	17.500	17.523	15.173	15.139
68	6.920	6.930	6.560	6.536

It may be obſerved in theſe examples, that the *deduced* values are *ſometimes* almoſt the ſame with the *correct* values; that *generally* they do not differ more than a 20th or 30th of a year's purchaſe; that in *joint* lives they differ leſs than in *ſingle* lives; and that they come equally near to one another *whatever* the rates of intereſt are.

The following obſervation will ſhew the reaſon of the circumſtance laſt mentioned.

The value *deduced* from the expectation coincides with the *correct* value when the rate of intereſt is little or nothing; and, conſequently, the difference between the two

values becomes then little or nothing; and to this it is continually tending as the intereſt is diminiſhed. On the contrary; the *in*creaſe of value occaſioned by the *de*creaſe of intereſt tends to make the difference greater. There is, therefore, in this caſe, the counter-action of two cauſes which always keep the difference nearly the ſame in all rates of intereſt.

The preceding rules ſeem to leave nothing wanting on this ſubject, except tables of the values of two joint lives *at any one rate of intereſt*, when the lives are either both male or both female lives. But the following rule for finding theſe values from the values in the two laſt Tables, will render the labour of compoſing ſuch tables almoſt needleſs.

RULE for computing from the values of two joint lives in Tables XLVI. and XLVII. the values of two joint lives both male or both female.

" Find in that column of Table XLIII.
" which ſhews the expectations of *lives in*
" *general*, two ages whoſe expectations come
" neareſt to the expectations of the two
" *male* or the two *female* lives propoſed.
" From theſe expectations deduce, by the
" rule in p. 176, the value of two joint
" lives

" lives at thoſe ages; and take the diffe-
" rence between this value and the *correct*
" value at thoſe ages in Tables XLVI.
" and XLVII.

" Deduce alſo, by the rule in p. 176,
" the value of the joint lives propoſed,
" from the expectations in Table XLIII. of
" male and female lives. The difference
" juſt found *added* to this laſt value, if the
" value before deduced from the expecta-
" tions of *lives in general* is *leſs* than the
" correct value, or *ſubtracted* from it if
" *greater*, will be nearly the correct value
" of the two joint lives propoſed."

EXAMPLE.

Let the two propoſed lives be both *female* lives, one aged 20 and the other aged 50.

The expectation of a *female* life aged 20 is, by Table XLII. 39.15. The expectation neareſt to it, in Table XLIII. ſhewing the expectations of lives in general, is 39.47, correſponding to a life aged 18.——In like manner; the expectation in the ſame Table neareſt to the expectation of a female life aged 50, is 19.09, correſponding to age 49.——The value (deduced from theſe expectations) of two joint lives aged 18 and 49, is, by the rule in p. 176, 10.245. The *correct* value, taken from Table XLVII. is

10.851, and the difference is .606, which difference *added* (ſince the former value is *leſs* than the latter) to 10.281 (the value of two joint female lives aged 50 and 20, deduced from the expectations by the rule in p. 176) makes 10.887, the *correct* value nearly of the joint *female* lives.

In order to find how near the values thus found come to the exact values, let the value of a *ſingle female* life aged 20 (reckoning intereſt at 4 *per cent.*) be computed in the ſame manner from the correct values given in Table XLV. of the values of lives in general.

The expectation in Table XLIII. neareſt to the expectation in Table XLII. of a *female* life aged 20, is 39.47, which, in Table XLIII. (ſhewing the expectation of lives in general) is the expectation of a life aged 18.——The value of a life aged 18, deduced from *this* expectation by the rule in p. 176, is 17.138. The correct value in that column of Table XLV. which ſhews the values of lives in general, is 17.897. The former value is the leaſt, and the difference is .759.——The value deduced by the ſame rule from 39.15 (the expectation of a *female* life aged 20) is 17.083, and the difference juſt found *added* to this value, makes it 17.842, which is very nearly the ſame with 17.872, the correct value in Table XLV.——The value deduced in the

ſame

ſame manner of a *male* life aged 20, is 17.363. The correct value (in Table XLV.) is 17.335.

Value by *this Rule* of

A *female* life aged 50, is 12.000——Correct value is 12.049
aged 60, is 9.018——Correct value is 9.039
Of a *male* life aged 30, is 15.722——Correct value is 15.751
aged 70. is 5.702——Correct value is 5.670

In calculating by this rule, when any other rate of intereſt than 4 *per cent.* is uſed; the values of the joint lives, at that rate of intereſt, (deduced from the expectations and from the values in Tables XLVI. and XLVII. at 4 *per cent.* by Prob. II. p. 176.) muſt be taken for the *correct* values.——It muſt likewiſe be remembered, that this Rule cannot be uſed when the youngeſt of the two joint lives is *leſs* than ten years of age. In other caſes, the values found by this Rule will be right *generally* within a 30th or 40th of a year's purchaſe, and *never*, I believe, wrong more than a 15th or 20th of a year's purchaſe.

THE

THE laſt Tables from p. 162, I reckon the moſt important in this collection, not only becauſe the only ones that give the ſeparate values of the lives of males and females, and becauſe derived from obſervations in their nature more correct; but on account of their particular uſe in furniſhing inſtruction to the numerous inſtitutions for granting annuities to widows. Mr. *Wargentin* informs me, that even in *Sweden* ſeveral ſocieties of this kind have become bankrupts for want of ſuch inſtruction. I think it, therefore, neceſſary to add the following Table.

TABLE XLVIII.

Shewing the Value of an Annuity for the Life of a Wife after the Death of her Huſband; deduced from the SWEDEN Obſervations on the ſeparate Probabilities of the duration of Life among Males and Females.

The Annuity 10*l.*——Intereſt 4 *per cent.*

Wife's Age.	Huſband's Age.	Value of the Annuity. Single Payment.	Annual Payment.	Wife's Age.	Huſband's Age.	Value of the Annuity. Single Payment.	Annual Payment.
		£.	£.			£.	£.
16	16	30.63	1.87	20	20	31.90	2.03
	22	35.92	2.26		26	37.28	2.46
	28	42.08	2.76		32	43.60	3.00
	34	49.04	3.38		38	51.52	3.80
	40	58.54	4.31		44	61.21	4.80
	46	68.62	5.46		50	73.05	6.31
	52	81.60	7.24		56	86.44	8.36
	58	96.25	9.82		62	102.14	11.79

TABLE XLVIII. continued.

Wife's Age.	Husband's Age.	Value of the Annuity. Single Payment.	Annual Payment.
		£.	£.
24	24	32.32	2.15
	30	37.97	2.62
	36	44.51	3.22
	42	53.79	4.18
	48	63.90	5.38
	54	76.50	7.21
	60	91.55	10.06
28	28	32.64	2.28
	34	38.25	2.77
	40	46.35	3.58
	46	55.16	4.57
	52	66.94	6.14
	58	80.54	8.45
	64	95.56	11.90
32	32	33.16	2.43
	38	39.52	3.04
	44	47.71	3.92
	50	58.13	5.22
	56	70.29	7.09
	62	84.95	10.05
	68	100.24	14.49
36	36	33.74	2.61
	42	41.81	3.86
	48	49.64	4.38
	54	61.71	6.04
	60	74.44	8.43
	66	88.76	12.00
42	42	34.62	3.00
	48	41.81	3.86
	54	51.63	5.25
	60	64.25	7.49
	66	77.69	10.75
	72	92.63	16.16
46	46	34.15	3.18
	52	42.54	4.29
	58	53.10	6.00
	64	65.65	8.65
	70	79.97	12.99
50	50	33.42	3.44
	56	41.75	4.70
	62	53.00	6.83
	68	65.62	10.11
54	54	31.89	3.63
	60	41.23	5.27
	66	51.94	7.70
	72	64.82	11.88
58	58	30.14	3.92
	64	39.04	5.75
	70	50.28	8.87

REMARKS.

REMARKS.

THE ſingle payments in this table are the exceſſes multiplied by 10 of the values of female lives in Table XLV. above the values of the joint lives of males and females in Tables XLVI. and XLVII. And the annual payments are the quotients ariſing from dividing the ſingle payments by the values of the joint lives increaſed by unity, agreeably to the rules in Vol I. p. 13, 14, and 15. The annual payments, therefore, ſuppoſe that the firſt is to be made immediately; and that they are to be continued during the joint duration of the lives of the wife and huſband. And both the *annual* and *ſingle* payments include the *whole value* of the annuity, and conſequently ſuppoſe that if one is preferred the other is excuſed.

One circumſtance a little curious appears in this Table. It ſhews, that the value in a ſingle payment of an annuity during the ſurvivorſhip of one life after another (when the difference of age is not very great) is leſs in the younger ages, and greateſt in the middle ages. This is owing to the high probabilities of living in the younger ages, in conſequence of which it happens that the ſurvivorſhip is poſtponed to a period ſo late as to *ſink* the value of the annuity more on that account than it is *raiſed* by the longer duration of the ſurvivorſhip.

The

The values in this Table would have been (ſuppoſing the ages of huſbands and wives equal or nearly equal) from an 8th to a 12th — or 13th lower than they are, had they been computed from the *means* between the values of the lives of males and females in Table XLV.; that is, from the values of lives in the kingdom of *Sweden* taken in the groſs, without diſtinguiſhing between males and females. There is, therefore, a deficiency to this amount in ſuch values when deduced from the common Tables of ſingle and joint lives.

In Vol. I. p. 124, an account has been given of an inſtitution in the dutchy of *Oldenberg*, which provides annuities for widows, at prices ſpecified in Tables correctly calculated by Mr. *Oeder*, from the values of ſingle and joint lives according to Mr. *Suſmilch's* Table of Mortality. Another inſtitution of the ſame kind at *Hamburgh*, has been deſcribed in p. 178 of the former Volume. And, lately, an account has been ſent me, by Mr. *Oeder*, of a new inſtitution for the ſame purpoſe, eſtabliſhed in *Denmark* and *Norway*, under the ſanction and guarantyſhip of his *Daniſh* Majeſty.

The Office for Equitable Aſſurances in *Chatham-Place*, *London*, includes alſo in its plan a like proviſion for widows. And theſe are all the annuity inſtitutions, with which

which I am acquainted, that are guided in this inſtance by the lights derived from correct obſervations and mathematical principles. But hitherto it has not been poſſible for any of them in calculating the contributions neceſſary to ſupport the annuities, to be governed by any regard to the longer duration of the lives of women. It has been juſt obſerved, that this renders the payments from an 8th to a 12th or 13th *too little* for ſuch annuities, when deduced from any tables which give (as all Tables have hitherto done) only the values of lives in general, without any diſcrimination between males and females. But it will be of uſe here to ſhew, by the following compariſons, the particular differences between the payments for ſuch annuities as determined *accurately* for a whole kingdom, and the payments required, without regarding the longer duration of the lives of females, by the Tables of the four inſtitutions juſt mentioned.

COMPARISON of the Values, in the preceding Table, of a Life Annuity to a Wife after her Husband, with the Values of the same Annuity in the Tables of the *Danish* and *Oldenberg* Institutions, calculated on the Supposition of the Improvement of Money at an Interest of 4 *per cent.*

Annuity 10*l.*

Wife's Age.	Husband's Age.	Value of the Annuity. By Table XLVIII. Single Payment.	By Table XLVIII. Annual Payment.	By *Oldenberg* Tables. Single Payment.	By *Oldenberg* Tables. Annual Payment.	By *Danish* Tables. Single Payment.
		£.	£.	£.	£.(a)	£.
20	20	31.90	2.03	29.82	2.11	
	26	37.28	2.46	34.34	2.60	35.74
	50	73.05	6.31	69.93	6.70	69.11
28	28	32.64	2.28	29.94	2.41	31.15
	34	38.25	2.77	36.30	2.84	35.50
	52	66.94	6.14	63.10	6.54	
42	42	34.62	3.00	30.72	3.34	30.00
	48	41.81	3.86	38.24	4.06	38.27
	60	64.25	7.49	55.84	7.18	57.00
35	35	33.55	2.55	31.36	2.74	31.45
	40	40.00	3.20	36.26	3.30	36.63
	60	76.09	8.59	67.44	8.36	68.49

(*a*) In the *Oldenberg*, and also in the *Hamburgh* Tables, these are *half*-yearly payments which I have doubled, and reckoned equivalent to *yearly* payments beginning immediately, and which therefore are over-rated, as may be learnt from the observations in p. 28, Vol. I. The Table for *Denmark* gives only the single payment.

COMPA-

COMPARISON of the Values in Table XLVIII. of a Life Annuity for a Wife after her Husband, with the Values of the ſame Annuity in the Tables of the *Hamburgh* and *Equitable Inſtitutions*, calculated at an Intereſt of 3 *per cent.*

Annuity 10 *l.*——Intereſt 3 *per cent.*

Wife's Age.	Husband's Age.	Value of the Annuity. By *Sweden* Table. Single Payment.	Annual Payment.	By *Hamburgh* Tables. Single Payment.	Annual Payment.	By Equitable Society. Single Payment.	Annual Payment.
		£. (*a*)	£. (*a*)	£.	£.	£. (*b*)	£. (*b*)
20	20	44.00	2.51	40.17	2.27	45.05	2.97
	26	50.62	3.01	47.47	2.85	49.82	3.40
	50	85.82	6.93	86.76	7.60	81.15	7.04
28	28	43 40	2.74	40.30	2.73	43.74	3.14
	34	50.40	3.33	48.08	3.52	49.14	3.67
	52	84.64	7.21	79.40	7.40	73.72	6.75
35	35	43.03	2.99	39.80	2.80	42.16	3.31
	40	50.44	3.70	45.81	3.54	47.25	3.86
	60	92.83	9.88	82.14	9.40	77.11	8.35

(*a*) In computing theſe payments, the values of lives at 3 *per cent.* according to the *Sweden* Tables, have been deduced from the values at 4 *per cent.* by the rules in p. 170, &c.

(*b*) Theſe payments may be eaſily deduced, either from the Tables in this collection of the values of ſingle and joint lives, according to the *Northampton* Obſervations, or from Table XXXVI.

For example. It appears from this laſt Table, that the annuity for a life aged 20 after another of the ſame age, to which either a ſingle payment of 27.96 *l.* or an annual payment of 1.848 *l.* during the joint lives will entitle an expectant, is 6.207 *l.*; from whence it will follow, by the rule of proportion, that the annuity being 10 *l.* the ſingle payment muſt be 45.05 *l.* and the annual payment 2.97 *l.*

From

From theſe compariſons it appears that, ſuppoſing intereſt at 4 *per cent*. and the *Sweden* Tables a proper ſtandard (and till ſimilar obſervations are made in other kingdoms they ought to be reckoned the propereſt) the payments required by the *Daniſh* eſtabliſhment are ſomewhat too little. The ſame appears to be true of the *ſingle* payments in the *Oldenburg* eſtabliſhment; but the *annual* payments in this eſtabliſhment appear to be more than the value (*a*).

In

(*a*) Agreeably to this obſervation, Mr. *Oeder*, in the examination mentioned in Vol. I. p. 126, found the ſingle payments deduced from Mr. *Suſmilch*'s Table of mortality to be frequently too little, but the annual payments almoſt always too great. This is to be accounted for in the following manner:

The values of ſingle and joint lives are greater by the *Sweden* Table of mortality, than by either Mr. *Suſmilch*'s or the *Northampton* Table; and had they been greater in the ſame proportion, the *difference* between them, that is, the value in one preſent payment of an annuity for the life of a woman after her huſband, would have been nearly the ſame according to all the Tables; and conſequently this difference, divided by the *greater* value of the joint lives according to the *Sweden* Table, would have given a leſs quotient; that is, a leſs value of the annuity in annual payments. But the value of the ſingle female life being greater in *proportion* by the *Sweden* Table than that of the joint lives, the difference is increaſed, but not ſo much as to produce, when divided by the greater value of the joint lives, a quotient equal to that produced by dividing a ſmaller difference reſulting from the other Tables by a ſmaller value of the joint lives.

The *Daniſh* eſtabliſhment makes the annuities payable only during widowhood, and on this account makes an abatement in the contributions; but it is impoſſible to determine properly what this abatement ought to be.——It has, I have ſaid, the advantage of being guarantyed by the King of *Denmark* for all his dominions. It has alſo the following ſecurities. All the military and naval, and other officers who receive their pay from the King's treaſury, are obliged, when appointed, to give

In the HAMBURGH eſtabliſhment it appears, that, if money is improved at no higher rate than 3 *per cent.* the *ſingle* payments are almoſt always too low, but the *annual* payments ſometimes too high. With reſpect to the EQUITABLE Society, it appears, that on the ſame ſuppoſition of no higher improvement of money than at an intereſt of three *per cent.* the ſingle payments are generally too little, but the annual payments generally too high; and that when compared with the values at 4 *per cent.* and the difference of age is not very great, they are near a *third* or a *quarter* too high. It ſeems, therefore, that in thoſe caſes of ſurvivorſhip where there was moſt reaſon to ſuſpect, that the NORTHAMPTON Tables might give values unfavourable to the Society, it gives them ſufficiently high; and that conſequently, even in theſe caſes, there is no reaſon for continuing that addition of 15 *per cent.* to all the values which has

up to this fund one month of their pay; and all ſubſcribers are obliged at admiſſion to contribute 10 *per cent.* more than the payments in the Tables.—I will add, that the calculations for this eſtabliſhment, like thoſe for the *Hamburgh* and *Oldenberg* eſtabliſhments, have been made with ſuch pains and ability from Mr. *Suſmilch's* Table of mortality (in his *Gottliche Ordnung*, Vol. II. p. 319) by two of the firſt *Daniſh* mathematicians (Mr. *Lous*, Profeſſor of Mathematicks and Navigation in the Academy of *Sea Cadets*; and Mr. *Bugge*, Profeſſor of Aſtronomy in the Univerſity of *Copenhagen*; and both of them Fellows of the Royal *Daniſh* Academy of Sciences) that there is not the leaſt danger of its ſharing the ſame fate with a former *Daniſh* eſtabliſhment deſcribed in Vol. I. p. 122.

been

been ordered by the Society.——Upon the whole; I cannot help thinking that this Society ought once more to lower its demands, and to content itſelf with the advantage it derives from computing by the *Northampton* Tables at ſo low an intereſt as 3 *per cent.* without making any additional charge, except, perhaps, ſuch a ſmall charge as that propoſed in Vol. I. p. 176, towards bearing the expences of management *.

In order to prove this more fully, I will here add a compariſon, in a few inſtances, of the *premiums* (excluſive of the additional charge) required for aſſurances on ſingle lives by this Society, with the values of the ſame aſſurances deduced from the SWEDEN Tables.

Values of the Aſſurance of 100*l.* on a Single Life.——Intereſt 3 *per cent.*

Age.	For *one* year by *Sweden* Tables.			For *ſeven* years by *Sweden* Tables.			For the *whole* Life by *Sweden* Tables.		
	Equit. Society Payment.	Male Payment.	Female Payment.	Equit. Society Annual Paymt.	Male Annual Payment.	Female Annual Payment.	Equit Society Annual Paymt.	Male Annual Payment.	Female Annual Payment.
20	1.36	.87	.71	1.47	.92	.76	2.18	1.80	1.64
28	1.53	1.03	.98	1.68	1.13	1.11	2.55	2.20	2.03
35	1.81	1.22	1.16	1.93	1.32	1.21	3.06	2.85	2.44
44	2.27	1.87	1.64	2.46	2.00	1.60	3 78	3.65	3.24

It appears from hence, that without the charge of 15 *per cent.* and reckoning intereſt ſo low as 3 *per cent.* the *premiums* for

* See note p. 105.

Aſſurances on Single Lives required by the Equitable Society are, in many caſes, above a *third*, and, in general, above a *quarter* greater than the true values for *mankind at large*, deduced from the SWEDEN Obſervations. And yet ſuch is the temptation to bad lives to ſeek admiſſion, ſuch the uncertainty what the rate of mortality in the Society may in the end prove, and ſuch the neceſſity on theſe accounts (as has been before obſerved) of ſecuring the permanency of the Society by erring rather on the ſide of *exceſs* than *defect*, that theſe premiums, were no addition made to them, could not reaſonably be thought exorbitant.

In the laſt compariſon there are two circumſtances which may deſerve the notice of this Society.

The price in annual payments of the aſſurance of a female life at 28 for ſeven years is, according to the *Swediſh* Tables, almoſt equal to the price of the ſame aſſurance at 35. And at 44 the *annual* payment for *ſeven* years is leſs than the *ſingle* payment for aſſuring only the firſt of theſe 7 years. Theſe circumſtances, inſtead of being, as they may ſeem, the effect of errors in the *Swediſh* Tables, ſhew a correctneſs not to be found in any other tables. Females whoſe ages are between 27 and 36 conſiſt chiefly of *child-bearing* women; and though, taking

ing the whole duration of marriage, the lives of *married* women may (agreeably to Mr. *Muret*'s Obſervations in *Switzerland*, hereafter mentioned) be *leſs* hazardous than the lives of *ſingle* women, yet at theſe ages they may be *more* ſo; and particularly in great towns and poliſhed ſocieties, where abſurd cuſtoms, wrong management, and a pernicious delicacy, render an event dangerous which is naturally ſafe*. According to Mr. *Sujmilch*'s obſervations in *Germany*, one birth in a hundred produces the death of the mother; but in *London* the proportion is much higher. This ſuggeſts the true reaſon of the firſt of the circumſtances I have mentioned.——With reſpect to the other, it muſt be conſidered, that at 44 the critical period raiſes the value of the aſſurance of a female life; but recovering after this period particular firmneſs, an aſſurance for ſeven or eight years becomes leſs in *annual* value than an aſſurance for only one or two years. See p. 148.

In p. 171. of the preceding volume, an account has been given of the mortality among the perſons aſſured by the Society for 12 years to 1780. I can now add, that during 14 years to *January* 1782, the number aſſured (excluſive of aſſurances on ſurvivor-

* In the *Equitable Society*, though eſtabliſhed near 30 years, and aſſuring the lives of women at all ages, I do not know an inſtance (except two which happened laſt year) of a claim's having been produced by *child-birth*. ED.

ſhips for different ſums not exceeding 2000*l.* on any ſingle life) has been 12,391, and that of this number 9890 have been perſons under 50 years of age, among whom the deaths have been fewer, in the proportion of 3 to 4, than thoſe which ſhould have happened according to the *Northampton* Table of Obſervations *, and correſpond beſt at every age to the mortality exhibited in the *Sweden* Table. Of the remaining aſſurances, 1997 have been on ſingle lives between 50 and 60, among which the mortality, compared with that exhibited in the *Northampton* Table, has been as 9 to 10. There have been in the ſame period 504 aſſurances of perſons between 60 and 70, and among them the mortality has been nearly equal to that in the *Northampton* Table.——This great ſucceſs at the outſet of the inſtitution, has been particularly favourable to it, and muſt ſtrengthen it for all future time; but it would be wrong to rely on the continu-

* During the laſt 22 years, from Jan. 1768 to Jan. 1790, the number of aſſurances on ſingle lives has been 30,998, of which number 24,083 have been on the lives of perſons under 50 years of age, among whom the deaths have been fewer than thoſe in the *Northampton* Table in the proportion of 3 to 5. Between the ages of 50 and 60 the number of aſſurances on ſingle lives has been 5182 and compared with the *Northampton* Table the number of deaths has been as 3 to 4. Between 60 and 70 years of age, the number of aſſurances on ſingle lives has been 1733, and among them the decrements compared with thoſe in the *Northampton* Table have been in the ratio of 7 to 6 nearly.—See a further account of this Society in the Introduction at the beginning of the 1ſt volume. ED.

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ance of it. Seaſons of uncommon mortality muſt come; and the increaſing credit and numbers of the Society will, as I have before obſerved, increaſe the danger of the intruſion of bad lives.

TABLE XLIX.

Shewing the Probabilities of the Duration of Human Life at all Ages, in a Kingdom at large; deduced from Obſervations in the Kurmark of BRANDENBURGH; and formed on the Suppoſition that a *Third* of a Kingdom conſiſts of Inhabitants of Towns, and *Two Thirds* of the Inhabitants of Country Pariſhes and Villages. See Mr. *Suſmilch's* GOTTLICHE ORDNUNG, VOL. III. Tables p. 33.

Decrements of Life in the Kurmark of BRANDENBURGH.

Age.	A In *Berlin*, the capital.	B In the other Towns.	C In the Country Pariſhes and Villages.	D $\frac{A+B+4C}{6}$
Still-born	40	34	44	42
Under 1	254	194	187	199
1— 5	185	196	138	156
0— 5	479	424	369	397
5—10	40	61	59	56
10—15	10	17	24	20
15—20	16	17	22	20
0—20	545	519	474	493
20—25	34	18	28	27
25—30	46	25	25	29
30—35	37	24	26	28
35—40	49	40	32	36
40—45	36	31	33	33
45—50	37	42	36	37
50—55	38	47	40	41
55—60	42	58	55	53
20—60	319	285	275	284

TABLE. XLIX. continued.

Age.	A In *Berlin*, the capital.	B In the other Towns.	C In the Country Parishes and Villages.	D $\frac{A+B+4C}{6}$
60— 65	31	46	63	55
65— 70	32	56	61	55
70— 75	27	35	58	49
75— 80	23	32	34	32
80— 85	11	16	22	19
85— 90	7	8	8	8
90— 95	3	2	3	3
95—100	2	1	1	1
Above 100	0	0	1	1
60—100	136	196	251	223
	1000	1000	1000	1000

From Column D the following Table has been formed.

Born 10,000—Still-born 42.					
Age.	Living.	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
0	958	199	1 of $4\frac{3}{4}$	29877	30.68
1	759	70	1 of 11	28918	
2	689	38	1 of 18	28159	
3	651	26	1 of 25	27470	
4	625	22	1 of 28	26819	
5	603	19	1 of 32	26194	42.93
6	584	14	1 of 42	25591	
7	570	10	1 of 57	25007	
8	560	8	1 of 70	24437	
9	552	5	1 of 110	23877	
10	547	4	1 of 137	23325	42.14

TABLE XLIX. continued.

Age.	Living.	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
11	543	4	1 of 136	22778	
12	539	4	1 of 135	22235	
13	535	4	1 of 134	21696	
14	531	4	1 of 133	21161	
15	527	4	1 of 132	20630	38.64
16	523	4	1 of 131	20103	
17	519	4	1 of 130	19580	
18	515	4	1 of 129	19061	
19	511	4	1 of 128	18546	
20	507	5	1 of 101	18035	34.52
21	502	5	1 of 100	17528	
22	497	5	1 of 99	17026	
23	492	5	1 of 98	16529	
24	487	5	1 of 97	16037	
25	482	6	1 of 80	15550	31.76
26	476	6	1 of 79	15068	
27	470	6	1 of 78	14592	
28	464	6	1 of 77	14122	
29	458	6	1 of 76	13658	
30	452	6	1 of 75	13200	28.70
31	446	6	1 of 74	12748	
32	440	6	1 of 73	12302	
33	434	6	1 of 72	11862	
34	428	6	1 of 71	11428	
35	422	7	1 of 60	11000	25.56
36	415	7	1 of 59	10578	
37	408	7	1 of 58	10163	
38	401	7	1 of 57	9755	
39	394	7	1 of 56	9354	
40	387	7	1 of 55	8960	22.65

TABLE XLIX. continued.

Age.	Living.	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
41	380	7	1 of 54	8503	
42	373	7	1 of 53	8193	
43	366	7	1 of 52	7820	
44	359	7	1 of 51	7454	
45	352	7	1 of 50	7095	19.65
46	345	7	1 of 49	6743	
47	338	7	1 of 48	6398	
48	331	7	1 of 47	6060	
49	324	7	1 of 46	5729	
50	317	8	1 of 40.	5405	16.55
51	309	8	1 of 39	5088	
52	301	8	1 of 38	4779	
53	293	9	1 of 32	4478	
54	284	9	1 of 31	4185	
55	275	10	1 of 27	3901	13.68
56	265	10	1 of 26	3626	
57	255	10	1 of 25	3361	
58	245	11	1 of 22	3106	
59	234	11	1 of 21	2861	
60	223	11	1 of 20	2627	11.28
61	212	11	1 of 19	2404	
62	201	11	1 of 18	2192	
63	190	11	1 of 17	1991	
64	179	11	1 of 16	1801	
65	168	11	1 of 15	1622	9.15
66	157	11	1 of 14	1454	
67	146	11	1 of 13	1297	
68	135	11	1 of 12	1151	
69	124	11	1 of 11	1016	
70	113	10	1 of 11	892	7.48

TABLE XLIX. continued.

Age.	Living	Decrements.	Proportion dying annually.	Sum of all the Living.	Expectations.
71	103	10	1 of 10	779	
72	93	10	1 of 9	676	
73	83	10	1 of 8	583	
74	73	9	1 of 8	500	
75	64	8	1 of 8	427	6.17
76	56	7	1 of 8	363	
77	49	6	1 of 8	307	
78	43	6	1 of 7	258	
79	37	5	1 of 7	215	
80	32	5	1 of 6	178	5.06
81	27	4	1 of 7	146	
82	23	4	1 of 6	119	
83	19	3	1 of 6	96	
84	16	3	1 of 5	77	
85	13	2	1 of 6	61	4.18
86	11	2	1 of 5	48	
87	9	2	1 of 4	37	
88	7	1	1 of 7	28	
89	6	1	1 of 6	21	
90	5	1	1 of 5	15	
91	4	1	1 of 4	10	
93	3	1			
95	2	1			
100	1	1			

REMARKS.

THIS Table is the ſame with that publiſhed in the laſt edition of Mr. *Suſmilch's Gottliche*

Gottliche Ordnung, with the addition of the *Expectations*, and an alteration in the arrangement of the number of the ſtill-born, which I have placed by itſelf, and deduced from the *whole* number born, in order to make the number *born alive* the *radix* of the Table.

This Table, it ſhould be further obſerved, has been formed without any regard to the correction explained in the Fourth Eſſay in the former Volume of this work; and, on this account, (as far as it has been deduced from the numbers dying at every age in the towns of *Brandenburg*) makes the probabilities of living too high in the firſt ſtages of life. But it ſhould be likewiſe attended to, that on *another* account, it makes them in a much greater proportion too low. I mean, on account of the great exceſs of the births above the burials in the country pariſhes and villages. The effect of ſuch an exceſs may be learnt from what is ſaid in p. 9, &c. of the Introduction to theſe Tables.

There is another Table of the probabilities of living at every age in a kingdom at large, in the Second Volume of Mr. *Suſmilch*'s *Gottliche Ordnung*, p. 319, which has been made the *baſis* of all the computations in GERMANY of the values of payments dependent on lives. This is the Table referred to in p. 189, and in the Note p. 193. It differs but little from this Table; and is liable

liable to the ſame objections. I muſt add, that the like is true of a table formed with the ſame view, and on the ſame principles, by Mr. FLORENCOURT, the ingenious author of a Mathematical Treatiſe on Political Arithmetick, publiſhed in *Germany*, in 1781.

Having occaſion to mention theſe two writers, I cannot help adding with regret, that being ignorant of the *German* language, I have found myſelf incapable of profiting by their works in the manner I wiſh.

In Tables 12th, 13th, 20th, 21ſt, and 24th, at the end of the Second Volume of Mr. SUSMILCH's *Gottliche Ordnung*, the decrements of life at all ages are given ſeparately for males and females in BERLIN for 14 years; in the pariſh of St. *Sulpice*, PARIS, for 30 years; and in ſeveral country pariſhes and villages in BRANDENBURGH for different periods of years. Theſe decrements are ſo far from giving a juſt repreſentation of human mortality, that a table of *obſervations* deduced from them would neceſſarily be very erroneous. They confirm, however, the difference in favour of females exhibited in the four preceding Tables; and therefore it will not be improper to inſert a ſummary of them.

DECRE-

DECREMENTS of Life.

Age.	In St. SULPICE Pariſh.		In BERLIN.		Country Pariſhes in BRANDENBURGH.	
	Males.	Females.	Males.	Females.	Males.	Females.
Still-born			360	253	45	39
1	5718	4615	2758	2370	420	383
1— 5	5925	6093	843	847	276	246
5—10	1597	1536	211	215	120	110
10—20	789	749	196	205	87	72
20—30	1293	1337	709	493	126	97
*30—45	2207	2315	1052	796	166	168
45—60	2026	2442	1023	746	280	234
60—70	1768	2177	443	506	237	207
70—80	1453	3505	337	417	148	183
80—90	648	1673	114	160	68	48
90—95	28	101	11	29	8	8
95—100	19	72	9	22	2	1
Above 100	0	0	1	4	7	2
Totals	24071	24467	8067	7063	1990	1798
*30—40			725	582	102	124
40—50			652	445	151	103
50—60			698	515	193	175

The decrements in the country pariſhes in *Brandenburgh* are too great in the firſt ſtages of life on account of the exceſs of the births above the burials, the former having been, in ſome of theſe pariſhes, more

more than *double* the latter. The decrements in *Berlin*, on the contrary, are too ſmall, for reaſons ſufficiently explained in the courſe of this work; but in the pariſh of St. *Sulpice*, *Paris*, they are particularly erroneous, for the reaſons mentioned in the Poſtſcript to the Firſt Eſſay, Vol. I. p. 291, 292.

THERE have been now given in this collection, tables of the duration and values of human life in great cities, in moderate towns, in country villages and pariſhes, and among the inhabitants of a whole kingdom, conſiſting of all *country* as well as *town* inhabitants. The accounts which have been given of the *data* from which they have been formed, and of the method of forming them, ſhew how far they are to be reckoned juſt repreſentations of the duration and values of lives in the different ſituations I have mentioned. But there is one remark which is applicable to all of them; and that is, that having been formed from obſervations on whole bodies of people of all ages and conditions, they cannot give a correct repreſentation of the duration and values of ſuch lives as form a body of *ſtate* annuitants, or of perſons on whoſe lives annuities have been purchaſed to commence either immediately or at any given future year. The reaſon is obvious. Such a body of annuitants are likely to conſiſt of a *ſelection* of the beſt lives from the common maſs; the intereſt of every perſon who purchaſes an annuity on any life requiring that he ſhould take care that it is a good life (*a*). Tables of mortality for ſuch lives

(*a*) The following account of the life-annuities ſold by our government, will, in ſome meaſure, prove the truth

lives have been publiſhed by Mr. *De Parcieux*, in *France*, from the liſts of the *French Tontines*; and by Mr. *Kerſſeboom*, in *Holland*, from ſome regiſters of *Dutch* annuitants. That nothing on this ſubject may be wanting which I am able to furniſh, I ſhall here inſert theſe Tables, with the addition of the expectations of life for every fifth year, according to each of them.

of this obſervation.———There were granted in 1745, 22,500 *l. per ann.* In *January*, 1782, they were reduced by deaths to 13,104 *l.* which is a reduction of two-fifths in 36 years, and a ſlower decreaſe than the higheſt of the preceding Tables of mortality ſhew in the ſame time among bodies of people, all 30 years of age. The ſame is true of the annuities ſold in 1746, which, in *Jan.* 1782, were reduced from 45,000 *l.* (their original amount) to 24,400 *l.* But the decreaſe has been ſloweſt in the annuities granted in 1757, which, in *Jan.* 1782, had fallen from 33,750 *l.* to 27,069 *l.*; that is, only a *fifth* in 24 years.

TABLE

TABLE L.

Shewing the Decrements and Expectations of Life among Bodies of Life-Annuitants, according to the Tables of Mortality publiſhed by Mr. *Kerſſeboom*, and by Mr. *De Parcieux* (*a*).

Age.	By Mr. KERSSEBOOM. Living.	Decr.	Expectat.	By Mr. DE PARCIEUX. Living.	Decr.	Expectat.
0	1400	275	34.47	10000	2550	34.79
1	1125	50	41.77	7450	362	45.52
2	1075	45	42.69	7088	265	46.82
3	1030	37	43.53	6823	205	47.62
4	993	29	44.14	6618	150	48.09
5	964	27	44.45	6468	123	48.19
6	947	17		6345	102	
7	930	17		6243	91	
8	913	9		6154	81	
9	904	9		6073	69	
10	895	9	42.71	6004	58	46.76
11	886	8		5946	49	
12	878	8		5897	43	
13	870	7		5854	39	
14	863	7		5815	37	

(*a*) The copy here given of Mr. *De Parcieux*'s Table is not that publiſhed by Mr. *De Moivre* at the end of his Book on the *Doctrine of Chances*; and by Mr. *Ferguſon* in his *Tables and Tracts*, &c. p. 289; but an improved copy publiſhed by Mr. *Florencourt* in *Germany*, at the end of his Treatiſe on *Political Arithmetick*.

A compariſon of the expectations will ſhew a conſiderable difference between this Table and Mr. *Kerſſeboom*'s; and one reaſon of this difference may be, that Mr. *Kerſſeboom*'s Table has been formed *partly* from obſervations on the mortality of the inhabitants of ſome *Dutch* villages.

TABLE L. continued.

Age.	By Mr. Kersseboom. Living.	Decr.	Expectat.	By Mr. De Parcieux. Living.	Decr.	Expectat.
15	856	7	39·55	5778	38	43·46
16	849	7		5740	41	
17	842	7		5699	44	
18	835	9		5655	47	
19	826	9		5608	50	
20	817	9	36.31	5558	52	40.08
21	808	8		5506	53	
22	800	8		5453	54	
23	792	9		5399	55	
24	783	11		5344	56	
25	772	12	33.27	5288	57	37.01
26	760	13		5231	58	
27	747	12		5173	57	
28	735	12		5116	56	
29	723	12		5060	55	
30	711	12	30.92	5005	54	33.96
31	699	12		4951	54	
32	687	12		4897	53	
33	675	10		4844	52	
34	665	10		4792	52	
35	655	10	28.36	4740	52	30.73
36	645	10		4688	51	
37	635	10		4637	49	
38	625	10		4587	49	
39	615	10		4538	48	
40	605	9	25.49	4490	49	27·30
41	596	9		4441	49	
42	587	9		4392	50	

TABLE L. continued.

Age.	By Mr. Kersseboom. Living.	Decr.	Expectat.	By Mr. De Parcieux. Living.	Decr.	Expectat.
43	578	9		4342	51	
44	569	9		4291	52	
45	560	10	22.34	4239	53	23.77
46	550	10		4186	54	
47	540	10		4132	55	
48	530	12		4077	56	
49	518	11		4021	57	
50	507	12	19.41	3964	59	20.24
51	495	13		3905	62	
52	482	12		3843	66	
53	470	12		3777	70	
54	458	12		3707	76	
55	446	12	16.72	3631	81	16.88
56	434	13		3550	85	
57	421	13		3465	88	
58	408	13		3377	91	
59	395	13		3286	95	
60	382	13	14.10	3191	99	13.86
61	369	13		3092	102	
62	356	13		2990	105	
63	343	14		2885	107	
64	329	14		2778	109	
65	315	14	11.56	2669	110	11.07
66	301	14		2559	111	
67	287	14		2448	112	
68	273	14		2336	113	
69	259	14		2223	114	
70	245	14	9.15	2109	116	8.34
71	231	14		1993	119	

TABLE L, continued.

Age.	By Mr. Kerseboom. Living.	Decr.	Expectat.	By Mr. De Parcieux. Living.	Decr.	Expectat.
72	217	14		1874	125	
73	203	14		1749	132	
74	189	14		1617	138	
75	175	15	6,81	1479	142	5·79
76	160	15		1337	139	
77	145	15		1198	134	
78	130	15		1064	128	
79	115	15		936	124	
80	100	13	5.05	812	115	4·73
81	87	12		697	107	
82	75	11		590	98	
83	64	9		492	88	
84	55	10		404	77	
85	45	9	3·38	327	66	3·45
86	36	8		261	55	
87	28	7		206	47	
88	21	6		159	42	
89	15	5		117	37	
90	10	3	2.47	80	30	1·79
91	7	2		50	22	
92	5	2		28	14	
93	3	1		14	8	
94	2	1,		6	3	
95	1			3	2	
96	0.6			1	1	
97	0.5			0	0	
98	0.4					
99	0.2					
100	0.0					

TABLE LI.

Shewing the Values of Single Lives according to the Probabilities of the Duration of Life in Mr. *De Parcieux*'s Table of Mortality.——See Mr. *Florencourt*'s Diſſertations on Political Arithmetick, p. 288.

Intereſt 5 *per cent.*

Age.	Value.	Age.	Value.	Age.	Value.	Age.	Value.
0	11.083	26	15.040	52	10.926	78	3.953
1	14.620	27	14.969	53	10.673	79	3.719
2	15.135	28	14.893	54	10.418	80	3.501
3	15.509	29	14.810	55	10.168	81	3.283
4	15.750	30	14.722	56	9.930	82	3.072
5	15.924	31	14.627	57	9.682	83	2.868
6	16.041	32	14.527	58	9.431	84	2.668
7	16.118	33	14.421	59	9.177	85	2.461
8	16.169	34	14.306	60	8.923	86	2.237
9	16.204	35	14.189	61	8.669	87	1.976
10	16.210	36	14.065	62	8.413	88	1.688
11	16.194	37	13.930	63	8.155	89	1.409
12	16.145	38	13.786	64	7.893	90	1.164
13	16.077	39	13.632	65	7.626		
14	15.994	40	13.466	66	7.351		
15	15.901	41	13.296	67	7.069		
16	15.807	42	13.116	68	6.778		
17	15.716	43	12.931	69	6.479		
18	15.631	44	12.738	70	6.171		
19	15.550	45	12.539	71	5.856		
20	15.474	46	12.333	72	5.540		
21	15.401	47	12.119	73	5.232		
22	15.328	48	11.897	74	4.942		
23	15.256	49	11.666	75	4.674		
24	15.184	50	11.425	76	4.429		
25	15.112	51	11.178	77	4.190		

From the values in this Table at 5 *per cent.* the values at all other rates of intereſt may be eaſily found by the rule in p. 170. But I am very happy that, on this occaſion, I can inform the public, that complete tables of the values of *ſingle* lives, deduced with perfect correctneſs (from the copy of Mr. *De Parcieux*'s Table of Mortality at the end of Mr. *De Moivre*'s Doctrine of Chances) for every rate of intereſt from 2 to 10 *per cent.* and alſo of *two joint* lives at $3\frac{1}{2}$ and $4\frac{1}{2}$ *per cent.* have been publiſhed by Mr. *Maſeres*, Curſitor Baron of the Exchequer, in a work on the principles of the doctrine of life-annuities.——To this work the ingenious author has added many calculations on the beſt means of redeeming the public debts; and I wiſh his name and abilities may be the means of engaging the attention of the kingdom effectually to this moſt important object.

IN p. 118, Vol. I. a ſcheme has been mentioned for providing for the Widows and Orphans of the Clergy within the Dioceſe of *Exeter*, and which the Reverend Mr. *Gandy* of *Plymouth*, had, with great public ſpirit, but without ſucceſs, endeavoured to carry into execution.

Much time and pains were employed in computing the neceſſary tables for this ſcheme; and as it is poſſible that in ſome future time they may be ſtill of uſe, I ſhall here inſert the chief of them.

TABLE LII.

Shewing the Values in Annual Payments during the Joint Lives (firſt Payment to be made at Admiſſion), and alſo in Single Payments, of a Life-Annuity of 10*l.* to be entered upon by a Wife at the Death of her Huſband.

INTEREST reckoned at 4 *per cent.*

Huſband's age.	Annual payment, ſuppoſing equal ages	Addition to annual payment for each year the age of the huſband exceeds the wife's	Equivalent compoſition, or ſingle payment	Addition to the ſingle payment for each year the huſband's age exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
25 or leſs	2:15:0	0:10	35:6:0	0:14:0
26	2:15:0	0:10	35:5:0	0:14:0
27	2:15:6	0:10	35:4:0	0:14:0
28	2:16:0	0:10	35:3:0	0:14:0
29	2:16:6	0:10	35:2:0	0:14:0
30	2:17:6	0:10	35:0:0	0:14:0
31	2:18:0	0:10	34:18:0	0:14:0

TABLE LII, continued.

Husband's age.	Annual payment, supposing equal ages.	Additional annual payments for each year the age of the husband exceeds the wife's	Single payment, supposing the annual excused.	Addition to the single payment for each year the age of the husband exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
32	2:18:6	0 : 11	34:16:0	0:14:6
33	2:19:0	0 : 11	34:14:0	0:15:0
34	2:19:6	1 : 0	34:12:0	0:15:6
35	3: 0:0	1 : 0	34:10:0	0:16:0
36	3: 0:0	1 : 0	34: 8:0	0:16:0
37	3: 0:6	1 : 1	34: 5:0	0:16:6
38	3: 1:0	1 : 1	34: 2:0	0:17:0
39	3: 1:6	1 : 2	33:18:0	0:17:6
40	3: 2:0	1 : 2	33:14:0	0:18:0
41	3: 2:6	1 : 3	33:10:0	0:18:6
42	3: 3:0	1 : 4	33: 6:0	0:19:0
43	3: 3:6	1 : 5	33: 2:0	0:19:6
44	3: 4:0	1 : 6	32:17:0	1: 0:0
45	3: 4:6	1 : 7	32:12:0	1: 1:0
46	3: 5:0	1 : 8	32: 6:0	1: 1:6
47	3: 5:6	1 : 9	32: 0:0	1: 2:0
48	3: 6:0	1 : 10	31:14:0	1: 2:6
49	3: 6:6	1 : 11	31: 8:0	1: 3:0
50	3: 7:0	2 : 0	31: 2:0	1: 3:6
51	3: 7:6	2 : 1	30:16:0	1: 4:0
52	3: 8:0	2 : 2	30: 9:0	1: 5:0
53	3: 8:6	2 : 3	30: 1:0	1: 6:0
54	3: 9:0	2 : 4	29:12:0	1: 7:0
55	3:10:0	2 : 6	29: 3:0	1: 8:0
56	3:10:6	2 : 7	28:14:0	1: 9:0
57	3:11:0	2 : 9	28: 4:0	1:10:0
58	3:11:6	2 : 11	27:14:0	1:11:0
59	3:12:0	3 : 1	27: 4:0	1:12:0
60	3:12:6	3 : 3	26:13:0	1:13:0
61	3:13:0	3 : 5	26: 2:0	1:14:0

In

In calculating this Table, the values of ſingle and joint lives were taken from the two Tables at the end of this volume, which were then reckoned the beſt guides. But a compariſon of theſe values, with thoſe in Table XLVIII. p. 186, will ſhew they want correction; and, particularly, that though when the ages of huſbands and wives are under 40, and nearly equal, the values in this Table are a little too high; yet, in other caſes, they are below, and, in ſome caſes, *much* below the proper values.

TABLE LIII.

Shewing the Values of a Life-Annuity of 5*l.* payable to a Wife after her Huſband, provided he lives *three* Years from the Time of purchaſing; and of an additional Annuity of 5*l.* provided he lives *five* Years from the Time of purchaſing.

INTEREST 4 *per Cent.*

Age.	Annual payment, ſuppoſing equal ages.	Additional annual payment for each year the age of the huſband exceeds the wife's	Single payment, ſuppoſing the annual excuſed.	Addition to the ſingle payment for each year the age of the huſband exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
25 or leſs	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
26	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
27	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
28	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
29	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
30	2 : 2 : 6	0 : 9	25 : 13 : 6	0 : 11 : 10
31	2 : 2 : 6	0 : 9	25 : 9 : 6	0 : 12 : 1
32	2 : 2 : 6	0 : 9	25 : 5 : 6	0 : 12 : 5
33	2 : 3 : 0	0 : 10	25 : 2 : 6	0 : 12 : 9
34	2 : 3 : 0	0 : 10	24 : 18 : 6	0 : 13 : 0
35	2 : 3 : 0	0 : 11	24 : 15 : 0	0 : 13 : 4
36	2 : 3 : 6	0 : 11	24 : 10 : 6	0 : 13 : 9
37	2 : 3 : 6	1 : 0	24 : 5 : 6	0 : 14 : 1
38	2 : 3 : 6	1 : 0	24 : 1 : 0	0 : 14 : 6
39	2 : 4 : 0	1 : 1	23 : 16 : 6	0 : 14 : 10
40	2 : 4 : 0	1 : 1	23 : 11 : 6	0 : 15 : 2
41	2 : 4 : 0	1 : 2	23 : 5 : 6	0 : 15 : 7
42	2 : 4 : 0	1 : 2	22 : 19 : 6	0 : 15 : 11
43	2 : 4 : 0	1 : 3	22 : 13 : 6	0 : 16 : 4
44	2 : 4 : 0	1 : 4	22 : 7 : 6	0 : 16 : 10
45	2 : 4 : 0	1 : 4	22 : 11 : 6	0 : 17 : 3
46	2 : 4 : 0	1 : 5	21 : 13 : 0	0 : 17 : 9

TABLE LIII. continued.

Age.	Annual payment, supposing equal ages.	Additional annual payment for each year the age of the husband exceeds the wife's.	Single payment, supposing the annual excused.	Addition to the single payment for each year the age of the husband exceeds the wife's.
	£. s. d.	s. d.	£. s. d.	£. s. d.
47	2: 4:0	1 : 6	21: 4:6	0:18: 3
48	2: 4:0	1 : 7	20:16:6	0:18:10
49	2: 4:0	1 : 8	20: 8:6	0:19: 3
50	2: 3:6	1 : 9	20: 0:6	0:19: 9
51	2: 3:6	1 : 10	19:11:6	1: 0: 2
52	2: 3:6	1 : 11	19: 2:6	1: 0: 8
53	2: 3:6	2 : 0	18:15:0	1: 1: 0
54	2: 3:6	2 : 0	18: 7:0	1: 1: 6
55	2: 3:0	2 : 1	17:18:6	1: 1:11
56	2: 3:0	2 : 2	17: 7:6	1: 2: 5
57	2: 2:6	2 : 3	16:16:6	1: 2:11
58	2: 2:0	2 : 5	16: 5:6	1: 3: 6
59	2: 2:0	2 : 7	15:14:6	1: 4: 0
60	2: 1:6	2 : 8	15: 3:6	1: 4: 6
61	2: 0:6	2 : 10	14: 8:6	1: 5: 2
62	1:19:6	3 : 0	13:14:0	1: 5:10

This Table has been computed by the Rule in Queſt. VII. Vol. I. p. 22, taking the probabilties of the duration of life as they are in Table V. p. 35; and the values of ſingle and joint lives as they are in the two Tables at the end of this Volume. The correct and legitimate Table would be a Table computed by the ſame rule from the SWEDEN Tables in this collection.

TABLE

TABLE LIV.

Shewing the Values of 100*l.* payable to ſuch Children, *under Age*, of a *married* Man, as ſhall happen to be living at the Time of his Deceaſe, provided he leaves no Widow.

INTEREST 4 *per cent.*

Age.	Annual payment during life.	Single payment, ſuppoſing the annual excuſed.	Age.	Annual payment during life.	Single payment, ſuppoſing the annual excuſed.
	l. *s.* *d.*	*l.* *s.* *d.*		*l.* *s.* *d.*	*l.* *s.* *d.*
25	0:10:0	8: 0:0	47	1: 3:6	14:18:0
26	0:11:0	8:10:0	48	1: 4:6	15: 6:0
27	0:11:6	9: 1:0	49	1: 5:6	15:15:0
28	0:12:0	9:10:0	50	1: 6:6	16: 4:0
29	0:12:6	9:18:0	51	1: 7:6	16:12:0
30	0:13:6	10: 6:0	52	1: 8:6	17: 0:0
31	0:14:0	10:14:0	53	1: 9:6	17: 8:0
32	0:14:6	11: 0:0	54	1:11:0	17:16:0
33	0:15:0	11: 5:0	55	1:12:6	18: 4:0
34	0:15:6	11: 9:0	56	1:13:6	18:13:0
35	0:15:6	11:13:0	57	1:15:0	19: 3:0
36	0:16:0	11:19:0	58	1:16:6	19:13:0
37	0:17:0	12: 4:0	59	1:18:6	20: 3:0
38	0:17:6	12:10:0	60	2: 0:6	20:13:0
39	0:18:0	12:15:0	61	2: 2:6	21: 3:0
40	0:18:6	13: 0:0	62	2: 5:0	21:13:0
41	0:19:0	13: 5:0			
42	0:19:6	13:10:0			
43	1: 0:0	13:15:0			
44	1: 1:0	14: 0:0			
45	1: 1:6	14: 5:0			
46	1: 2:6	14:11:0			

METHOD

METHOD of CALCULATION.

LET the age be reckoned 35.——The value (intereſt being at 4 *per cent.*) of 100 *l.* payable at the death of a perſon aged 35, provided he ſurvives another perſon of the ſame age, is *l.* 14.55, by Mr. *Simpſon*'s Problem quoted in Queſtion XII. Vol. I. p. 39, and by the correction explained in Vol. I. p. 34 and 62: deducing the values of the longeſt of the two lives from the two Tables at the end of this volume, by the rule in p. 79.

This gives the value ſought for this Table, on the ſuppoſition that it is *certain*, that a married man will at his death leave children under age. If one *tenth* of thoſe who die widowers leave either no children, or none under age, then this value muſt be diminiſhed, on that account, one tenth. And if, beſides, one in five of all who are left widowers marry a ſecond time wives not older than themſelves, one *half* at leaſt of whom, (that is. one *tenth* of all that are left widowers) muſt be reckoned to die in a 2d or 3d marriage; then the ſame value muſt be diminiſhed again another tenth; that is, a *fifth* in all; and this will make it *l.* 11.64, (or 11*l.* 13*s.* nearly) which is the value in a *ſingle payment* given in the Table.——Divide *l.* 11.64 by 14.98 (the value

value increaſed by unity of a life aged 25 by Table I. at the end of this Volume) and the quotient will be .777 (or 15*s.* 6*d.*) which is the value in annual payments during the ſingle life, the firſt payment to be made immediately.

In this Table no allowance has been made for the inequality of age between a man and his wife, and for the chances of ſurvivorſhip being, on this and other accounts, much againſt him in marriage. The values in it, therefore, are probably much too high.

Had the value juſt determined been deduced from the *Sweden* Tables for males and females taken collectively, it would have been in the ſingle payment 10*l.* 16*s.*; in the annual payment 13*s.* 7*d.*——Had the wife been reckoned 29 (the huſband being 35), it would have been in the ſingle payment 9*l.* 4*s.* 6*d.*; in the annual payment 11*s.* 7*d.* ———A ſociety, therefore, for relieving orphans on this plan, might ſafely adopt lower payments than thoſe in this Table; nor would there be any danger from the admiſſion of bad lives.

TABLE

TABLE LV.

Shewing the present Value of an Annuity of 10*l.* for five Years; 20*l.* for the next succeeding five Years; and 30*l.* for the whole of Life after Ten Years; payable *quarterly*; and to commence at FIFTY-FIVE Years of Age.—See the Reference to this and the following Table in Vol. I. p. 144.

Age of the purchaser.	Value of the annuity in one present payment.			Value of the annuity in annual payments, to be continued till 55, the 1st payment to be made immediately.		
	£.		s.	£.		s.
20	38	:	6	2	:	4
21	40	:	7	2	:	7
22	42	:	8	2	:	10
23	44	:	9	2	:	13
24	46	:	11	2	:	16
25	48	:	13	3	:	0
26	51	:	3	3	:	4
27	53	:	14	3	:	8
28	56	:	6	3	:	13
29	58	:	18	3	:	18
30	61	:	11	4	:	4
31	64	:	16	4	:	11
32	68	:	1	4	:	18
33	71	:	7	5	:	5
34	74	:	13	5	:	13
35	78	:	0	6	:	1
36	81	:	16	6	:	11
37	85	:	12	7	:	2
38	89	:	9	7	:	13
39	94	:	0	8	:	6
40	98	:	11	9	:	0
41	103	:	16	10	:	0
42	109	:	0	11	:	0
43	114	:	4	12	:	3
44	121	:	0	13	:	13
45	128	:	8	15	:	9

TABLE LVI.

Shewing the Values of an Annuity of 10*l.* for five Years; 20*l.* for the next ſucceeding Five Years; and 30*l.* for the whole of Life after Ten Years; payable *quarterly*, and to commence at SIXTY Years of Age.——See Vol. I. p. 144.

Age of the purchaſer.	Value of the annuity in one preſent payment.			Value of the annuity in annual payments, to be continued till the age of 60, the firſt payment to be made immediately.		
	£.		ſ.	£.		ſ.
20	22	:	13	1	:	5
21	23	:	18	1	:	6
22	25	:	3	1	:	8
23	26	:	8	1	:	10
24	27	:	13	1	:	12
25	28	:	19	1	:	14
26	30	:	10	1	:	16
27	32	:	2	1	:	18
28	33	:	13	2	:	0
29	35	:	4	2	:	3
30	36	:	18	2	:	6
31	38	:	12	2	:	9
32	40	:	8	2	:	12
33	42	:	5	2	:	15
34	44	:	2	2	:	19
35	46	:	0	3	:	3
36	48	:	10	3	:	8
37	51	:	0	3	:	13
38	53	:	10	3	:	19
39	56	:	5	4	:	5
40	59	:	0	4	:	12
41	61	:	10	5	:	0
42	64	:	10	5	:	8
43	68	:	0	5	:	18
44	72	:	10	6	:	14
45	77	:	0	7	:	10
46	81	:	10	8	:	4
47	86	:	0	9	:	0
48	90	:	10	9	:	16
49	96	:	0	11	:	0
50	102	:	0	12	:	10

Theſe two laſt Tables have been calculated by the rules in Vol. I. p. 17, 18, &c.

The probabilities of the duration of life have been ſuppoſed *nearly* the ſame with thoſe in the *Northampton* Table of mortality.

The intereſt of money has been reckoned at 3 *per cent.*; and it muſt be further remembered, that the values in *each* of the 2d and 3d columns are the *whole* values.

ACCOUNT of the Values of the Renewal of Leaſes, and of the Method of computing them.

TABLE LVII.

Shewing the Fines due on the Renewal of a Leaſe of 21 Years after 5, 7, 9, or 11 Years have elapſed.

Years unexpired.	Values of the Renewal. At 4 per cent.	At 6 per cent.	At 8 per cent.	At 10 per cent.
16	$2\frac{39}{100}$ } Years purchaſe.	$1\frac{66}{100}$ } Years purchaſe.	$1\frac{16}{100}$ } Years purchaſe.	$\frac{82}{100}$ } Years purchaſe.
14	$3\frac{46}{100}$	$2\frac{47}{100}$	$1\frac{77}{100}$	$1\frac{28}{100}$
12	$4\frac{64}{100}$	$3\frac{38}{100}$	$2\frac{48}{100}$	$1\frac{83}{100}$
10	$5\frac{92}{100}$	$4\frac{2}{3}$	$3\frac{3}{10}$	$2\frac{1}{2}$

The value in every caſe of this kind is the difference between the value (in Tables II. and LIX.) of the whole term, and the value (in the ſame Tables) of the unexpired part of the term.

If leaſes are held by lives (the value of their renewal is the difference between the value of all the lives (including the life or lives to be added) and the value of the exiſting life or lives.——For example.

The value of the renewal of a leaſe held by two lives after one has dropped is (ſuppoſing the exiſting life a male life aged 50, and the life to be added a female life aged 20) the *difference* between 18.575 (the value by

by Table XLVII. and the rule in p. 79, of the longeſt of the two lives) and 11.267 the value by Table XLV. of a ſingle male life aged 50. That is, 7.308, or $7\frac{3}{10}$ of a year's purchaſe nearly, reckoning intereſt at 4 *per cent.*——Again, the value of the renewal of a leaſe held by *three* lives, after *one* has dropped, is (ſuppoſing the two exiſting lives aged 50 and 56, and the life to be added aged 20) the *difference* between 19.537 (the value of the longeſt of the three lives by the column for lives in general in Table XLV. and by Tables XLVI. and XLVII. and the rule in p. 97) and 13.809 (the value by the ſame Tables and the Rule in p. 79, of the longeſt of two lives aged 50 and 56). This difference is 5.728, or $5\frac{3}{4}$ years purchaſe; which, therefore, is the fine due for ſuch a renewal, reckoning intereſt at 4 *per cent.*

N. B. If the values of ſuch renewals are wanted at any rates of intereſt higher or lower than thoſe for which the values of ſingle and joint lives are given in the preceding Tables, they muſt be deduced from the values given in the Tables by the Rules in p. 170.

It would be an *endleſs* labour to compute tables ſhewing the value of ſuch renewals in all caſes; and theſe directions render it an *unneceſſary* labour.

Sometimes a right may be purchaſed to put in, on the firſt vacancy among the lives by which an eſtate is held, ſuch a new life as the purchaſer ſhall chuſe.—In order to find the preſent value of ſuch a right, it is neceſſary to aſſume ſome given value for the life to be nominated, and this aſſumed value multiplied by the difference between the value of the exiſting life, if there is but one (or the value of the joint continuance of the exiſting lives, if there are two or more) and the perpetuity; and the product, divided by the perpetuity, will give the anſwer.

EXAMPLE.

Let there be but one exiſting life, and let it be a male life, its age 50, and conſequently its value (by Table XLV. p. 162.) 10.298, reckoning intereſt at 5 *per cent.*—Let the life to ſucceed it be reckoned a life of the greateſt poſſible value, that is, a female life aged 9, and conſequently worth (by Table XLV.) 16.343 year's purchaſe at 5 *per cent.*—The difference between 20 (the perpetuity) and 10.298 multiplied by 16.343, is 158.54; which product, divided by 20, gives 7.927, the anſwer.

If there are two exiſting lives, one male and the other female, and both 50, the value of their joint continuance will be (by Table XLVI. p. 165) 8.707; the difference between

between which value and the perpetuity is 11.293, which multiplied by 16,342, and the product divided by the perpetuity, gives 7.114 the anſwer in this caſe, or the number of years purchaſe which ought to be paid for a right of reneving a leaſe now held by two lives both aged 50, by putting in the beſt life in the room of the firſt of the two lives that ſhall happen to drop.

The rule for finding the value is the ſame, if the right to be ſold is the right of preſentation to a church living at the death of the preſent incumbent.

The eſtate meant in theſe rules is the nett *ſurplus* rent after deducting all taxes and repairs.

TABLE LVIII.

The preſent Value of 1 *l.* to be received at the End of any Number of years not exceeding 100, at the Rates of 2, 2½, 7, 8, 9, and 10 *per cent.* Compound Intereſt; being a Supplement to Table I. p. 18.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	.980392	.975609	.934579	.925925	.917431	.909090
2	.961168	.951814	.873438	.857338	.841679	.826446
3	.942322	.928599	.816297	.793832	.772183	.751314
4	.923845	.905595	.762895	.735029	.708425	.683013
5	.905730	.883854	.712986	.680583	.649931	.620921
6	.887971	862296	.666342	.630169	.596267	.564473
7	.870560	.841265	.622749	.583490	.547034	.513158
8	.853490	.820746	.582009	.540268	.501866	.466507
9	.836755	.800728	.543933	.500248	.460427	.424097
10	.820348	.781198	.508349	.463193	.422410	.385543
11	.804263	.762144	.475092	.428882	.387532	.350493
12	.788493	.743555	.444011	.397113	.355534	.318630
13	.773032	.725420	.414964	.367697	.326178	.289664
14	.757875	.707727	.387817	.340461	.299246	.263331
15	.743014	.690465	.362446	.315241	.274538	.239392
16	.728445	.673624	.338734	.291890	.251869	.217629
17	.714162	.657195	.316574	.270268	.231073	.197844
18	.700159	.641165	.295863	.250249	.211993	.179858
19	.686430	.625527	.276508	.231712	.194489	.163508
20	.672971	.610270	.258419	.214548	.178430	.148643
21	.659775	.595386	.241513	.198655	.163698	.135130
22	.646839	.580864	.225713	.183940	.150181	.122845
23	.634155	.566697	.210946	.170315	.137781	.111678
24	.621721	.552875	.197146	.157699	.126404	.101525
25	.609530	.539390	.184249	.146017	.115967	.092296
26	.597579	.526234	.172195	.135201	.106392	.083905
27	.585862	.513399	.160930	.125186	.097607	.076277
28	.574374	.500877	.150402	.115913	.089548	.069343
29	.563112	.488661	.140562	.107327	082154	.063039
30	.552070	.476742	.131367	.099377	.075371	.057308
31	.541245	.465114	.122773	.092016	.069147	.052098
32	.530633	.453770	.114741	.085200	.063438	.047362
33	.520228	.44270	.107234	.078888	.058200	.043056
34	.510028	.431905	.100219	.073045	053394	.039142

TABLE LVIII. continued.

Years	2 per cent.	2½ per cent	7 per cent.	8 per cent.	9 per cent.	10 per cent.
35	·530027	·421371	·093662	·067634	·048986	·035584
36	·490223	·411093	·087535	·062624	·044941	·032349
37	·480610	·401067	·081808	·057985	·041230	·029408
38	·471187	·391284	·076456	·053690	·037826	·026734
39	·461948	·381741	·071455	·049713	·034702	·024304
40	·452890	·372430	·066780	·046030	·031837	·022094
41	·444010	·363346	·062411	·042621	·029208	·020086
42	·435304	·354484	·058328	·039464	·026797	·018260
43	·426768	345838	·054512	·036540	·024584	·016600
44	·418400	·337403	·050946	·033834	·022554	·015091
45	·410196	·329174	·047613	·031327	·020692	·013719
46	402153	·321145	·044498	·029007	·018983	·012472
47	·394268	·313312	·041587	·026858	017416	·011338
48	·386537	·305671	038866	024869	·015978	·010307
49	·378958	·298215	·036324	·023026	·014658	·009370
50	·371527	·290942	·033947	·021321	·013448	·008518
51	·364243	·283846	·031726	·019741	012338	·007744
52	·357101	·276922	·029651	018279	·011319	·007040
53	·350099	·270168	·027711	·016925	·010384	·006400
54	·343234	·263579	·025898	·015671	·009527	·005818
55	·336504	257150	·024204	014510	·008740	·005289
56	·329906	·250878	·022620	·013435	·008018	·004808
57	·323437	·244759	·021140	·012440	·007356	·004371
58	·317095	·238789	·019757	·011519	·006749	·003973
59	·310877	·232965	·018465	010665	·006192	·003612
60	·304782	·227283	·017257	·009875	·005680	·003284
61	·298806	·221740	016128	·009144	·005211	·002985
62	·292947	·216331	·015073	·008466	·004781	·002714
63	·287203	·211055	·014087	·007839	·004386	·002467
64	·281571	·205907	·013165	·007259	004024	·002243
65	·276050	·200885	·012304	·006721	·003692	·002039
66	·270637	·195985	·011499	·006223	·003387	·001853
67	·265331	·191205	·010745	005762	·003107	·001685
68	·260128	·186542	·010043	005335	·002851	·001532
69	·255028	·181992	·009386	·004940	·002615	·001392
70	·250027	·177553	·008772	004574	·002399	·001266
71	·245125	·173223	·008198	·004235	·002201	·001151
72	·240318	·168998	·007662	·003921	·002019	·001046
73	·235606	·164876	·007161	·003631	·001852	·000951
74	·230986	·160854	·006692	003362	·001699	·000864
75	·226457	·156931	·006254	·003113	·001559	·000780

TABLE LVIII. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
76	.222017	.153103	.005845	.002882	.001430	.000714
77	.217664	.149369	.005463	.002669	.001312	.000649
78	.213396	.145726	.005105	.002471	.001204	.000590
79	.209211	.142172	.004771	.002288	.001104	.000537
80	.205109	.138704	.004459	.002118	.001013	.000488
81	.201087	.135321	.004167	.001961	.000929	.000443
82	.197145	.132021	.003895	.001816	.000853	.000403
83	.193279	.128800	.003640	.001682	.000782	.000366
84	.189489	.125659	.003402	.001557	.000718	.000333
85	.185774	.122594	.003179	.001442	.000658	.000303
86	.182131	.119604	.002971	.001335	.000604	.000275
87	.178560	.116687	.002777	.001236	.000554	.000250
88	.175059	.113841	.002595	.001144	.000508	.000227
89	.171626	.111064	.002425	.001059	.000466	.000207
90	.168261	.108355	.002267	.000981	.000428	.000188
91	.164962	.105712	.002118	.000908	.000392	.000171
92	.161727	.103134	.001980	.000841	.000360	.000155
93	.158556	.100619	.001850	.000779	.000330	.000141
94	.155447	.098165	.001729	.000721	.000303	.000128
95	.152399	.095770	.001616	.000667	.000278	.000116
96	.149411	.093434	.001510	.000618	.000255	.000106
97	.146481	.091155	.001411	.000572	.000234	.000096
98	.143609	.088932	.001319	.000530	.000214	.000087
99	.140793	.086763	.001233	.000490	.000197	.000079
100	.138032	.084647	.001152	.000454	.000180	.000072

TABLE LIX.

The preſent Value of an Annuity of 1 *l.* for any Number of Years not exceeding 100, at the ſeveral Rates of 2, 2½, 7, 8, 9, and 10 *per cent.* being a Supplement to Table II. p. 21.

Years	2 per cent	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	.9803	.9756	.9345	.9259	.9174	.9090
2	1.9415	1.9274	1.8080	1.7832	1.7591	1.7355
3	2.8838	2.8560	2.6243	2.5770	2.5312	2.4868
4	3.8077	3.7619	3.3872	3.3121	3.2397	3.1698
5	4.7134	4.6458	4.1001	3.9927	3 8896	3.7907
6	5.6014	5.5081	4.7665	4.6228	4.4859	4.3552
7	6.4719	6.3493	5.3892	5.2063	5.0329	4.8684
8	7.3254	7.1701	5.9712	5.7466	5.5348	5.3349
9	8.1622	7.9708	6.5152	6.2468	5.9952	5.7590
10	8.9825	8.7520	7.0235	6.7100	6.4176	6.1445
11	9.7868	9.5142	7.4986	7.1389	6.8051	6.4950
12	10.575	10.257	7.9426	7.5360	7.1607	6.8136
13	11.348	10.983	8.3576	7.9037	7.4869	7.1033
14	12.106	11.690	8.7454	8.2442	7.7861	7.3666
15	12.849	12.381	9.1079	8.5594	8.0606	7.6060
16	13.577	13.055	9.4466	8.8513	8.3125	7.8237
17	14.291	13.712	9.7632	9.1216	8.5436	8.0215
18	14.992	14.353	10.059	9.3718	8.7556	8.2014
19	15.678	14.978	10.335	9.6035	8.9501	8.3649
20	16.351	15.589	10.594	9.8181	9.1285	8.5135
21	17.011	16.184	10.835	10.016	9.2922	8.6486
22	17.658	16.765	11.061	10.200	9.4424	8.7715
23	18.292	17.332	11.272	10.371	9.5802	8.8832
24	18.913	17.884	11.469	10.528	9.7066	8.9847
25	19.523	18.424	11 653	10.674	9.8225	9.0770
26	20.121	18.950	11.825	10.809	9.9289	9.1609
27	20.706	19.464	11.986	10.935	10.026	9.2372
28	21.281	19.964	12.137	11.051	10.116	9.3065
29	21.844	20.453	12.277	11.158	10.198	9.3696
30	22.396	20.930	12.409	11.257	10.273	9.4269
31	22.937	21.395	12.531	11.349	10.342	9.4790
32	23.468	21.849	12.646	11.434	10.406	9.5263
33	23.988	22.291	12.753	11.513	10.404	9.5694
34	24.498	22.723	12.854	11.586	10.517	9.6085
35	24.998	23.145	12.947	11.654	10.566	9.6441
36	25.488	23.556	13.035	11.717	10.611	9.6765
37	25.969	23.957	13.117	11.775	10.652	9.7059

TABLE LIX. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
38	26.440	24.348	13.193	11.828	10.690	9.7320
39	26.902	24.730	13.264	11.878	10.725	9.7569
40	27.355	25.102	13.331	11.924	10.757	9.7790
41	27.799	25.466	13.394	11.967	10 786	9.7991
42	28.234	25.820	13.452	12.006	10.813	9 8173
43	28.661	26.166	13.506	12 043	10.837	9.8339
44	29.079	26.503	13.557	12.077	10.860	0.8490
45	29.490	26.833	13.605	12.108	10.881	9.8628
46	29.892	27.154	13.650	12.137	10.900	9.8752
47	30.286	27.467	13.691	12.164	10.917	9.8866
48	30.673	27.773	13.730	12.189	10.933	9.8969
49	31.052	28.071	13.766	12.212	10.948	9.9062
50	31.423	28.362	13.800	12.233	10.961	9.9148
51	31.787	28.646	13.832	12.253	10.974	9.9225
52	32.144	28.923	13.862	12.271	10.985	9 9295
53	32.495	29.193	13.889	12.288	10.995	9.9359
54	32.838	29.456	13.915	12.304	11.005	9.9418
55	33.174	29.713	13.939	12.318	11.013	9.9471
56	33.504	29.964	13.962	12.332	11.022	9.9519
57	33.828	30.209	13.983	12.344	11.029	9.9562
58	34.145	30.448	14.003	12.356	11.036	9.9602
59	34.456	30.681	14.021	12.366	11.042	9.9638
60	34.760	30.908	14.039	12.376	11.047	9.9671
61	35.059	31.130	14.055	12.385	11.053	9.9701
62	35.352	31.346	14.070	12.394	11.057	9.9728
63	35.639	31.557	14.084	12.402	11.062	9.9753
64	35.921	31.763	14.097	12.409	11.066	9.9775
65	36.197	31.964	14.109	12.415	11.070	9.9796
66	36.468	32.160	14.121	12.422	11.073	9.9814
67	36.733	32.351	14.132	12.427	11.076	9.9831
68	36.993	32.538	14.142	12.433	11.079	9.9846
69	37.248	32.720	14.151	12.438	11.082	9.9860
70	37.498	32.897	14.100	12.442	11.084	9.9873
71	37.743	33.071	14.168	12.447	11.086	9.9884
72	37.984	33.240	14.176	12.450	11.088	9.9895
73	38.219	33.404	14.183	12.454	11.090	9.9904
74	38.450	33.565	14.190	12.457	11.092	9.9913
75	38.677	33.722	14.196	12.461	11.093	9.9921
76	38.899	33.875	14 202	12.463	11.095	9.9928
77	39.116	34.025	14.207	12.466	11.096	9.9935
78	39.330	34.170	14.212	12.469	11.097	9.9940
79	39.539	34.313	14.217	12.471	11.098	9.9946

TABLE LIX. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
80	39.744	34.451	14.222	12.473	11.099	9.9951
81	39.945	34.587	14.226	12.475	11.100	9.9955
82	40.142	34.719	14.230	12.477	11.101	9.9959
83	40.336	34.847	14.233	12.478	11.102	9.9963
84	40.525	34.973	14.237	12.480	11.103	9.9966
85	40.711	35.096	14.240	12.481	11.103	9.9969
86	40.893	35.215	14.243	12.483	11.104	9.9972
87	41.071	35.332	14.246	12.484	11.104	9.9974
88	41.247	35.446	14.248	12.485	11.105	9.9977
89	41.418	35.557	14.251	12.486	11.105	9.9979
90	41.586	35.665	14.253	12.487	11.106	9.9981
91	41.751	35.771	14.255	12.488	11.106	9.9982
92	41.913	35.874	14.257	12.489	11.107	9.9984
93	42.072	35.975	14.259	12.490	11.107	9.9985
94	42.227	36.073	14.261	12.490	11.107	9.9987
95	42.380	36.169	14.262	12.491	11.108	9.9988
96	42.529	36.262	14.264	12.492	11.108	9.9989
97	42.675	36.353	14.265	12.492	11.108	9.9990
98	42.819	36.442	14.266	12.493	11.108	9.9991
99	42.960	36.529	14.268	12.493	11.108	9.9992
100	43.098	36.614	14.269	12.494	11.109	9.9992
Perp.	50.000	40.000	14.286	12.500	11.111	10.000

TABLE LX.

Shewing the Sum to which 1 *l.* Principal will increaſe at 2, 2½, 7, 8, 9, and 10 *per cent.* Compound Intereſt, in any Number of Years not exceeding 100; being a Supplement to Table III. p. 25.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.02000	1.02500	1.07000	1.08000	1.09000	1.10000
2	1.04040	1.05062	1.14490	1.1664	1.1881	1.21000
3	1.06120	1.07689	1.22504	1.25971	1.29502	1.33100
4	1.08241	1.10381	1.31079	1.36048	1.41158	1.46410
5	1.10408	1.13140	1.40255	1.46932	1.53862	1.61051
6	1.12616	1.15969	1.50073	1.58687	1.67710	1.77156
7	1.14868	1.18868	1.60578	1.71382	1.82803	1.94871
8	1.17165	1.21840	1.71818	1.85093	1.99256	2.14358
9	1.19509	1.24886	1.83845	1.99900	2.17189	2.35794
10	1.21899	1.28008	1.96715	2.15892	2.36736	2.59374
11	1.24337	1.31208	2.10485	2.33163	2.58042	2.85311
12	1.26824	1.34488	2.25219	2.51817	2.81266	3.13842
13	1.29360	1.37851	2.40984	2.71962	3.06580	3.45227
14	1.31947	1.41297	2.57853	2.93719	3.34172	3.79749
15	1.34586	1.44829	2.75903	3.17216	3.64248	4.17724
16	1.37278	1.48450	2.95216	3.42594	3.97030	4.59497
17	1.40024	1.52161	3.15881	3.70001	4.32763	5.05447
18	1.42824	1.55965	3.37993	3.99601	4.71712	5.55991
19	1.45681	1.59865	3.61652	4.31570	5.14166	6.11590
20	1.48594	1.63861	3.86968	4.66095	5.60441	6.72749
21	1.51566	1.67958	4.14056	5.03383	6.10881	7.40024
22	1.54597	1.72157	4.43040	5.43654	6.65860	8.14027
23	1.57689	1.76461	4.74052	5.87146	7.25787	8.95430
24	1.60843	1.80872	5.07236	6.34118	7.91108	9.84973
25	1.64060	1.85394	5.42743	6.84847	8.62308	10.8347
26	1.67341	1.90029	5.80735	7.39635	9.39915	11.9181
27	1.70688	1.94780	6.21386	7.98806	10.245	13.1099
28	1.74102	1.99649	6.64883	8.62710	11.1671	14.4209
29	1.77584	2.04640	7.11425	9.31727	12.1721	15.8630
30	1.81136	2.09756	7.61225	10.0626	13.2676	17.4494
31	1.84758	2.15000	8.14511	10.8676	14.4617	19.1943
32	1.88454	2.20375	8.71527	11.737	15.7633	21.1137
33	1.92223	2.25885	9.32533	12.676	17.182	23.2251
34	1.96067	2.31532	9.97811	13.6901	18.7284	25.5476
35	1.99988	2.37320	10.6765	14.7853	20.4139	28.1024
36	2.03988	2.43253	11.4239	15.9681	22.2512	30.9126

TABLE LX. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
37	2.08068	2.49334	12.2236	17.2456	24.2532	34.0039
38	2.12229	2.55568	13.0792	18.6252	26.4367	37.4043
39	2.16474	2.61957	13.9948	20.1152	28.8159	41.1447
40	2.20803	2.68506	14.9744	21.7245	31.4094	45.2592
41	2.25220	2.75219	16.0226	23.4624	34.2362	49.7851
42	2.29724	2.82099	17.1442	25.3394	37.3175	54.7636
43	2.34318	2.89152	18.3443	27.3666	40.6761	60.2400
44	2.39005	2.96380	19.6284	29.5559	44.3369	66.2640
45	2.43785	3.03790	21.0024	31.9204	48.3272	72.8904
46	2.48661	3.11385	22.4726	34.4740	52.6767	80.1795
47	2.53634	3.19169	24.0457	37.2320	57.4176	88.1974
48	2.58707	3.27148	25.7289	40.2105	62.5852	97.0172
49	2.63881	3.35327	27.5299	43.4274	68.2179	106.718
50	2.69158	3.43710	29.4570	46.9016	74.3575	117.390
51	2.74541	3.52303	31.5190	50.6537	81.0496	129.129
52	2.80032	3.61111	33.7253	54.7060	88.3441	142.042
53	2.85633	3.70139	36.0861	59.0825	96.2951	156.247
54	2.91346	3.79392	38.6121	63.8091	104.961	171.871
55	2.97173	3.88877	41.3150	68.9138	114.408	189.059
56	3.03116	3.98599	44.2070	74.4269	124.705	207.965
57	3.09178	4.08564	47.3015	80.3811	135.928	228.761
58	3.15362	4.18778	50.6126	86.8116	148.162	251.637
59	3.21669	4.29247	54.1555	93.7565	161.496	276.801
60	3.28103	4.39978	57.9464	101.257	176.031	304.481
61	3.34665	4.50978	62.0026	109.357	191.874	334.929
62	3.41358	4.62252	66.3428	118.106	209.142	368.422
63	3.48185	4.73809	70.9868	127.554	227.965	405.265
64	3.55149	4.85654	75.9559	137.759	248.482	445.791
65	3.62252	4.97795	81.2728	148.779	270.845	490.370
66	3.69497	5.10240	86.9619	160.682	295.222	539.407
67	3.76887	5.22996	93.0492	173.536	321.792	593.348
68	3.84425	5.36071	99.5627	187.419	350.753	652.683
69	3.92113	5.49473	106.532	202.413	382.321	717.951
70	3.99955	5.63210	113.989	218.606	416.730	789.746
71	4.07954	5.77290	121.968	236.094	454.235	868.721
72	4.16114	5.91722	130.506	254.982	495.117	955.593
73	4.24436	6.06515	139.641	275.381	539.677	1051.15
74	4.32925	6.21678	149.416	297.411	588.248	1156.26
75	4.41583	6.37220	159.876	321.204	641.190	1271.89
76	4.50415	6.53151	171.067	146.900	698.898	1399.08
77	4.59423	6.69480	183.042	174.652	761.798	1538.99

TABLE LX continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
78	4.686111	6.86217	195.851	404.625	830.360	1692.89
79	4.779784	7.03372	209.564	436.995	905.093	1862.18
80	4.87543	7.20956	224.234	471.954	986.551	2048.40
81	4.97294	7.38980	239.930	509.711	1075.34	2253.24
82	5.07240	7.57455	256.725	550.488	1172.12	2478.56
83	5.17385	7.76391	274.696	594.527	1277.61	2726.42
84	5.27733	7.95801	293.925	642.089	1392.59	2999.06
85	5.38287	8.15696	314.500	693.456	1517.93	3298.96
86	5.49053	8.36088	336.515	748.933	1654.54	3628.86
87	5.60034	8.56991	360.071	808.847	1803.45	3991.75
88	5.71235	8.78415	385.276	873.555	1965.76	4390.92
89	5.82660	9.00376	412.245	943.439	2142.68	4830.02
90	5.94313	9.22885	441.102	1018.91	2335.52	5313.02
91	6.06199	9.45957	471.980	1100.42	2545.72	5844.32
92	6.18323	9.69606	505.018	1188.46	2774.83	6428.75
93	6.30690	9.93846	540.370	1283.53	3024.57	7071.63
94	6.43303	10.1869	578.196	1386.22	3296.78	7778.79
95	6.56169	10.4416	618.669	1497.12	3593.49	8556.67
96	6.69293	10.7026	661.976	1616.89	3916.91	9412.34
97	6.82679	10.9702	708.314	1746.24	4269.43	10353.58
98	6.96332	11.2444	757.897	1885.94	4653.68	11388.93
99	7.10259	11.5255	810.949	2036.81	5072.51	12527 82
100	7.24464	11.8137	867.716	2199.76	5529.04	13780.61

TABLE LXI.

Shewing the Sum to which an *Annuity* of 1*l.* will increase at 2, 2½, 7, 8, 9, and 10 *per cent.* Compound Interest, in any Number of Years not exceeding 100; being a Supplement to Table IV. p. 28.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.02000	2.02500	2.07000	2.08000	2.09000	2.10000
3	3.06040	3.07562	3.21490	3.24640	3.27810	3.31000
4	4.12160	4.15251	4.43994	4.50611	4.57312	4.64100
5	5.20404	5.25632	5.75073	5.86660	5.98471	6.10510
6	6.30812	6.38773	7.15329	7.33592	7.52333	7.71561
7	7.43428	7.54743	8.65402	8.92280	9 20043	9.48717
8	8.58296	8.73611	10.2598	10 6366	11 0284	11.4358
9	9.75462	9 95451	11 .9779	12.4875	13.0210	13.5794
10	10.9497	11.2033	13.8164	14.4865	15.1929	15.9374
11	12.1687	12.4834	15.7835	16.6454	17.5602	18.5311
12	13.4120	13.7955	17.8884	18.9771	20.1407	21.3842
13	14.6803	15.1404	20.1405	21.4952	22.9533	24.5227
14	15.9739	16.5189	22.5504	24.2149	26.0191	27.9749
15	17.2934	17.9319	25.1290	27.1521	29.3609	31.7724
16	18.6392	19.3802	27.8880	30.3242	33.0033	35.9497
17	20.0120	20.8647	30.8402	33.7502	36.9737	40.5447
18	21.4123	22.3863	33.9990	37.4502	41.3013	45.5991
19	22.8405	23.9460	37.3789	41.4462	46.0184	51.1590
20	24.2973	25.5446	40.9954	45.7619	51.1601	57.2750
21	25.7833	27.1832	44.865	50.4229	56.7645	64.0025
22	27.2989	28.8628	49.0057	55.4567	62.8733	71.4027
23	28.8449	30.5844	53.4361	60.8932	69.5319	79.5430
24	30.4218	32.3490	58.1766	66.7047	76.7898	88.4973
25	32.0302	34.1577	63.2490	73.1059	84.7008	98.3470
26	33.6709	36.0117	68.6764	79 9544	93.3239	109.181
27	35.3443	37.9120	74.4838	87.3507	102.723	121.099
28	37.0512	39.8598	80.6976	95.3388	112.968	134.209
29	38.7922	41.8562	87.3465	103.965	124.135	148.630
30	40.5680	43.9027	94.4607	113.283	136.307	164.494
31	42.3794	46.0002	102.073	123.345	149.575	181.943
32	44.2270	48.1502	110.218	134.213	64.036	201.137
33	46.1115	50 3540	118.933	145.95	179.800	222.251
34	48.0338	52.6128	128.258	158.626	196.982	245.476
35	49.9944	54.9282	138 236	172.316	15.710	271.024
36	51.9943	57.2014	148.913	187.102	236.124	299.126

TABLE LXI. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent.
37	54.0342	59.7339	160.337	203.070	258.375	330.09
38	56.1149	62.2272	172.561	220.315	282.629	364.043
39	58.2372	64.7829	185.640	238.941	309.066	401.447
40	60.4019	67.4025	199.635	259.056	337.882	442.592
41	62.6100	70.0876	214.609	280.781	369.291	487.851
42	64.8622	72.8398	230.632	304.243	403.528	537.636
43	67.1594	75.6608	247.776	329.583	440.845	592.400
44	69.5026	78.5523	266.120	356.949	481.521	652.640
45	71.8927	81.5161	285.749	386.505	525.858	718.904
46	74.3305	84.5540	306.751	418.426	574.186	791.795
47	76.8171	87.6678	329.224	452.900	626.862	871.974
48	79.3535	90.8595	353.270	490.132	684.280	960.172
49	81.9405	94.1310	378.998	530.342	746.865	1057.18
50	84.5791	97.4843	406.528	573.770	815.083	1163 90
51	87.2709	100.921	435.985	620.671	889.441	1281.29
52	90.0164	104.444	467.504	671.325	970.490	1410.42
53	92.8167	108.055	501.230	726.031	1058.83	1552.47
54	95.6730	111 756	537.316	785.114	1155.13	1708.71
55	98.5865	115.550	575.928	848.923	1260.09	1880.59
56	101.558	119.439	617.243	917.837	1374.50	2069.65
57	104.589	123.425	661.450	992.264	1499.20	2277.61
58	107.681	127.511	708.752	1072.64	1635.13	2506.37
59	110.834	131.699	759 364	1159.45	1783.29	2758.01
60	114.051	135.991	813.520	1253.21	1944 79	3034.81
61	117.332	140.391	871.466	1354.47	2120.82	3339.29
62	120.679	144 901	933.469	1463.82	2312 69	3674.22
63	124.092	149.523	999.812	1581.93	2521 84	4042.65
64	127.574	154.261	1070.79	1709.48	2749 80	4447.91
65	131.126	159.118	1146.75	1847.24	2998.28	4893.70
66	134.748	164.096	1228.02	1996.02	3269.13	5384.07
67	138.443	169.198	1314.95	2156.71	3564.35	5923 48
68	142.212	174.428	1408.03	2330.24	3886.14	6516.83
69	146.056	179.789	1507.60	2517.66	4236.90	7169.51
70	149.977	185.284	1614.13	2720.08	4619.22	7887.46
71	153.977	190.916	1728.12	2938.68	5035.95	8677.21
72	158.057	196.689	1850.09	3174.78	5490.18	9545.93
73	162.218	202.606	1980.59	3429.76	5985.30	10501.53
74	166.462	208.671	2120.24	3705.14	6524 98	11552.68
75	170.791	214.888	2269.65	4002.55	7113.23	12708.95
76	175.207	221.260	2429.53	4323.76	7754.42	13980 85
77	179.711	227.792	2600.60	4670.66	8453.37	15379 93

TABLE LXI. continued.

Years	2 per cent.	2½ per cent.	7 per cent.	8 per cent.	9 per cent.	10 per cent
78	184.305	234.486	2783.64	5045.31	9215.12	16918.92
79	188.992	241.348	2979.49	5449.94	10045.4	18611.82
80	193.771	248.382	3189.06	5886.93	10950.5	20474.00
81	198.647	255.592	3413.29	6358.89	11937.1	22522.40
82	203.620	262.982	3653.22	6868.60	13012.4	24775.64
83	208.692	270.556	3909.95	7419.08	14184.5	27254.20
84	213.866	278.320	4184.65	8013 61	15462.2	29980.62
85	219.143	286.278	4478.57	8655.70	16854.8	32979.69
86	224.526	294.435	4793.07	9349.16	18372.7	36278.65
87	230.017	302.796	5129.59	0098.0	20027.2	39907 52
88	235.617	311.366	5489.66	10906.9	21830.7	43899.27
89	241.330	320.150	5874.93	11780.4	23796.5	48290.20
90	247.156	329.154	6287.18	12723.9	25939.1	53120.22
91	253 099	338.383	6728.28	13742.8	28274.7	58433 25
92	259.161	347.842	7200.26	14843.2	30820.4	64277.57
93	265.345	357.538	7705.28	16031.7	33595.2	70706.33
94	271.651	367.477	8245.65	17315.2	36619.8	77777.96
95	278.084	377.664	8823.85	18701.5	39916.6	85556.76
96	284.646	388.105	9442.52	20198.6	43510.1	94113.43
97	291.339	398.808	10104.5	21815.5	47427.0	103525.8
98	298.166	409.778	10812.8	23561.7	51696 4	113879.3
99	305.129	421.023	11570.7	25447.7	56350.1	125268.3
100	312.232	432.548	12381.6	27484.5	61422.6	137796.1

THE four *laſt* Tables are to be conſidered as continuations of the four *firſt* Tables; and they have been added to this collection, *partly* becauſe it will be found ſometimes neceſſary to determine the values and amounts of ſums and annuities at the *higher* and *lower* rates of intereſt ſpecified in them; but *chiefly* becauſe they furniſh with the means of determining eaſily theſe values and amounts for the moſt common *half*-yearly as well as *yearly* rates of intereſt; Mr. SMART, in his very uſeful and comprehenſive Tables, having given theſe *half*-yearly values and amounts improperly.

It is very obvious, that the amount at any given *yearly* intereſt of any given annuity payable *half*-yearly, is the ſame with the amount of *half* that annuity at *half* the intereſt, and payable a *double* number of times. The amount, for inſtance, at 4 *per cent.* of an annuity of 10*l.* payable *yearly* for 30 years, is, by Table IV. p. 28, *l.* 560.849.

If it is payable *half*-yearly, its amount will be the ſame with the amount at 2 *per cent*, of an annuity of 5*l.* payable for 60 years, which, by Table LXI. is *l.* 570.257.

——In like manner; the amount at 5 *per cent.* of an annuity of 50*l.* for 40 years, payable *half*-yearly, is the ſame with the amount at 2½ *per. cent.* of an annuity of 25*l.* for

for 80 years, which, by Table LXI. appears to be *l.* 6209.567. The amount at 5 *per cent.* of the ſame annuity, payable *yearly*, appears, by Table IV. to be *l.* 6039.988.

Farther. The amount of 10*l.* *principal* put out to *yearly* intereſt at 4 *per cent.* and forborne for 30 years, is (by Table III. p. 25) *l.* 32.433. But if it is put out to 4 *per cent.* *half*-yearly intereſt, its amount will be the ſame with the amount of the ſame principal, bearing *half* the intereſt in double the time; that is, it will, in the preſent inſtance, be the ſame with the amount of 10*l.* bearing 2 *per cent* intereſt in 60 years, which, by the laſt Table but one, appears to be *l.* 32.810.

Theſe amounts can be thus determined from theſe Tables only, when the term for which they are wanted does not exceed 50 years, or 100 *half* years.

In order to find them for any longer term, the following method muſt be taken:

"If the amount required is the amount "not of an *annuity*, but of a ſum—find firſt "the *half*-yearly amount for 50 years; after "which find the *half*-yearly amount of that "amount for the remainder of the term, and "this laſt will be the amount deſired."

EXAMPLE.

Let the amount be required, at 4 *per cent.* of 10*l.* in 80 years, ſuppoſing the intereſt payable *half-yearly*.

Anſ. The amount in 50 years, determined in the manner juſt deſcribed, is 72.446; and the amount of 72.446 in 30 years, determined in the ſame way, is *l.*237.676, which is the amount required.

This amount, ſuppoſing the intereſt payable yearly, is *l.*230.049.

But if the amount required is the amount of an *annuity* improved at any given rate of compound intereſt payable *half*-yearly, it will be neceſſary, after finding the ſum which is the amount for 50 years, to find the yearly intereſt that ſum will carry at the given rate; and the amount for the remainder of the term, of this intereſt increaſed by the annuity, added to the amount for 50 years, will be the amount required.

EXAMPLE.

Let the amount be required, at 4 *per cent.* of 10*l. per ann.* in 80 years, ſuppoſing the annuity payable *half*-yearly.

Anſ. The amount in 50 years (being the ſame with the amount of 5*l. per ann.* in 100 years, at 2 *per cent.*) is, by Table LVIII. *l.*1561.116.——The yearly intereſt of *l.*1561.116, at 4 *per cent.* is *l.*62.446, which increaſed by 10*l.* makes *l.*72.446; and the amount of *l.*72.446 *per ann.* payable *half*-yearly in 30 years (or of *l.*36.223 in 60 *half* years)

years) is *l.* 4620.96, which added to *l.* 1561.116, makes *l.*6182.076 the amount required.

This amount, ſuppoſing the annuity payable yearly, is *l.* 5982.665.

N. B. Theſe amounts for any given term and rate of intereſt are the ſame with the *debts* bearing that intereſt, which will be gradually ſunk in that term by any given annuity appropriated to the redemption of the debt.—It appears, therefore, from the laſt example, that a ſinking fund of a million *per ann.* never diverted, would pay off, in 82 years, a public debt of 598 millions, bearing 4 *per cent.* intereſt, ſuppoſing it applied to that purpoſe *yearly*; but that if applied *half*-yearly, it would pay off, in the ſame time, a debt of 618 millions. See p. 34.

Theſe examples ſhew the method of finding, by the preceding Tables, the values at any rate of intereſt of annuities payable for any given terms, ſuppoſing them payable *half*-yearly; and likewiſe the values of any ſums payable at the end of any terms, ſuppoſing a *half*-yearly inſtead of an *yearly* diſcount allowed. But in ſuch caſes, theſe Tables will be of no uſe, if the terms exceed 50 years, or 100 *half*-years; and it will be neceſſary to have recourſe to the *theorems* at the beginning of the *third* of the following Additional Eſſays, by which, with the help of logarithms, it is eaſy, in

all caſes, to compute the difference between the values of annuities (including *life*-annuities) as they are payable yearly, half-yearly, or quarterly.

With reſpect to life-annuities, it may be proper to obſerve here particularly, that their values deduced from the *complements*, that is, from twice the *expectations* according to any given table of mortality (by the rules in p. 170, and p. 172), and payable *half*-yearly or *quarterly*, is the ſame with the values of *half* or a *quarter* the annuities at *half* or a *quarter* the yearly intereſt, deduced (by the ſame rules) from *double* or *quadruple* the complements; and that the difference between the *yearly* values and theſe *half*-yearly or *quarterly* values added to the *true* yearly values according to the ſame table of mortality, will give, with almoſt perfect correctneſs, the *half*-yearly or *quarterly* values according to that table.

EXAMPLE.

Let the *different* values be required of an annuity on a ſingle life aged 50, according as it is payable *half*-yearly or quarterly, reckoning intereſt at 4 *per cent.* and the probabilities of the duration of life, as they are in Table XLII. p. 150.

Anſ. The *complement* (that is, twice the *expectation*) by Table XLII. of a life aged 50 is 36.92.——The value of an annuity payable *yearly* on a life at this age deduced at 4 *per cent.* from this *complement*, is by the rule in p. 170, 11.533.——The value deduced from *double* this complement (that is, of a life whoſe complement is ſuppoſed 73.84) at 2 *per cent.* is 23.466 (*a*), the half of which

(*a*) In computing in this caſe, by the rule here referred to, it is neceſſary to find the value at 2 *per cent.* of an annuity certain payable for 73.84 years. This value (by the Firſt Theorem in the Third Additional Eſſay) is $50 - \frac{1}{.02 \times \overline{1.02}^{73.84}}$. By logarithms it is eaſy to find that $\overline{1.02}^{73.84}$ is 4.3154; and, conſequently, that this expreſſion is 38.416, which multiplied, according to the rule in p. 170, by 51 (the perpetuity increaſed by unity), and the product divided by 73.84 will give 36.533, which quotient ſubtracted from 50, (the perpetuity) leaves 23.466.

In like manner; the value, by the ſame theorem, at 1 *per cent.* for an annuity certain for a number of years equal to four times the complement (that is, to 147.68 years) is $100 - \frac{1}{.01 \times \overline{1.01}^{147.68}}$, which is equal to 76,994; and the product of 76.994 into 101, divided by 147.68, gives 52.654, which, ſubtracted from 100, leaves 47.345, the quarter of which is 11.836.

It is neceſſary to add here, that in computing the yearly value of any life-annuity from the expectation by the rule in p. 170, the value of an annuity certain for a number of years equal to twice the expectation (or the complement) may be always taken from Table II. p. 21, when the complement is any *whole* number of years; and alſo,

which is 11.733.——The difference is .200. And this difference, added to 11.658 (the *true* value by Table XLIII. of an annuity payable yearly on the ſuppoſed life), makes 11.858; which is the true value of the annuity payable *half*-yearly.

The value of the ſame annuity deduced (by the rule in p. 170) from *quadruple* the complement at 1 *per cent.*; that is, the value at 1 *per cent.* of a life whoſe complement is ſuppoſed to be 147.68 years, is 47.345, the quarter of which is 11.836. The difference between this value and 11.533 is .303, which added as before to 11.658, makes 11.961, the true value of the annuity payable quarterly.

In the ſame way the values are to be computed (by the ſecond rule in p. 172) of annuities payable *half-yearly* or *quarterly* on any two joint lives.

alſo, that when it is not any whole number of years, it may be taken for the correſpondent arithmetical mean between the two neareſt yearly values in the Table. Thus; in the example given above, the value at 4 *per cent.* of an annuity certain for 36 years, by Table II. p. 21, is 18.908. The value for 37 years is 19.142. The difference is .234; and this difference multiplied by .92 (the fractional part of the complement) and added to the leaſt of theſe two values, gives 19.123 for the value of an annuity certain for 36.92 years.

The exact value by the firſt Theorem is $25 - \frac{1}{1.04 \times 1.04^{36.92}}$, which is equal to 19.123, and the ſame with the former value

If

If the annuity is a life-annuity (*a*) ſecured on land, the value is to be computed by the directions in the Third Additional Eſſay.———If ſuch an annuity is payable *half*-yearly, as is moſt common, its value, in the preſent inſtance, will be 11.858 (the half-yearly value juſt determined) increaſed by the quotient of 19.206 (the value of an annuity certain and payable *half*-yearly for a number of *half* years, equal to *four* times the *expectation*, or *twice* the *complement*; that is, 73.84) divided by *four* times the complement. This quotient is .130; and the value, therefore, is 11.989.

The following compariſon will ſhew, in ſome meaſure, what additions ſhould be made, at all ages, to the yearly values of life-annuities, on account of theſe different modes of payment.

(*a*) It ſhould be remembered, that all the values of life-annuities in the preceding Tables ſuppoſe that, when the annuitant dies, nothing can be claimed for the time that has paſt ſince the laſt payment became due. If a payment proportioned to that time may be claimed; that is, if the annuity is payable to the laſt moment of life, it is called an annuity *ſecured on land*.

TABLE LXII.

Shewing the Additions to the Values of Life-Annuities on account of their being payable *half-yearly*, or *quarterly*, or *half-yearly* and ſecured on Land.

Intereſt 4 *per cent.*

Age	Yearly value by Table XLIII.	Half-yearly value.	Exceſs above yearly value.	Quarterly value.	Exceſs above yearly value.	Value ſecured by land and payable half-yearly.	Exceſs above yearly value.
10	18.891	19.018	.127	19.089	.198	19.085	.194
20	17.603	17.746	.143	17.819	.216	17.824	.221
30	16.006	16.168	.162	16.249	.243	16.259	.253
40	14.034	14.216	.182	14.307	.273	14.324	.290
50	11.658	11.858	.200	11.961	.300	11.989	.331
60	8.789	9.014	.225	9.119	.330	9.170	.381
70	5.783	6.019	.236	6.136	.353	6.204	.421
75	4.534	4.770	.236	4.892	.358	4.967	.433

Intereſt 5 *per cent*

Age							
10	16.169	16.278	.109	16.331	.162	16.332	.163
20	15.260	15.383	.123	15.445	.183	15.447	.187
30	14.080	14.223	.143	14.293	.213	14.299	.219
40	12.558	12.720	.162	12.801	.243	12.812	.254
50	10.634	10.820	.186	10.914	.280	10.933	.299
50	8.184	8.403	.219	8.498	.314	8.544	.360
70	5.494	5.719	.225	5.835	.341	5.891	.397
75	4.345	4.575	.230	4.692	.347	4.762	.417

Theſe exceſſes are the ſame from whatever tables of mortality the yearly values are deduced.——They are alſo nearly the ſame (ſuppoſing equal *yearly* values) whether the yearly values are the values of ſingle, or of joint lives, or of any number of lives.

A SUPPLE-

A

SUPPLEMENT, (a)

CONTAINING

Additional Obſervations on the Duration of Human Life in different Situations; and on the Population of the Kingdom.

SINCE the firſt publication of this work, I have had the pleaſure of reading an ingenious Memoir on the State of Population in the *Pais de Vaud*, a diſtrict of the province of *Bern* in *Switzerland*. The author of this memoir is Mr. *Muret*, the firſt miniſter at *Vevey*, a town in that diſtrict, and ſecretary to the Oeconomical Society there. It forms the firſt part of the *Bern* Obſerva-

(a) This ſupplement was an addition to this Treatiſe in the Second and Third Editions of it. I have in the preſent Edition added to it a *Poſtſcript*, containing a review of the arguments for and againſt the increaſing population of the kingdom.

tions

tions for the year 1766; and a good abſtract of it may be found in the 69th article of a work entitled, *De re Ruſtica*, or the *Repoſitory*. It contains an account of many facts which appear to me curious and important; and which confirm the obſervations I have made in the Firſt and Fourth Eſſays in the Firſt Volume of this Treatiſe.—Some of theſe facts I will here recite.

In the Firſt Eſſay I have aſſerted, that there is a much greater difference between the probabilities of the duration of life in *great towns* and in *county pariſhes*, than is commonly ſuſpected; and, as one proof of this, I have obſerved, that though in *London* the greateſt part of the natives die under three years of age, in the country the greater part live to marry. Mr. *Muret*'s Obſervations and Tables give a diſtinct demonſtration of this, by ſhewing, that in the province of *Vaud*, the greater part of the inhabitants live many years beyond the age of maturity.—But to be a little more explicit.

The diſtrict of *Vaud*, in *Switzerland*, contains 112,951 inhabitants of all ages; 25,773 *families*; 38,328 married perſons: and the annual medium of *births*, for 10 years before 1766, had been 3155; of *weddings*, 808; of *deaths*, 2504.——It appears, therefore, that the married are very nearly a *third* part of the inhabitants, that the number of perſons

to a family is $4\frac{1}{3}$; and that one in 45 of the inhabitants die annually. It may be further learnt (by dividing half the number of the married by the annual medium of weddings), that the *expectation* of marriage in this country is 23 years and $\frac{1}{2}$; and (from the proportions of the births, weddings, and deaths) (*a*) that the greater part of thoſe who are born live to marry. But of this fact there is, I have juſt intimated, a more particular and diſtinct proof.—From a Table given by Mr. *Muret*, of the rate of human mortality in this country (derived from regiſters kept in 43 pariſhes, of the ages at which the inhabitants die), it appears, that one *half* of all that are born live beyond 41 years of age.—The examination of this Table will, undoubtedly, be a gratification to the reader; and, therefore, I have choſen to make it a part of theſe additions. See p. 259. I have alſo given a Table which I have formed from a regiſter in *Suſmilch's* works, of the ages at which the inhabitants of a country pariſh in BRANDENBURGH died, during 50 years, ended at 1759.—And I have further thought proper to add, as contraſts to theſe Tables, two Tables exhibiting the probabilities of life at VIENNA and BERLIN. See p. 260, 261, and 262.

The following obſervations concerning theſe Tables ſhould be attended to.

(*a*) See the note, p. 264, &c. Vol. I.

The

The Table for the country of VAUD, though it gives the probabilities of life in its firſt ſtages very high; and, at ſome ages, more than double to the probabilities of life in great cities; yet, certainly, gives them too low. For, firſt, it has juſt appeared, that in this country the births exceed conſiderably the deaths. The emigrations, likewiſe, from it are very numerous, as will be preſently obſerved: And the neceſſary effect of theſe two cauſes is, to make the regiſters give the number of deaths in the firſt ſtages of life too great in compariſon of the deaths in the laſt ſtages. A Table formed from ſuch regiſters muſt give the probabilities of life too low, according to the obſervations in the Fourth Eſſay; and, in the introduction to the preceding Collection of Tables.

After 40, the probabilities of living in this country decreaſe very faſt; and, after 65, they appear to be rather lower than is common. Mr. *Muret* has taken notice of this fact, and aſcribes it to the particular prevalency of drunkenneſs in his country. He had, he ſays, once the curioſity to examine the regiſter of deaths in one town, and to mark thoſe whoſe deaths might be imputed to drunkenneſs; and he found the number ſo great, as to incline him to believe, that hard drinking kills more of mankind, than pleuriſies and fevers, and all the moſt malignant diſtempers.

The

The former of theſe obſervations is applicable to the Table for the country pariſh in *Brandenburgh*; for it appears from *Suſmilch*'s account (*a*), that the births there exceed the deaths more than in the country of VAUD; nor is it to be imagined, that there are not likewiſe many emigrations from it, particularly, to BERLIN and the King of *Pruſſia*'s armies.

From the Tables for VIENNA and LONDON, compared with the Table for BERLIN, it appears, that the laſt of theſe towns, though much the ſmalleſt, has at ſome ages even a worſe effect on the duration of life, than either of the former: And the reaſon, perhaps, may be, that the inhabitants there are much more crowded together. See p. 295, Vol. I. Between the ages of 30 and 35, and alſo between 42 and 52, there is an irregularity in the BERLIN Table, which, very probably, would not have appeared in it, had it been formed from the bills for a longer term of years.

From the age of 25 to 45, VIENNA appears, in the Tables, to be leſs unfavourable to life than LONDON; but it cannot be depended upon that this is the truth, for the VIENNA Table may give the probabilities of living at theſe ages higher, only becauſe the recruits from the country come to it later,

(*a*) See the remarks on the Table in p. 207 in the preceding collection.

or in greater numbers, after 30 and 40, than in LONDON. A like effect would alſo ariſe from a greater number of migrations in old age from LONDON than from VIENNA.

In forming the Tables for VIENNA and BERLIN, I have applied the correction explained in the Fourth Eſſay, and demonſtrated there to be neceſſary; and, in making this correction, I have ſuppoſed, agreeably to the proportion of the births to the burials, that a fifth of all who die in theſe cities, are perſons who removed to them at 20 years of age.—Notwithſtanding this correction, the Table for BERLIN gives the probabilities of life between 10 and 20 ſo high, and in ſuch diſproportion to the probabilities of life immediately after 20, as to exceed all the bounds of credibility. The true reaſon of this may be learnt from what has been ſaid in p. 295, Vol. I. of the rapid increaſe of BERLIN.

TABLE

TABLE I. *(a)*

Shewing the Probabilities of Life in the Diſtrict of VAUD, SWITZERLAND, formed from the Regiſters of 43 Pariſhes, given by Mr. *Muret*, in the Firſt Part of the BERN Memoirs for the Year 1766.

Age.	Living	Decr.	Age.	Living	Decr.	Age.	Living	Decr.
0	1000	189	31	558	5	62	286	12
1	811	46	32	553	5	63	274	12
2	765	30	33	548	4	64	262	12
3	735	20	34	544	5			
4	715	14				65	250	14
			35	539	6	66	236	16
5	701	13	36	533	6	67	220	18
6	688	11	37	527	7	68	202	18
7	677	10	38	520	7	69	184	16
8	667	8	39	513	7			
9	659	6				70	163	15
			40	506	6	71	153	13
10	653	5	41	500	6	72	140	11
11	648	5	42	494	6	73	129	10
12	643	4	43	488	6	74	119	10
13	639	4	44	482	6			
14	635	4				75	109	11
			45	476	7	76	98	13
15	631	5	46	469	8	77	85	14
16	626	4	47	461	10	78	71	13
17	622	4	48	451	10	79	58	12
18	618	4	49	441	10			
19	614	4				80	46	10
			50	431	9	81	36	7
20	610	4	51	422	8	82	29	5
21	606	4	52	414	8	83	24	4
22	602	5	53	406	9	84	20	3
23	597	5	54	397	9			
24	592	5				85	17	3
			55	388	11	86	14	3
25	587	5	56	377	13	87	11	2
26	582	5	57	364	16	88	9	2
27	577	5	58	348	17	89	7	2
28	572	5	59	331	17			
29	567	4				90	5	1
			60	314	15			
30	563	5	61	299	13			

(a) All the Bills, from which this and the following Tables are formed, give the numbers dying under 1 as well as under 2 years; and, in the numbers dying under 1 are included, in the country pariſh in *Brandenburg* and at *Berlin*, all the ſtill-borns. All the bills alſo give the numbers dying in every period of five years.

TABLE II.

Shewing the Probabilities of Life in a Country Parish in BRANDENBURGH, formed from the Bills for 50 Years, from 1710 to 1759, as given by Mr. SUSMILCH, in his *Gottliche Ordnung*, p. 43.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	1000	225	31	482	5	62	260	12
1	775	57	32	477	5	63	248	12
2	718	31	33	472	5	64	236	12
3	687	23	34	467	5	65	224	11
4	664	22	35	462	6	66	213	11
5	642	20	36	456	6	67	202	12
6	622	15	37	450	6	68	190	12
7	607	12	38	444	6	69	178	12
8	595	10	39	438	6	70	166	13
9	585	8	40	432	5	71	153	15
10	577	7	41	427	5	72	138	16
11	570	6	42	422	5	73	122	15
12	564	5	43	417	5	74	107	14
13	559	5	44	412	6	75	93	13
14	554	5	45	407	6	76	80	12
15	549	5	46	400	6	77	68	9
16	544	5	47	394	6	78	59	8
17	539	4	48	388	7	79	51	7
18	535	4	49	381	7	80	44	6
19	531	4	50	374	7	81	38	6
20	527	5	51	367	8	82	32	6
21	522	5	52	359	8	83	25	6
22	517	5	53	351	8	84	21	5
23	512	5	54	343	9	85	15	4
24	507	5	55	334	10	86	11	3
25	502	4	56	324	10	87	8	2
26	498	3	57	314	10	88	6	2
27	495	3	58	304	11	89	4	1
28	492	3	59	293	11	90	3	1
29	489	3	60	282	11	91	2	1
30	486	4	61	271	11	92	1	1

TABLE III.

Shewing the Probabilities of Life at VIENNA, formed from the Bills for Eight Years, as given by Mr. SUSMILCH, in his *Gottliche Ordnung*, Page 32, Tables.

Age.	Living.	Decr.	Age.	Living.	Decr.	Age.	Living.	Decr.
0	1495	682	31	364	6	62	129	6
1	813	107	32	358	5	63	123	7
2	706	61	33	353	6	64	116	7
3	645	46	34	347	7	65	109	8
4	599	33	35	340	8	66	101	8
5	566	30	36	332	8	67	93	8
6	536	20	37	324	8	68	85	7
7	516	11	38	316	9	69	78	7
8	505	9	39	307	9	70	71	6
9	496	7	40	298	8	71	65	5
10	489	6	41	290	7	72	60	5
11	483	5	42	283	6	73	55	4
12	478	5	43	277	6	74	51	4
13	473	6	44	271	7	75	47	5
14	467	6	45	264	8	76	42	5
15	461	6	46	256	9	77	37	5
16	455	7	47	247	9	78	32	5
17	448	6	48	238	9	79	27	4
18	442	6	49	229	9	80	23	3
19	436	6	50	220	8	81	20	2
20	430	5	51	212	7	82	19	2
21	425	5	52	205	7	83	16	2
22	420	5	53	198	7	84	14	2
23	415	6	54	191	7	85	12	2
24	409	6	55	184	8	86	10	2
25	403	6	56	176	8	87	8	2
26	397	6	57	168	9	88	6	2
27	391	7	58	159	8	89	4	1
28	381	7	59	151	8	90	3	1
29	377	7	60	143	7	91	2	1
30	370	6	61	136	7	92	1	1

TABLE IV.

Shewing the Probabilities of Life at BERLIN, formed from the Bills for Four Years, from 1752 to 1755, given by Mr. SUSMILCH (*a*), in his *Gottliche Ordnung*, Vol. II. page 37, Tables.

Age.	Living.	Decr.	Age	Living.	Decr.	Age.	Living.	Decr.
0	1427	5 4	33	361	7	65	112	6
1	903	151	34	354	7	66	106	7
2	752	61				67	99	7
3	691	73	35	347	8	68	92	6
4	618	45	36	339	9	69	86	6
			37	330	10			
5	573	21	38	320	10	70	80	6
6	552	15	39	310	10	71	74	6
7	536	13				72	68	6
8	523	9	40	300	10	73	62	5
9	514	7	41	290	9	74	57	5
			42	281	8			
10	507	5	43	274	7	75	52	5
11	502	4	44	266	7	76	47	5
12	498	4				77	42	5
13	494	4	45	259	7	78	37	5
14	490	4	46	252	7	79	32	4
			47	245	7			
15	486	4	48	238	7	80	28	4
16	482	5	49	231	7	81	24	3
17	477	5				82	21	2
18	472	5	50	224	7	83	19	2
19	467	6	51	217	7	84	17	2
			52	210	7			
20	461	6	53	203	8	85	15	2
21	455	6	54	195	8	86	13	2
22	449	6				87	11	2
23	443	7	55	187	8	88	9	2
24	436	8	56	179	8	89	7	1
			57	171	8			
25	428	9	58	163	9	90	6	1
26	421	9	59	154	9	91	5	1
27	412	9				92	4	1
28	403	9	60	145	8	93	3	1
29	394	9	61	137	7	94	2	1
			62	130	6			
30	385	9	63	124	6			
31	376	8	64	118	6			
32	363	7						

(*a*) This writer has alſo given the bills of the pariſh of St. *Peter's* at BERLIN, for 24 years; and a Table formed from them, agrees nearly with this.

The

THESE Tables exhibit, in a ſtriking light, the *difference* between the duration of human life, in *great cities* and in the *country*. I will here lay ſome of the chief particulars of it before the reader, deſiring him to take with him this conſideration, that, for the reaſons I have explained, they can be erroneous only by giving the difference much too *little*.

Proportion of Inhabitants dying annually in

Pais De Vaud.	Country Pariſh in Brandenburg.	Vienna.	Berlin.
1 in 45	1 in 45	1 in 19½	1 in 26½ (*a*)

Ages to which half the born live.

Pais De Vaud.	Country Pariſh in Brandenburg.	Vienna.	Berlin.
41	25½	2	2¾

Proportion of Inhabitants (*b*) who reach 80 Years of Age.

Pais De Vaud.	Country Pariſh in Brandenburg.	Vienna	Berlin.
1 in 21½	1 in 22½	1 in 41	1 in 37

The numbers *born* at BERLIN, during the 4 years abovementioned, were, *males*, 9219; *females*, 8743; or 21 to 20.

The numbers that died under 2 years of age, were, *males*, 3118; *females*, 2623; or 7 to 6.

The numbers that died upwards of 80 years of age, were, *males*, 135; *females*, 215; or 5 to 8.

The numbers that died between 91 and 105, were, *males*, 21; *females*, 55.

(*a*) See p. 295 Vol. I. This proportion, were there either no increaſe, or but a ſlow increaſe at BERLIN, would probably be found to be much the ſame with that in VIENNA and LONDON.

(*b*) It ſhould be recollected here, that a conſiderable part of thoſe who die turned of 80 years of age in great towns,

The Probalities of living one Year in

Odds.	Pais De Vaud.	Country Pariſh in Brandenburg.	Vienna.	Berlin.
At birth	4½ to 1	3½ to 1	1⅕ to 1	1¼ to 1
Age 12	160 to 1	112 to 1	84 to 1	123 to 1
25	117 to 1	110 to 1	66 to 1	50 to 1
30	111 to 1	107 to 1	56 to 1	44 to 1
40	83 to 1	78 to 1	36 to 1	32 to 1
50	49 to 1	50 to 1	27 to 1	30 to 1
60	23 to 1	25 to 1	19 to 1	18 to 1

EXPECTATIONS of Life.

	Pais De Vaud.	Country Pariſh in Brandenburg	Vienna.	Berlin.
At birth	37 yrs	32½ years	16¼ yrs	18 yrs
Age 12	44⅕	44	35¾	35½
25	34¾	35½	28⅓	27⅓
30	31¼	31½	25½	25¼
35	27½	28	22½	22¼
40	24	25	20½	20¼
45	20½	21½	17¼	18¼
50	17½	18	16⅓	16⅓
55	14½	15	13½	14
60	12	12¼	11¾	12½

towns, are *emigrants* from the country, who came to them in full maturity, after eſcaping the weakneſs of infancy. And that alſo in general theſe *emigrants* conſiſt of the more hearty and robuſt part of the kingdom. On both theſe accounts the number of inhabitants (including *aliens* as well as *natives*) attaining old age in great towns ought to be much greater than in the country. In *London*, *Vienna*, and *Berlin*, it ought to be nearly *double*; but we ſee, that, in reality, it is ſcarcely *half*. There are no obſervations from which the proportion of *natives* in great towns, who live to 80, can be deduced with correctneſs, except thoſe made at *Stockholm*; and theſe prove, that of females one in a 100, and of males one in 300, live to 80.——See Vol. I. p. 273; and this Volume, p. 13; and Table XLIV, p. 158.

From

From this compariſon (*a*) it appears with how much truth great cities have been called the *graves* of mankind. It muſt alſo convince all who will conſider it, that, according to the obſervation at the end of the Fourth Eſſay in the former Volume, it is by no means ſtrictly proper to conſider our diſeaſes as the original intention of nature. They are, without doubt, in general, our own creation. Were there a country, where the inhabitants led lives entirely natural and virtuous, few of them would die without meaſuring out the whole period of preſent exiſtence allotted them; pain and diſtempers would be unknown among them; and death would come upon them like a ſleep, in conſequence of no other cauſe than gradual and unavoidable decay.—Let us then, inſtead of charging our Maker with our miſeries, learn more to accuſe and reproach *ourſelves*.

The reaſons of the baleful influence of great towns, as it has been now exhibited, are plainly,

(*a*) A more diſtinct and ſtriking compariſon of this kind may be drawn from the Tables for *London* and the pariſh of *Holy-Croſs*; and from the Tables for *Stockholm* and *Sweden* at large in the preceding collection of Tables. See the Introduction to theſe Tables.

Firſt,

Firſt, The irregular modes of life, the luxuries, debaucheries, and pernicious cuſtoms, which prevail more in towns than in the country.

Secondly, The foulneſs of the air in towns, occaſioned by uncleanlineſs, ſmoak, the perſpiration and breath of the inhabitants, and putrid ſteams from drains, church-yards, kennels, and common-ſewers.—It is, in particular, well known that air, ſpoiled by breathing, is rendered ſo noxious, as to kill, inſtantaneouſly, any animal that is put into it. There muſt be cauſes in nature (*a*) continually operating, which reſtore the air after being thus ſpoiled. But in towns it is, probably, conſumed faſter than it can be adequately reſtored; and the larger the town is, or the more the inhabitants are crowded together, the more this inconvenience muſt take place.

(*a*) A celebrated and excellent philoſopher has for ſome time been employed in enquiring into theſe cauſes; and, among other curious and important facts, he has diſcovered, that one of theſe cauſes is the vegetation of plants, and the action of light upon them. See the Fourth and Fifth Volumes of Dr. *Prieſtley*'s Experiments on Air; and an Oration on preſenting him with a prize-medal, delivered by Sir *John Pringle*, Preſident of the Royal Society.——See, likewiſe, *Experiments on Vegetables, diſcovering their Power of purifying common Air in Sunſhine*, &c. by Dr. *Ingenhouſz*, Counſellor of the Court, and Body Phyſician to their Imperial and Royal Majeſties, F. R. S. &c.

But

But I muſt proceed to ſome more of Mr. *Muret*'s obſervations.——At the end of the Fourth Eſſay in the former Volume, &c. I have given an account of ſeveral facts which prove the probabilities of life to be higher among females than males. Agreeably to this it appears, that in the diſtrict of VAUD, half the *females* don't die till the age of 46 and upwards, though half the *males* die under 36. This great difference is in ſome meaſure owing to the military and commercial emigrations among the males; but it appears undeniably, that their greater mortality contributes likewiſe to it. The number of *males* who died, for a courſe of years, in 39 pariſhes of this diſtrict, was 8170; of *females* 8167; of whom the numbers that died under one year of age were 1817 *males*, and 1305 *females*; and under 10 years of age, 3099 *males*, and 2598 *females*. In the *beginning* of life, therefore, and before any emigrations can take place, the rate of mortality among *males* appears to be much greater than among *females*: And this is rendered yet more certain, by the account Mr. *Muret* gives of the proportion of the deaths among males and females in the *firſt* year of life at VEVEY. In this town, he acquaints us, that for 20 years ending in 1764, there died in the firſt month, of *males* 135, to 89 *females*; and, in the firſt year, 225 to 162.——To the ſame effect it appears

appears, from a Table given by *Suſmilch* (*a*), that in BERLIN 203 *males* die in the firſt month, and but 168 *females*; and in the firſt year, 489 to 395; and alſo, from a Table of *Struyck*'s, that in HOLLAND, 396 *males* die in the firſt *year*, to 306 *females*.—What is moſt of all remarkable is, that theſe accounts ſhew, that both at VEVEY and BERLIN the *ſtill-born males* are to the *ſtill born females*, as 30 to 21, or nearly in the proportion given by the accounts referred to in Vol. I. p. 364.

The whole number of inhabitants at VEVEY in 1764, was 3350. Of theſe 1931 were females, and only 1419 males. Sixty-ſix were *widowers*, and 200 *widows*. The number of *bachelors*, above 16 years of age, was 529; and of *virgins*, above 14 years of age, 734. See Mr. *Muret*'s Tables, p. 124.

Mr. *De Parcieux* at PARIS, and Mr. *Wargentin* in SWEDEN, have obſerved, that not only *women* live longer than *men*, but that *married* women live longer than *ſingle* women. The regiſters examined by Mr. *Muret* confirm this; and it appears in ſome of them, that, of equal numbers of *ſingle* and *married* women between 15 and 25, more of the former died than of the latter, in the proportion of 2 to 1. This is a difference ſo great, that it muſt, I ſuppoſe, have been in ſome degree accidental. The fact, how-

(*a*) See *Suſmilch's Gottliche Ordnung*, Vol. II. p. 317, &c.

ever,

ever, in general, when underſtood with abatements for that part of female life which is moſt expoſed to the dangers of child-bearing, is highly probable; for firſt, the women who marry are likely to be a ſelect body, conſiſting of the more healthy and vigorous part of the ſex. And ſecondly, it is reaſonable to expect that in this, as well as in all other inſtances, the conſequences of following nature muſt be favourable.

The facts recited here, and at the end of the Fourth Eſſay, prove (*a*), that there is a difference between the mortality of males and females.—I muſt however obſerve, that it may be doubted, whether this difference, ſo unfavourable to males, is *natural*; and the following facts will prove, that I have reaſon for ſuch a doubt.

It appears, from ſeveral regiſters in *Suſmilch*'s works, that this difference is much leſs in the *country pariſhes* and *villages* of BRANDENBURG, than in the *towns*: And, agreeably to this, it appears likewiſe, from the accounts of the ſame curious writer, that the number of males in the country comes much nearer to the number of females.

In 1056 ſmall *villages* in BRANDENBURG, the *males* and *females*, in 1748, were 106,234,

(*a*) This is put out of all doubt in the preſent Edition of this work, by the Tables in the preceding collection, deduced from the *Cheſter* and *Sweden* obſervations.

and

and 107,540, or to one another as 100 to 101$\frac{1}{5}$. In twenty ſmall *towns* they were 9544, and 10,333; or as 100 to 108$\frac{1}{4}$. In BERLIN they were, excluſive of the garriſon, 39,116 and 45,938; or as 100 to 117$\frac{1}{2}$.

At the time the accounts, mentioned in p. 276, Vol. I. were taken of the inhabitants in the province of NEW-JERSEY in AMERICA, they were diſtinguiſhed particularly into *males* and *females* under and above 16.

In 1738, the number of
Males under 16 was, 10639. Females 9700
Males above 16 —— 11631. Females 10725

In 1745, theſe numbers were,
Males under 16 —— 14523. Females 13754
Males above 16 —— 15087. Females 13704

The inference from theſe facts is very obvious. They ſeem to ſhew ſufficiently, that human life in males is more brittle than in females, only in conſequence of adventitious cauſes, or of ſome particular debility, that takes place in poliſhed and luxurious ſocieties, and eſpecially in great towns (*a*).

(*a*) See on this ſubject the remark at the end of Table XLIV. p. 161.

It will not be amiſs to inſert here the following accounts of the mortality of *ſummer* compared with that of *winter*, that is of the four months, *June*, *July*, *Auguſt*, and *September*, compared with *December*, *January*, *February*, and *March*.

The deaths for 60 years at VEVEY in the former months, were to the deaths in the latter, as 2140 to 1697,

or

or 5 to 4. (See Mr. *Muret*'s Tables, p. 100). In LONDON and at PARIS, this proportion is nearly the ſame. At EDINBURGH, as 4 to 3. In 25 country towns and pariſhes mentioned by Dr. *Short* (*New Obſervations*, p. 142) as 50 to 41.——The ſick admitted into the *Hotel Dieu* at *Paris*, for 40 years, from 1724 to 1763, were, in the former months, 314,824; in the latter, 238,522, or as 4 to 3. See *Recherches ſur la Population*, &c. par *M. Meſſance*, p. 181.——It is remarkable that the births alſo in winter to thoſe in ſummer, are, at VEVEY, as 5 to 4; in LONDON, as 8 to 7; in the country towns and pariſhes juſt mentioned as 7 to 6.

Annual average of births and deaths in all SWEDEN *for* 13 *years.—See the Memoirs of the Royal Academy of Sciences at* Stockholm, *publiſhed at* Paris, 1772, p. 20, &c.

	Births	Deaths
In the four ſummer months —	28080	18880
In the four winter months —	31327	20690
In *April* and *May* — —	14078	12274
In *October* and *November* —	17178	8612

Annual average of births and deaths in STOCKHOLM *for five years.* Ibid.

	Births	Deaths
Summer ——— —	933	1515
Winter ——— ——	870	1139
April and *May* ——	426	739
October and *November* ———	469	645

Whole number of births and deaths at Gainſborough *for* 20 *years ended at* 1771.

	Births	Deaths
Summer ———	779	590
Winter — —	811	765
April and *May* ———	427	390
October and *November* ———	410	345

Whole

Whole number of deaths at WARRINGTON *in* Lancaſhire, *for eight years ended at* 1780.

	Deaths
Summer	692
Winter	968
April and *May*	508
October and *November*	280

Whole number of deaths at MANCHESTER *for nine years ended at* 1780.

	Births	Deaths
Summer	3308	1788
Winter	3608	2427
April and *May*	1956	1098
October and *November*	1736	1022

Whole number of deaths at ECCLES *near* MANCHESTER, *for five years ended at* 1779.

	Births	Deaths
Summer	440	415
Winter	521	455
April and *May*	314	226
October and *November*	212	234

The deaths at CHESTER, for the years 1772, 1773, and 1774, were, in ſummer 340; in winter, 478; in *April* and *May*, 185; in *October* and *November*, 274. And they were more numerous in *Autumn* than *Spring*, only becauſe in one of theſe years the ſmall pox carried off 90 perſons in *October* and *November*.

Of POPULATION; *the general Cauſes which promote or obſtruct it; and the* preſent *State of it in* ENGLAND *compared with its State* formerly.

FROM the proportion of the births to the deaths in the diſtrict of VAUD, as mentioned in p. 254, it follows, by the rule in the Note, Vol. I. p. 278, that the inhabitants ought to double their own number in 120 years. But the fact is, that ſo many migrate into foreign armies and with commercial views, that their increaſe is ſcarcely ſenſible. Mr. *Muret*, after obſerving this, enters into a general account of the cauſes which obſtruct population in his country. Among theſe he inſiſts particularly on LUXURY and the ENGROSSING OF FARMS. I wiſh his obſervations on theſe ſubjects were not applicable to the preſent ſtate of this kingdom: But, perhaps, there is no kingdom in the world to which they are *ſo* applicable.—In conſequence of the eaſy communication, lately created, between the different parts of the kingdom, the LONDON faſhions and manners and pleaſures, have been propagated every where; and almoſt every diſtant town and village now vies with the capital in all kinds of expenſive diſſipation

and amuſement. This enervates and debilitates; and, together with our taxes, raiſes every where (*a*) the price of the means of ſubſiſtence, checks marriage, and brings on poverty, dependance, and venality.—With reſpect, particularly, to the cuſtom of *engroſſing farms*, Mr. *Muret* obſerves, with the higheſt reaſon, that a large tract of land, in the hands of one man, does not yield ſo great a return, as when in the hands of ſeveral, nor does it employ ſo many people; and, as a proof of this, he mentions two pariſhes in the diſtrict of VAUD, one of which (once a little village) having been bought by ſome rich men, was ſunk into a ſingle *demeſne*; and the other (once a ſingle *demeſne*), having fallen into the hands of ſome peaſants, was become a little village.—How many facts of the former kind can this country now furniſh?—And there is reaſon to apprehend they will go on increaſing.—The cuſtom of engroſſing farms eaſes *landlords* of the trouble attending the neceſſities of little tenants and the repairs of cottages.—A great farmer, by having it more in his power to

(*a*) The price of corn, in particular, has for ſome time been complained of by the poor as oppreſſively high, though far from being ſo high as it generally was at the end of the laſt century. This is a ſtriking fact which implies that the *lower* part of the nation are now more diſtreſſed than ever. The conſequence has been a reduction of their number; and this is the effect that muſt go on increaſing, with increaſing luxury and taxes.

ſpeculate

ſpeculate and command the markets, and by drawing to himſelf the profits which would have ſupported ſeveral farmers, is capable, with leſs culture, of paying a higher rent. Our ſuperiors, therefore, find their account in this evil —But it is, indeed, erecting *private* benefit on *public* calamity; and, for the ſake of a temporary advantage, giving up the nation to depopulation and diſtreſs.——We have, for many years, been feeling the truth of this obſervation (*a*).

Dr. *Davenant* (the beſt, while not venal, of all political writers), tells us, that at *Michaelmas*, in the year 1685, it appeared

(*a*) "Thoſe who contribute towards the deſtruction "of ſmall farms" (ſays a gentleman of great knowledge and experience in this way) "can have very little re- "flexion. If they have, their feelings are not to be "envied. Where this has been the practice, we ſee a "vaſt number of families reduced to poverty and miſery, "the poor rates much increaſed, the ſmall articles of "proviſion greatly diminiſhed in quantity and number, "and conſequently augmented in price."—See *Hints to Gentlemen of Landed Property*, printed for Mr. *Dodſley* in 1776, p. 223, &c. &c.; where the pernicious tendency of large farms ſeems abundantly proved. There are thouſands of pariſhes, he ſays, which, ſince little farms have been ſwallowed up in greater, do not ſupport ſo many cows as they did by 50 or 60 in a pariſh; and the inhabitants have decreaſed in proportion.——He concludes his obſervations on this ſubject with expreſſing "his anxious wiſhes that the deſtructive practice of en- "groſſing farms may be carried no farther, the ſtab al- "ready given by it to plenty and population having "greatly affected the proſperity of this country."

by a ſurvey of the hearth-books (*a*), that the number of houſes in all ENGLAND and WALES was 1,300,000, of which 554,631 were houſes of only one chimney. See Dr. *Davenant*'s Works, Vol. II. p. 203.—In his *Eſſay on Ways and Means*, &c. Vol. I. p. 33, he gives a particular account of the number of houſes in every county, according to the *hearth-books* of Lady-day, 1690; and the ſum total then was 1,319,215.—At the *reſtoration* it appeared by the ſame hearth-books, that the number of houſes in the kingdom (*b*), was 1,230,000.—In the interval, therefore, between the *reſtoration* and the *revolution*; the people of ENGLAND had increaſed above 300,000; and "of SMALLER "TENEMENTS, Dr. *Davenant* obſerves (*c*), "there had been, from 1666 to 1688, about "70,000 new foundations laid."—But what a reverſe has taken place ſince?—In 1759 the number of houſes in ENGLAND and WALES was 986,482; of which not more than 330,000 were houſes having leſs than

(*a*) At this time there was a tax of two ſhillings on every *fire-hearth*; which was taken off at the REVOLUTION, becauſe reckoned "not only a great oppreſſion to "the poorer ſort, but a badge of ſlavery on the whole "people, expoſing every man's houſe to be entered into "and ſearched at pleaſure by perſons unknown to him." *Preamble to the Act for taking away the revenue ariſing by hearth-money.* 1 William and Mary, Chap. 10.

(*b*) Continuation of *Rapin*, Vol. I. p. 53.

(*c*) Dr. *Davenant*'s Works, Vol. I. p. 370.

ſeven

ſeven windows; and 282,429 were *cottages* not charged on account of poverty.——In 1765, notwithſtanding the increaſe of buildings in LONDON, the number of houſes was reduced to 980,692 (*a*); of which 276,149 were *cottages* not charged. According to theſe accounts then, our people have, ſince the year 1690, decreaſed near a *million and a half*.—And the waſte has fallen principally on the inhabitants of cottages; nor indeed could it fall any where more unhappily; for, from cottages our navies and armies are ſupplied, and the lower people are the chief ſtrength and ſecurity of every ſtate (*b*).——What

(*a*) See *Conſiderations on the Trade and Finances of this Kingdom*, p. 95, 97, 98, printed for *Wilkie*, 1766. See alſo Vol. I. p. 246, &c. of this Treatiſe; and my *Appeal to the Public on the Subject of the National Debt*, p. 86, &c.—It deſerves particular notice, with reſpect to the accounts here given of the number of houſes in 1759 and 1765, that, being returns made by the ſurveyors of the houſe and window-duties throughout all ENGLAND and WALES, they are ſubject to no ſuch deficiencies as thoſe in the account of the number of houſes in LONDON, taken by Mr. *Maitland* from the *pariſh* books, and mentioned in the Note, Vol. I. p. 246.—The reaſon is, that no landlord or tenant can ever conſent that any *two or more houſes* belonging to him, ſhould be charged by the aſſeſſors of the window-tax as *ſingle* houſes; becauſe, in this caſe, he would be taxed too high, and pay more than the law required.——The number of houſes, therefore, ſubject to the houſe and window-duty, given in the above returns, muſt probably be the full number of ſuch houſes in the kingdom.

(*b*) Cottagers are indiſputably the moſt beneficial race of people we have: "They are bred up in greater ſim-

What renders this calamity more alarming is, that the inhabitants of the cottages thrown down in the country, fly to LONDON and other towns, there to be corrupted and periſh (*a*).—I know I ſhall be here told that

" plicity, live more primitive lives, more free from vice " and debauchery, than any other ſet of men of the lower " claſs; and are beſt formed and enabled to ſuſtain the " hardſhips of war, and other laborious ſervices. Great " towns are deſtructive both to morals and health, and " the greateſt drains we have; for where many of the " lower ſort of people crowd together, as in *London*, " *Norwich*, *Birmingham*, and other manufacturing towns, " they are obliged to put up with bad accommodations, " and an unwholeſome and confined air, which breeds " contagious diſtempers, debilitates their bodies, and " ſhortens their lives.——Since, therefore, it is appa- " rent that all ſuch towns muſt cauſe a diminution or " waſte of people, we cannot be at a loſs to trace the " ſpring which feeds theſe channels. The country muſt " be the place; and cottages and ſmall farms the chief " nurſeries which ſupport population."—*Hints to landed Gentlemen*, p. 243, 244.——In what follows a repreſentation is made of the miſery of cottagers in their preſent ſtate, and propoſals offered for better accommodating and encouraging them, which do honour to Mr. *Kent*'s public ſpirit and humanity.

(*a*) Dr. *Davenant* ſays, from Mr. *King*'s Obſervations, " that the ſupply of LONDON alone takes up above *half* " the neat increaſe of the kingdom."—Is it then to be wondered at, that the ſupply of the waſte in *all* the towns of the kingdom, added to that increaſe of luxury and taxes, and of the drain to our *armies*, and *navies*, and *foreign ſettlements*, which has taken place within theſe 70 years, ſhould have ſo far exceeded the increaſe of the kingdom, as to produce the depopulation I have mentioned?—It has been aſſerted by political calculators, that no population can bear more than one ſoldier for every hundred ſouls.

that the *Revenue* thrives. But this is not a circumſtance from which any encouragement can be drawn. It thrives, by a cauſe that is likely in time to deſtroy both itſelf and the kingdom; I mean, by an increaſe of luxury (*a*), producing ſuch an increaſe of *conſumption* and *importation* as ſecretly *accelerates* ruin, while *at preſent* (as far as the Revenue is concerned) it overbalances the effects of depopulation.——What remedies can be applied in ſuch circumſtances?——The anſwer is obvious.

ſouls. This is ſaying a great deal too much; but were it true, the number of our ſoldiers and ſailors, even in *peace*, would alone be ſufficient to reduce us to nothing in a little time.

A flouriſhing commerce, though favourable to population in ſome reſpects, is, I think, on the whole, extremely unfavourable; and, while it flatters, may be deſtroying: particularly, by increaſing luxury, the worſt enemy of population, as well as of public virtue; and, by calling off too many perſons from agriculture to unhealthy trades and the ſea-ſervice.—Suppoſe 100,000 ſoldiers and ſailors, added to other burdens, to have been formerly the whole number the nation could bear without decreaſing. In ſuch circumſtances, it is plain, that any cauſes which doubled or tripled that number, would depopulate with rapidity.

(*a*) For example. In LONDON, thoſe who uſed to ſatisfy themſelves with *one* houſe, or perhaps *half* a houſe, muſt now have *two* houſes. Thoſe who uſed to live plain, muſt now live high; and thoſe who uſed to *walk*, muſt now be *carried*. This is the reaſon of the increaſe of conſumption and of buildings in LONDON, and not an increaſe of the inhabitants, for the number of inhabitants is certainly (if any regard is due to the bills) leſs now than it was fifty years ago.

Enter immediately into a deciſive enquiry into the ſtate of population in the kingdom.—Promote agriculture.—Drive back the inhabitants of towns into the country.—Eſtabliſh ſome regulations for preſerving the lives of infants.—Diſcourage luxury, and celibacy, and the engroſſing of farms.—Let there be entire liberty; and maintain public peace by a government founded, not in *conſtraint*, but in the *reſpect* and the *hearts* of the people.—But above all things, if it be not now too late; "find out means of avoiding the miſe-"ries of an impending bankruptcy, and of "eaſing the nation of that burden of debts "and taxes under which it is ſinking."

I will here enter a little more minutely into the conſideration of ſome of the heads now mentioned, and of the preſent compared with the former ſtate of the body of the people in this kingdom.

One of the moſt obvious diviſions of the ſtate of mankind is, into the *wild* and the *civilized* ſtate. In the former, man is a creature rude, ignorant, and ſavage; running about in the woods; and living by hunting, or on the ſpontaneous productions of the earth. In this ſtate, the means of ſubſiſtence being ſcarce, and a large quantity of ground neceſſary to ſupport a few, there can never be any conſiderable increaſe.—In the latter

ſtate,

ſtate, man is a creature fixed on one ſpot, employing himſelf in cultivating the ground, and enjoying the advantages of ſcience, arts, and civil government. Of this laſt ſtate there are many different degrees or ſtages, from the moſt ſimple to the moſt refined and luxurious. The firſt or the ſimple ſtages of civilization, are thoſe which favour moſt the increaſe and the happineſs of mankind: For in theſe ſtates, agriculture ſupplies plenty of the means of ſubſiſtence; the bleſſings of a natural and ſimple life are enjoyed; property is equally divided; the wants of men are few, and ſoon ſatisfied; and families are eaſily provided for.——On the contrary. In the refined ſtates of civilization property is engroſſed, and the natural equality of men ſubverted; artificial neceſſaries without number are created; great towns propagate contagion and licentiouſneſs; luxury and vice prevail; and, together with them, diſeaſe, poverty, venality, and oppreſſion. And there is a limit at which, when the corruptions of civil ſociety arrive, all liberty, virtue, and happineſs muſt be loſt, and complete ruin follow.—Our *American* colonies are at preſent, for the moſt part, in the firſt and the happieſt of the ſtates I have deſcribed; and they afford a very ſtriking proof of the effects of the different ſtages of civilization on population. In the inland parts of NORTH-AMERICA, or the back ſettlements, where the

the modes of living are moſt ſimple, and almoſt every one occupies land for himſelf, there is an increaſe ſo rapid as to have hardly any parellel. Along the ſea-coaſt, where trade has begun to introduce refinement and luxury, the inhabitants increaſe more ſlowly: And in the maritime towns (if I may judge from the bills of mortality at BOSTON, mentioned in p. 268, Vol. I.) they do not increaſe at all (*a*).

But to confine my thoughts to my own country.—Here, it is too evident that we are far advanced into that laſt and worſt ſtate of ſociety, in which falſe refinement and luxury multiply wants, and debauch, enſlave, and depopulate.—Among the evils of this ſtate, and the cauſes of depopulation, I have mentioned the accumulation of property.

"Only revive, ſays Mr. *Suſmilch*, the "laws of *Licinius*, forbidding any Roman "to hold more than ſeven *jugera* of land; "or that of *Romulus*, which limited every "Roman to two *jugera*, and you will ſoon "convert a barren deſart into a buſy and "crowded hive."—The doubts of ſome ingenious men on this ſubject, have, indeed, greatly ſurprized me. I can ſcarcely think

(*a*) Along the ſea-coaſt they double their own number in about 35 years; but in the back-ſettlements, in 15 years. See *Eſſay* I. Vol. I. p. 276; and p. 109 of *A Diſcourſe on Chriſtian Union*, by Dr. STYLES, now the worthy Preſident of the College of YALE in CONNECTICUT.

of

of a more evident maxim, than that "the "diviſion of property promotes population." —Let a tract of ground be ſuppoſed in the hands of a multitude of little proprietors and tenants, who maintain themſelves and families by the produce of the ground they occupy, by ſheep kept on a common, by poultry, hogs, &c.; and who, therefore, have little occaſion to purchaſe any of the means of ſubſiſtence. If this land gets into the hands of a few great farmers, the conſequence muſt be, that the little farmers will be converted into a body of men who earn their ſubſiſtence by working for others, and who will be under a neceſſity of going to market for all they want (*a*). And, ſubſiſtence in this way being difficult, families of children will become burdens, marriage will be avoided, and population will decline.——

(*a*) "Every ſpeculative Engliſhman," ſays Mr. *Kent*, "who travels through the *Auſtrian Netherlands*, is aſto-"niſhed at the great population of that country, and at "the ſight of the markets, which are plentiful beyond "deſcription. Upon enquiring into the internal ſtate and "regulation of the country, he finds that there are no "large farms, no claſs of men who paſs under the cha-"racter of gentlemen farmers, acquiring large fortunes "merely by ſuperintending the buſineſs of farming; but "that the whole country is divided into much ſmaller "portions than land is with us, and occupied by a ſet of "laborious people, who in general work for themſelves, "and live very much on a footing of equality."——See *Hints to Gentlemen of Landed Property*, p. 217.

At

At the ſame time there will, perhaps, be more labour, becauſe there will be more compulſion to it. More bread will be conſumed, and, therefore, more corn grown; becauſe there will be leſs ability of going to the price of other food. Pariſhes, likewiſe, will be more loaded, becauſe the number of poor will be greater. And towns and manufactures will increaſe, becauſe more will be driven to them in queſt of places and employments.—This is the way in which the engroſſing of farms naturally operates: And this is the way in which, for many years, it has been actually operating in this kingdom.

It deſerves particular notice, that the obſervations now ſuggeſted ſhew, that the very cauſes which produce depopulation among us, may, for ſome time, promote tillage; and I will take this opportunity to add, that they will alſo account for the following fact.—In the year 1697, wheat was at 3*l.* a quarter, and other grain proportionably dear. But there was no clamour, and the exportation went on. See a valuable and uſeful pamphlet, entitled, *Three Tracts on the Corn Trade*, page 100, 107, 145. At preſent, though the quantity of money (or of what paſſes for money) is doubled, when wheat is below this price, and in general before any grain, except oats, gets above the

the prices at which the law uſed to allow a bounty on exportation, there is an alarm, the poor are ſtarving, and the exportation is prohibited. I referred to this fact in the Note, p. 274; and the true reaſon of it ſeems to be, that the high price of bread was not, at the time I have mentioned, of eſſential conſequence to the lower people, becauſe they could live more upon other food which was then cheap; and becauſe alſo being more generally occupiers of land, they were leſs under a neceſſity of purchaſing bread. Whereas now, being forced by greater difficulties, and the high price of all other food, to live principally or ſolely on bread, if that is not cheap, they are rendered incapable of maintaining themſelves.

In confirmation of this account, I will beg leave to mention, that though during the whole laſt century, corn (wheat, rye, oats, and barley) was generally dearer than it has been, at an average, for 40 years to 1773; yet fleſh-meat was about half its preſent price: And that, in an *Act of Parliament* of the 25th of Henry VIII. beef, veal, pork, and mutton are mentioned as the food of the poor, and their price limited to about a halfpenny a pound. Beef and pork, in particular, were ſold in LONDON at two pounds and a half, and three pounds for a penny; at the ſame time that wheat was

was at 7*s.* and 8*s.* a quarter (*a*), and bore the ſame proportion to the price of fleſh as it would bear now, were it at about 4*l.* a quarter. See *Chronicon Pretioſum*, p. 116.—

It

(*a*) Even ſo far back as the year 1463, the price of wheat was reckoned not too high at 6*s* 8*d.* per quarter; nor that of barley at 3*s.* and rye at 4*s.*; for it was in that year enacted, that the *importation* of theſe three ſorts of grain ſhould not be allowed till they got above theſe prices. See Mr. *Anderſon's Chronological Deduction of Commerce*, Vol. I. p. 280.

By a ſtatute of 1 *Philip* and *Mary*, 1553, leave was given to *export* theſe three kinds of grain till they roſe to theſe prices. *Ib.* p. 387.

By an ordinance in 1563, the exportation prices were fixed to 10*s.* per quarter for *wheat*; 8*s.* for *rye*, *peaſe*, and *beans*; and 6*s.* 8*d.* for *malt*.—And in 1593, to 1*l.* for *wheat*; 13*s.* 4d. *peaſe* and *beans*; and 12*s.* *barley* and *malt*. Ib. p. 401 and 442.

PRICES per QUARTER.

		Of Wheat.			Of Malt.			Of Oats.		
		l.	*s.*	*d.*	*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
In	1491,	0	14	8—0	0	0	—0	00	0	
	1494,	0	4	0—0	0	0	—0	00	0	
	1504,	0	5	8—0	0	0	—0	00	0	
	1512,	0	6	2—0	4	0	—0	2	0	
	1521,	1	0	0—0	0	0	—0	00	0	
From 1553 to—	1556,	0	8	0—0	5	0	—0	00	0	
Beforeharveſt, in	1557,	2	13	4—2	4	0	—0	00	0	
After harveſt, in	1557,	0	8	0—0	5	0	—0	10	0	
	1560,	0	8	0—0	5	0	—0	5	0	
Beforeharveſt, in	1574,	2	16	0—0	0	0	—0	00	0	
After harveſt, in	1574,	1	4	0—0	0	0	—0	00	0	
	1587,	3	4	0—0	0	0	—0	00	0	

A dearth

It appears, indeed, that our anceſtors took great care to keep the price of fleſh low for the poor; and this was one of the reaſons of the many proclamations publiſhed by Queen *Elizabeth*,

		Of Wheat. *l. s. d.*	Of Malt. *l. s. d.*	Of Oats. *l. s. d.*
A dearth occaſioned by exceſſive exportation; & in 1596 by great rains	1594,	2 16 0——0 0 0	——0 00 0	
	1595,	2 13 4——1 0 0	——0 00 0	
	1596,	4 0 0——1 6 8	——0 00 0	
	1597,	5 4 0——2 6 4	——0 00 0	
AVERAGE PRICE,				
From 1606 to —	1706,	1 18 6——1 2 0	——0 00 0	
From 1707 to —	1765,	1 12 6——1 1 9	——0 00 0	
From 1766 to —	1772,	2 3 6——0 0 0	——0 19 0	

See Bp. *Fleetwood's Chronicon Pretioſum*, from p. 113 to p. 124; and *Three Tracts on the Corn Trade*, p. 98, &c.

With theſe prices of *corn* let us compare the prices of *fleſh*, at two or three different periods.

In 1512, the price of wheat was from 5*s.* 8*d.* to 6*s.* 8*d.* in *Yorkſhire*. See the *Regulations and Eſtabliſhment of the Houſhold of Henry Algernon Percy, the fifth Earl of Northumberland, at his Caſtles of Wreſill and Lekingfield, in Yorkſhire*, begun Anno Dom. 1512, page 2, 4. Let us call the mean price 6*s.* 2*d.* The price of malt was 4*s.* and of oats 2*s.* We may therefore reckon, that the *nominal* price of grain at this time was about a ſeventh of its *nominal* price for the laſt 40 years.

The price of a fat ox at the ſame time, and in the ſame county, was 13*s.* 4*d.*; of a lean ox, 8*s.*; of a weather, 1*s.* 8*d.*; of a calf, 1*s.* 8*d.*; of a hog, 2*s.* Ib. p. 5, 6, 7.—The nominal price of meat, therefore, was no more than about a 15th of its preſent price, and bore the ſame proportion to the price of corn that it would now bear, were it at *half* its preſent price.—A like inference may be drawn from comparing the following prices:

Wheat, in 1549, was about 12*s.* per quarter in LONDON. Malt, 10*s.* Barley, 9*s.* Rye, 6*s.* 6*d.* Oats, 4*s.*

—A

Elizabeth, *James* I. and *Charles* I. againſt eating fleſh in Lent and on fiſh days; and againſt the erection of new buildings in *London*, and the reſidence in it of the nobility and gentry.

—A middling ox, 1*l.* 18*s.* A weather, 3*s.* Butter, *three farthings* and a *penny* a pound. Cheeſe, a *halfpenny* a pound. See *Maitland*'s Hiſtory of London, page 143, 144.

"In 1574, there was a great dearth, and wheat was "before harveſt, at 2*l.* 16*s.* per quarter; and beef at "*Lammas* ſo dear, as to be ſold at twopence-halfpenny "a pound." See *Chronicon Pretioſum*, p. 123. That is, beef compared with wheat, was at leaſt one half cheaper than it is now.

In 1445, wheat was at 4*s.* 6*d.* per quarter. In 1447, at 8*s.* In 1448, at 6*s.* 8*d.* In 1449, 5*s.*—A bullock, in 1445, 5*s.* A ſheep, 2*s.* 5*d.*½. A hog, 1*s.* 11*d.*½. ——Fine cloth for ſurplices, in 1446, 8*d.* per ell. Cloathing for a year, at the ſame period, of a common ſervant of huſbandry, 3*s.* 4*d.* Of a chief carter and ſhepherd, 4*s.* Of a bailiff of huſbandry, 5*s.* Ib. page 108, 109, 160.—*Cloathing*, therefore, at this time, ſeems to have been cheaper in compariſon of the price of corn than even fleſh.

The weight of ſilver coin *formerly*, to the weight of ſilver coin of the ſame denomination *now*, was from 1461 to 1509, as 62 to 37½. From 1509 to 1543, as 62 to 45. From 1552 to 1600, as 62 to 60. And from 1600 to the preſent time as 62 to 62. But nothing depends on this in the preſent enquiry; the object of which is, not the proportion of the prices of the different articles of ſubſiſtence *now* to their prices *formerly*, but the proportion TO ONE ANOTHER of their prices *now*, in compariſon with the ſame proportion *formerly*. And this may be as well deduced from the *nominal* as from the *abſolute* prices.—Thus. The price of bread now is nearly the ſame that it was 100 years ago; but, in *compariſon* with the price of beef and mutton, it is at leaſt *one half* cheaper.

The

The reaſon now aſſigned accounts farther for the great variations in the price of grain which uſed to take place formerly. Theſe were ſuch as could not be now endured; but, bread being then leſs a neceſſary article of ſubſiſtence, they were leſs felt and regarded.

I have taken for granted, in thoſe obſervations, that the quantity of ground brought under tillage in this kingdom is now more than ever it was. This is generally believed; and, if true, the cauſes of it have been thoſe I have mentioned, in conjunction with the encouragement given to the growth of corn by the bounty on exportation, and the increaſe of luxury occaſioning an increaſe of horſes, and rendering even the poor averſe to all bread except that made of the (*a*) fineſt flour. But, perhaps, the fact may not be ſo certain as ſome think it. At leaſt, there is reaſon to apprehend, that whatever the increaſe of tillage might have been for 50 or 60 years after the Revolution, it is now at an end.—I have lately received an account of a large common field in *Leiceſterſhire*, which uſed to produce annually 800 quarters of corn, beſides maintaining 200 cattle; but

(*a*) Bread made of *bran*, and even of *peaſe* and *beans*, was formerly not uncommon among the lower people. But no diſtreſſes could force them now to eat ſuch bread, or even to live upon *rice*, though the food of a conſiderable part of the reſt of mankind. See the *Earl of Northumberland's Houſehold Book*, Preface, p. 13, &c.

which now, in conſequence of being incloſed and getting into few hands, produces little or no corn; and maintains no more cattle than before, though the rents are conſiderably advanced.—This is only one inſtance among many of an evil that has been prevailing for ſome time, and which is the general effect of the laws for incloſing open fields.—In *Northamptonſhire* and *Leiceſterſhire*, incloſing has greatly prevailed; and moſt of the new-incloſed lordſhips, ſays a very ſenſible writer, "are turned into paſturage; in "conſequence of which, many lordſhips "have not now 50 acres ploughed yearly, "in which 1500, or at leaſt 1000 were "ploughed formerly; and ſcarce an ear of "corn is now to be ſeen in ſome that bore "hundreds of quarters.——And ſo ſeverely "are the effects of this felt, that worſe "wheat has been lately ſold in theſe coun-"ties on an average, at 7*s.* and 7*s.* 6*d.* the "*Wincheſter* buſhel, for many months to-"gether, than uſed to be ſold at 3*s.* 6*d.* "and 4*s.* And 5*s* and 5*s.* 6*d.* has been "given for malt that has been uſually "bought there at little more than half-a-"crown." See a pamphlet, entitled, *An Enquiry into the Reaſons for and againſt incloſing Open Fields*; by the Rev. Mr. *Addington*. Publiſhed in 1772 for Mr. *Buckland*, *Paternoſter Row*.—In the counties of *Northampton* and *Leiceſter*, ſays the ſame writer, p. 43, "the

"the decreaſe of the inhabitants in almoſt "all the incloſed villages in which they "have no conſiderable manufacture, is ob-"vious to be remarked by every one who "knew their ſtate 20 or 30 years ago, and "ſees them now; and that to a degree that "cannot but give every true friend to his "country the moſt ſenſible concern. The "ruin of former dwelling-houſes, barns, ſta-"bles, &c. ſhew every one who paſſes "through them that they were once better "inhabited. A hundred houſes and families "have in ſome places, dwindled into eight "or ten.—The landholders, in moſt pa-"riſhes that have been incloſed only 15 "or 20 years, are very few in compariſon "of the numbers who occupied them in "their open field ſtate. It is no uncommon "thing to ſee four or five wealthy graziers "engroſſing a large incloſed lordſhip, which "was before in the hands of 20 or 30 "farmers, and as many ſmaller tenants or "proprietors. All theſe are hereby thrown "out of their livings with their families, "and many other families which were em-"ployed and ſupported by them." *Ib.* p. 37. See an account of *Norfolk*, in ſome reſpects ſimilar to this, in my *Appeal to the Public on the Subject of the National Debt*, p. 93, &c. I can ſcarcely think of any thing that ſhould be more alarming than ſuch accounts.—

How aſtoniſhing is it that our parliament, inſtead of applying any remedy to theſe evils, ſhould chuſe to promote them, by paſſing every year, bills almoſt without number, for new incloſures? (*a*)

The device, ſays Lord *Bacon*, (*Eſſays, civil and moral*, Sect. 20.) "of King Henry VII. "was profound and admirable, in making "farms and houſes of huſbandry of a "ſtandard; that is, maintained with ſuch "a proportion of land to them, as may "breed a ſubject in convenient plenty and "no ſervile condition, and to keep the "plough in the hands of the *owners* and "not *hirelings*."—"Incloſures," ſays the ſame great writer (in his Hiſtory of the Reign of *Henry the Seventh*), "began at "that time (or in 1489) to be more fre-"quent, whereby arable land was turned

(*a*) I have here in view incloſures of *open fields and lands* already improved. It is acknowledged by even the writers in defence of incloſures, that theſe diminiſh tillage, increaſe the monopolies of farms, raiſe the prices of proviſions, and produce depopulation. Such incloſures, therefore, however gainful they may be at preſent to a few individuals, are undoubtedly pernicious.—On the contrary. Incloſures of *waſte lands and commons* would be uſeful, if divided into ſmall allotments, and given up to be occupied at moderate rents by the poor. But if, beſides leſſening the produce of fine wool, they bear hard on the poor by depriving them of a part of their ſubſiſtence, and only go towards increaſing farms already too large, the advantages attending them may not much exceed the diſadvantages.

"into

" into paſture, which was eaſily managed " by a few herdſmen. This bred a decay " of people. In remedying this inconve- " nience, the King's wiſdom and the Par- " liament's was admirable. *Incloſures* they " would not forbid; and *tillage* they would " not compel; but they took a courſe to " take away *depopulating incloſures*, and *de-* " *populating paſturage* by conſequence. The " ordinance was, that all houſes of huſban- " dry, with 20 acres of ground to them, " ſhould be kept up for ever, together with " a competent proportion of land to be oc- " cupied with them, and in no wiſe to be " ſevered from them. By theſe means, the " houſes being kept up, did, of neceſſity, " enforce a dweller; and the proportion of " land for occupation being alſo kept up, " did, of neceſſity, enforce that dweller not " to be a beggar (*a*)." The ſtatute here mentioned was renewed in King *Henry* the Eighth's time; and every perſon who converted tillage into paſture ſubjected to a forfeiture of half the land, till the offence was removed. See Mr. *Anderſon's Chronological Deduction of Commerce*, Vol. I. page 347. ——In a law of the 25th of the ſame reign, it is ſet forth, " that many farms, and great " plenty of cattle, particularly ſheep, had " been gathered into few hands, whereby

(*a*) See Lord Bacon's Works, Vol. III. p. 431.

" *the rents of lands had been increaſed, and* " *tillage very much decayed*; churches and " towns pulled down; the price of provi- " ſions exceſſively enhanced, and a mar- " vellous number of people rendered inca- " pable of maintaining themſelves and fa- " milies; and, therefore, it was enacted, " that no perſon ſhould keep above 2000 " ſheep, *nor hold more than two farms.*" Ib. p. 363.—In the 3d of *Edw.* VI. a bill was brought in for the benefit of the poor, for rebuilding decayed farm houſes, and maintaining tillage *againſt too much incloſing*. Parliamentary Hiſt. Vol. III. p. 247.—In the year 1638, there was a ſpecial commiſſion from *Charles* I. for enforcing the ſtatute of the 30th of *Elizabeth*, by which no cottage was allowed in any country place, without at leaſt four acres of land to it, to prevent the increaſe of the poor, by ſecuring to them a maintenance; nor were any inmates allowed in any cottage, to ſecure the full cultivation of the land, by diffuſing the people more over it. See *Rymer's Fæd.* 20. 256, and 340.—By an Act in *Cromwell's* time, no new houſe was to be built within ten miles of LONDON, unleſs there were four acres of land occupied by the tenant. *Parliamentary Hiſtory*, Vol. XXI.

Such was the policy of former times.—*Modern* policy is, indeed, more favourable to the higher claſſes of people; and the conſequence of it may in time prove, that the whole

whole kingdom will conſiſt of only *gentry* and *beggars*, or of *grandees* and *ſlaves*.

I cannot conclude this Supplement without adding one farther obſervation which has ſtruck me on the preſent ſubject.—As in former times the number of the occupiers of land was greater, and all had more opportunities of working for *themſelves*, it is reaſonable to conclude, that the number of people willing to work for *others*, muſt have been ſmaller, and the price of day-labour higher. This is now the caſe in our *American* colonies; and this likewiſe, upon enquiry, I find to have been the caſe in this country formerly.——The *nominal* price of day-labour is at preſent no more than about *four* times, or at moſt *five* times higher than it was in the year 1514. But the price of corn (*a*) is *ſeven* times, and of fleſh-meat and raiment about *fifteen* times higher. See the Note, p. 286. - So far, therefore, has the price of labour been from advancing in proportion to the increaſe in the expences of living, that it does not appear that it bears now *half* the proportion to thoſe expences that it did bear formerly (*b*).

Upon

(*a*) See *Chronicon Pretioſum*, Chap. V. From whence, compared with the account in Chap. IV. of the price of corn and other commodities, for the laſt 600 years, abundant evidence for what I have here obſerved, may be collected.

(*b*) "The balance at preſent is conſiderably againſt "the labourer; and yet the landlord and tenant derive

 "ulti-

Upon the whole. The circumſtances of the lower ranks of men are altered in almoſt every reſpect for the worſe. From little occupiers of land, they are reduced to the ſtate of *day-labourers* and *hirelings*; and at the ſame time their ſubſiſtence in that ſtate is become more difficult, in conſequence of the cauſe juſt aſſigned; and alſo of *luxury*, which has extended its influence even to them, though ſtarving, and rendered *tea, fine wheaten bread*, and other delicacies, neceſſary to them, which were formerly unknown among them.—— Such a change cannot but draw after it important conſequences. They are the lower people chiefly who pay the taxes of a ſtate, fight its battles, carry on its commerce, and maintain its ſplendor. In every country, the higher ranks are a very ſmall body, compared with them. Even in this country, where their numbers are probably much leſſened, they are ſtill more the majority than is commonly imagined; for, from the returns made by the ſurveyors of the houſe and window-duties, it appears, that near THREE-FOURTHS of all the houſes in the kingdom are houſes not having more than *ſeven* windows.

" ultimately no advantage from hence.——The great " increaſe in the poor rates may be accounted for in a " few words. The riſe upon land and its produce, " is at leaſt 60 *per cent.*; the riſe upon labour not " above 20 *per cent.* The difference is of courſe againſt " the working hands; and when their earnings are in- " ſufficient for the abſolute neceſſaries of life, they muſt " inevitably fall upon the pariſh."—*Hints to Gentlemen of Landed Property*, p. 273.

POST-

POSTSCRIPT,

CONTAINING

Review of the Controverſy relating to the State of Population in England *and* Wales *ſince the Revolution.*

THE obſervations, in the preceding Supplement, on the population of this kingdom, are the ſame with thoſe which have been publiſhed in the former editions of this work. A more particular account of the evidence which ſeems to prove a progreſſive decreaſe in our population, has been given in an ESSAY on this ſubject firſt publiſhed at the end of Mr. *Morgan*'s Treatiſe on the *Doctrine of Annuities and Aſſurances on Lives and Survivorſhips*, and ſince republiſhed with the addition of an *Appendix*, containing remarks on Mr. EDEN's objections in his fifth letter to Lord CARLISLE. Theſe publications have been lately followed by others on the ſame ſubject; particularly, Mr. *Wales's Enquiry into the preſent State of the*

the Population of ENGLAND *and* WALES; and Mr. *Howlett's* Examination of Dr. *Price's* Eſſay on the Population of *England*; and a pamphlet entitled *The Uncertainty of the preſent Population of this Kingdom, deduced from a candid Review of the Accounts lately given of it by Dr.* PRICE *on the one Hand, and Mr.* EDEN, *Mr.* WALES, *and Mr.* HOWLETT, *on the other.*

In the Preface to the ESSAY juſt mentioned, fearing that I might have expreſſed my conviction too ſtrongly, I referred myſelf to the candour of the Public, and deſired that my aſſertions might not be regarded any farther than they were ſupported by undeniable facts.——The proſpect of an increaſing depopulation is ſo diſcouraging, that nothing but the faireſt overbalance of evidence ſhould engage us to admit it. I thought ſuch evidence did exiſt, and, therefore, ſtated it; believing that ſatisfaction ought never to be founded on impoſition, and that by endeavouring to apprize the kingdom of its true ſtate, I might be doing it an important ſervice.——The ingenious Author of the pamphlet laſt mentioned, writes in the character of one who doubts, and wiſhes only to know how things are; but Mr. *Wales* and Mr. *Howlett* zealouſly maintain, in oppoſition to the arguments I have produced, that our population is increaſing faſt. My intention in this *Poſtſcript*

is

is to give as fair and yet as brief an account as I can of the preſent ſtate of this diſpute, by reciting the evidence offered on both ſides, and making ſuch remarks upon it as ſhall appear to me neceſſary.

The principal evidence to prove that our population has declined, is taken from the compariſon ſtated in page 276 of this Volume (but more particularly in the ESSAY), between the number of houſes in the kingdom at different periods from the Revolution to the preſent time.

Houſes in *England* and *Wales* at Lady-day 1690 — —	1,319,215	including 554,641 having only one hearth.

	Charged and chargeable.	Excuſed for poverty.	Total.
Houſes in 1750	729,048	. . .	. . .
in 1759 (a)	704,053	282,429	986,482
in 1761	704,543	276,149	980,692
in 1777	701,473	251,261	952,734

The number of houſes at Lady-day 1690, is ſtated diſtinctly by Dr. *Davenant* for every county (ſee his Works, Vol. I. p. 38); and repreſented by him as an important inſtruction derived from the hearth-books then

(a) This year was the firſt in which an order was given to return the cottages excuſed for poverty.——The chargeable or uninhabited houſes in this year, and in 1761 and 1777, were 24,904, 25,628, and 19,396 reſpectively. See the Eſſay on the Population of *England* and *Wales*, printed for Mr. *Cadell*, p. 10 and 12.

exiſting,

exiſting, and containing accounts fairly kept and ſtated. *Ib.* p. 136, 373.

The numbers for the ſubſequent years are given from the returns to the tax-office of the ſurveyors of the houſe and window-duties in every diſtrict in the kingdom, made by the order of government in thoſe years.

A compariſon of theſe numbers with thoſe given by Dr. *Davenant*, affords an evidence which, as far as it can be truſted, is full and deciſive.——I know of nothing which has been urged againſt Dr. *Davenant*'s account, except that by *houſes* he meant *families*; but it has been obſerved, that the difference between the number of *families* and *houſes* in the kingdom, is by no means conſiderable enough to account for the exceſs in Dr. *Davenant*'s total; and that, were the contrary true, it is evident he muſt have meant *houſes*, becauſe he has divided this total into two numbers (namely, 1,208,000 and 111,215) the firſt of which he ſuppoſes to be the number of houſes having *ground* about them; and the ſecond, the houſes not having ground about them.

The principal objections which have been made to the other accounts are the following.

Firſt; the cottages are included in them, and theſe being excuſed, and no account kept of them, the ſurveyors could not be correct in returning them.

This

This is certainly true. But it ſhould be remembered, that the ſame objection holds againſt the returns of the cottages made from the hearth-tax; that if in any inſtance ſuch returns have been made from *conjecture*, they are more likely to exceed the truth, than to fall ſhort of it; and that it is quite incredible that theſe returns ſhould be ſo deficient as not to give above two out of five of the true number; or that the cottages of the poor ſhould be almoſt equal to all the other houſes in the kingdom, which muſt be the caſe if there has been no decreaſe.——I have been, however, aſſured that in ſome diſtricts, the returns of the cottages have been made from actual ſurveys, and may be depended on.——And, if in other diſtricts, they have been made careleſsly, or perhaps in ſome not at all, an allowance on this account of an omiſſion of *half* the cottages would ſtill leave the number of houſes ſhort of what it was formerly.

According to the returns, the decreaſe in the cottages has been much more conſiderable than in the other houſes; and, in the interval between the two laſt returns, amounted to 24,888. , Such an authority only as the returns of the cottages, gives no ſufficient reaſon for believing this. But there are two facts which give it credibility. The firſt is, that acknowledged deſtruction of cottages which has been the conſe-

conſequence of the increaſe of large farms. And the other is, that decreaſe of the houſes *charged* having ſeven windows or leſs, amounting to 24,651, which took place in the ſame interval of time. See the account of this decreaſe in the Eſſay on the population of *England* and *Wales*, p. 11.——To this nothing has been oppoſed but a ſtrange objection of Mr. *Howlett*'s, implying, that, on account of the diſtreſſes of the poor, it is not poſſible that theſe houſes and the cottages ſhould decreaſe together.

The ſame writer has endeavoured to diſcredit all the returns to the tax-office, by obſerving, in p. 60, that they have repreſented the number of houſes as diminiſhed (ſince 1755) in ſome places where it is known they have increaſed. He inſtances in *Thaxted* in *Eſſex*, conſiſting of 350 houſes; two pariſhes in the ſame county and one in *Kent*, conſiſting between them of only 206 houſes; and *Maidſtone*, conſiſting of 1106 houſes. He gives no other proof that theſe places have not decreaſed than a bare aſſertion; and if I may judge from his principal inſtance (or *Maidſtone*), his account of the returns for theſe places deſerves no regard. According to him, the return of the houſes for this town in 1777 was 633, and leſs by 23 than in 1755: Whereas the number returned in that year of inhabited houſes only paying the houſe and window-duties, and therefore

therefore excluſive of all the other houſes (which were included in the general return for the county) was 727; as any one may know who can either enquire at the tax-office, or will conſult the accounts printed by the Houſe of Commons in 1781.

Mr. *Howlett*, after making this objection to the tax-office accounts, informs the public (p. 62), from the authority of ſome ſurveyor of the window duties, that *doubtleſs* there was no return at all of the cottages in 1777.——It is difficult to account for ſo groſs an error. In the firſt ſeſſion of the preſent parliament, Lord MAHON moved the Houſe of Commons for an account of all the returns to the tax-office of the houſes in the kingdom. In conſequence of this motion, the general return for 1777 was, among other returns, laid by the commiſſioners of the tax-office before parliament. This return was afterwards printed, and it diſtinctly ſpecifies the number of cottages, as well as of other houſes, in *every* county; and it is the ſame with the return for 1777 which I have given at the beginning of this Poſtſcript, but more at large in the Eſſay on the Population of *England* and *Wales*.

After finding Mr. *Howlett* ſo miſtaken in this and ſome other inſtances (*a*), I might, I think, be excuſed were I to ſave myſelf

(*a*) See Vol. I. p. 255, and 258, 259, 260.

the

the trouble of taking any farther notice of him. There are, however, ſome other miſtakes into which he has fallen, ſtill more important and palpable, which in what follows it will be proper to mention.

In this argument, a great deal depends on the proportion of the houſes *charged* and *chargeable* (and conſequently entered in the books of the aſſeſſors) to the whole number of houſes in the kingdom. The return in 1777 makes this proportion to be as 701,473 to 952,734, or as 3 to 4 nearly. See p. 299. A compariſon of this proportion with the like proportion in a great variety of pariſhes and towns in different parts of the kingdom, aſcertained by careful enumerations, would ſhew how far it deviates from truth, and what addition ought to be made to the excuſed houſes, in order to obtain the whole number of houſes.—— I am not poſſeſſed of many ſuch accounts. Thoſe which I think moſt to be depended on are the following.

	Total of Houſes.	Houſes charged.
Beccles in *Suffolk* — — —	468	297
Bungay — — — —	326	220
Henham, *Sotherton*, *Shipmeadow*, *Weſton*, and two other pariſhes in *Suffolk* —	135	106
	929	623

Wenhaſton

	Total of Houſes.	Houſes charged.
Brought over — —	929	623
Wenhaſton (*a*) in *Suffolk* —	76	73
Southwold, *Aldeburgh*, *Orford*, and *Gorleſtone*, pariſhes in *Suffolk* — — —	720	563
Remainder of the diſtrict in *Suffolk* in which theſe pariſhes are — —	5906	4859
Warrington in *Lancaſhire*, with its vicinity —	1941	558
	9572.	6676

(*a*) Only 56 houſes have been reckoned in this pariſh; but in the office accounts 73 houſes are charged, in conſequence of the diviſion of ſeveral cottages deemed ſingle houſes, into two or three ſeparate dwellings, holding ſo many families.——One of the excuſed houſes in this pariſh (and alſo in *Bungay*) is an alms-houſe, and in this account reckoned but one houſe, though conſiſting of ſeveral apartments, and therefore capable of being reckoned 5 or 6 houſes; and in all accounts of this kind it ſhould be remembered, that ſome differences will ariſe, as a houſe or cottage containing two or more families, having no communication, is reckoned a ſingle or two or more houſes.

Weſton pariſh conſiſts only of 21 houſes, *Shipmeadow* of 11, *Henham* of 15, and *Sotherton* of 24. It is not conceivable that any pariſhes ſhould have been always ſo ſmall; and yet there are multitudes of ſuch pariſhes in *Suffolk*, *Norfolk*, *Northamptonſhire*, *Suſſex*, *Kent*, and ſome other counties, and ſome of them provided with large churches. In *Norfolk*, particularly, the dilapidated churches in *ſome* places, and their diſproportionate ſize in *others*, prove that it muſt have been formerly more populous. Even NORWICH itſelf bears evident marks of having been once a much more conſiderable city.

	Total of Houſes.	Houſes charged.
Brought over —	9572	6676
Sandwich in *Kent* (*a*) —	578	349
Chriſtleton in *Lancaſhire*, by an exact ſurvey in 1780	102	72
Firſt totals —	10,252	7097
Add *Sudbury* diviſion —	7740	4122
Second totals —	17,992	11,219

Accounts collected by Mr. *Wales*. See his *Enquiry*, p. 39, 43, 47, &c.

	Total of Houſes.	Houſes charged.
The two diviſions of *Agbridge* and *Morley* in the Weſt-Riding of *Yorkſhire*	21,929	12,832
Twenty-eight villages in *Northamptonſhire* —	1024	706
Weſthall, *Wangford*, *Holton*, *Spexhall*, *Swilland*, *Tuddenham*, *Weſterfield*, *Wiſſet*, *Witneſham*, *Blythford*, and *Bramfield*, pariſhes in *Suffolk* — —	391	351
	23,344	13,889

(*a*) According to an accurate account taken by Mr. *Boys* in 1776. The number of inhabitants was 2252, or 3,8 to a houſe; though *three* workhouſes containing 33 perſons, and *two* hoſpitals containing 21 perſons, are reckoned as only five families.

Aſhill,

	Total of Houſes.	Houſes charged.
Brought over —	23,344	13,889
Aſhill, *Clapton*, *Ilminſter*, and *Wayford*, in *Somerſetſhire*	388	134
Third totals —	23,732	14,023
Add the *Second* totals —	17,992	11,219
Fourth total —	41,724	25,242

If we may judge from the firſt totals, which are thoſe alone in which from my own enquiry I can confide, and which (including in them a town with its vicinity full of the pooreſt manufacturers, where the proportion of charged houſes is lower than I have found it any where elſe) may not poſſibly be an improper guide in this caſe, the proportion of charged to the whole number of houſes will be as 7097 to 10,252. And, ſince the charged and chargeable houſes are known by the returns in 1777 to have been then 701,473, the whole number of houſes in the kingdom will come out 1,013,000, or nearly a million, as I have reckoned it. If we add to theſe totals thoſe for SUDBURY and its neighbourhood, where alſo (becauſe full of poor manufacturers) the proportion of charged houſes is particularly low, the number of houſes in the kingdom will come out 1,125,000.——If we judge

by the accounts Mr. *Wales* has collected, this number will come out 1,187,000.—— If we judge by all theſe accounts taken together it will come out 1,159,000.

All theſe determinations ſhew a great diminution in the number of houſes ſince the *Revolution*; nor (ſuppoſing Dr. *Davenant*'s account right, or even not *very* wrong) is it poſſible to reckon it equal now to what it was then without contradicting all probability.

A confirmation of this might be derived from Mr. *Howlett*'s accounts, could they be truſted. He has (in his Examination of Dr. *Price*'s Eſſay, p. 139, &c.) given a liſt of towns and pariſhes in 20 different counties, in which the total of houſes is 29,262 by *enumeration*, and 17,225 by the *returns* of the ſurveyors. The laſt of theſe totals includes in it only the *charged* houſes; and it gives a proportion of theſe to all the houſes in the kingdom, which makes their number 1,191,000. But the truth is, that Mr. *Howlett*'s account of the returns of the ſurveyors cannot at all be depended on; and the following particulars will abundantly prove this.

The numbers returned for *Beccles*, *Bungay*, *Shipmeadow*, *Mettingham*, and *Homersfield* in *Suffolk*, were in 1780 (*a*), according to him, 169, 260, 7, 21, and 21 for theſe places reſpectively.——I am aſſured, on the

(*a*) There was no return in this year.

contrary,

contrary, that the numbers (when the laſt general return was made in 1777) were 297, 220, 11, 27, and 23 returned as charged; and 171, 106, 0, 3, and 11, returned as *excuſed*.——Then umbers returned for *Northampton*, *Maidſtone*, *Cheſter*, and *Shrewſbury*, he makes to be 768, 623, 1227, and 967 reſpectively; whereas it appears, from the accounts printed by the Houſe of Commons in 1781, that the numbers returned to the tax-office for theſe towns in 1777, were, 706, 727, 1244, and 904, excluſive of the *uninhabited*, and *excuſed* houſes which were likewiſe returned, but included in the totals for the counties.

But Mr. *Howlett* has here fallen into a ſtill greater miſtake; for, through haſte or inattention, he has taken the numbers in his liſt (being in reality only the number of houſes *taxed* given very inaccurately) for the whole of the numbers (*a*) *returned*, including *uninhabited* and *excuſed* houſes; and, arguing upon this miſtake, he makes the houſes in the kingdom 1,609,555; which is above a third more than, by computing in his own way,

(*a*) "The number of houſes in Mr. *Howlett*'s liſt ſaid "to be returned for *Tenterden* in *Kent*, is 96, the total "198. A correſpondent, on whoſe veracity I can de- "pend, aſſures me that theſe 198 houſes are all in the "pariſh duplicate; and that the 96 are thoſe which are "*charged*."—*Uncertainty of the Population of this Kingdom*, p. 24.

he muſt have found them had he not fallen into this miſtake (*a*).

It

(*a*) Mr. *Howlett*, in conſequence of thus over-rating the number of houſes, and allowing 5 and two-fifths to a houſe, makes the inhabitants of *England* and *Wales* to be near nine millions. The proportion of inhabitants to houſes may be, in ſome meaſure, collected from the Table in p. 6th of the Eſſay on the *Population of England and Wales*, which has been reprinted with ſome additions at the end of the Firſt Eſſay in the preceding Volume of this work. To the towns and pariſhes in that Table I will here add SANDWICH in KENT, where, by an accurate ſurvey in 1776, the houſes were found to be 578, and the inhabitants 2252, or $3\frac{9}{16}$ to a houſe; and alſo EASTRY in the ſame county, where, in 1774, the houſes were 141, and the inhabitants 656, or $4\frac{1}{2}$ to a houſe.—The total of houſes in that Table, with theſe added, is 45,217; and of inhabitants 231,842, which makes 5 and an eighth to a houſe.

Mr. *Howlett* has inſerted in his *Examination*, &c. p. 144, the houſes and inhabitants in *Birmingham*, *Norwich*, *Manchester*, *Nottingham*, and *Liverpool*, juſt as I had given them in the Eſſay on the *Population of England*, &c. but with ſuch additions as to bring out the allowance juſt mentioned 5 and two-fifths to a houſe. But had Mr. *Howlett* choſen to add to his own liſt the *whole* of my liſt in the Eſſay, as well as that part of it juſt mentioned which gives the higheſt allowance, he would have found (taking 4338 for the number of houſes at *Manchester* and *Salford* in 1773, and not 4268 as he makes it) the total of houſes to be 41,030, and of inhabitants 244,422; and conſequently the allowance to a houſe not to be ſo much as five and one-fifth to a houſe.

Mr. *Howlett*'s additions, with SANDWICH and EASTRY, and the additions which have been made (in the Table in the Firſt Volume, p. 298) to the Table in the Eſſay on the *Population of England and Wales*, will make the total of houſes 52,036, and of inhabitants 268,568, and the allowance 5 and a ſixth.

It

It is neceſſary to obſerve, that the method here uſed of deducing the total of houſes

It ſhould be conſidered, that theſe totals, conſiſting chiefly of the houſes and inhabitants in five of the moſt populous towns in the kingdom, give moſt probably a proportion of inhabitants to houſes too high for the kingdom at large. If we throw out BIRMINGHAM and the town of MANCHESTER, the remainder will perhaps make a properer mixture of great and ſmall towns and country pariſhes; and the totals (or 41,675 and 210,158) will give $5\frac{1}{16}$ to a houſe. If LIVERPOOL is likewiſe thrown out, the totals will give leſs than 5 to a houſe.

In the Table juſt referred to I have given the number of houſes and inhabitants at *Birmingham* from a ſurvey in 1770; when the houſes were 6025, and the inhabitants 30,804; of whom 15,363 were males, and 15,441 females.——I have lately been informed that, according to a very accurate ſurvey of *Birmingham* in autumn 1782, the houſes (excluſive of the hamlet of *Deretend*) were then 8125, of which 291 were uninhabited. From the ſame account I learn, that the annual average of burials at *Birmingham* (excluſive of *Deretend*) for four years to 1774, was 1116; and for ſix years to 1780, was 1342. ——The number of inhabitants in 1770, divided by the *firſt* of theſe averages, makes the proportion dying annually at *Birmingham* to be one in $27\frac{1}{3}$; which, being very nearly the ſame with the proportion dying annually at *Liverpool* and *Mancheſter*, cannot probably be far from right: and this number (or $27\frac{1}{3}$) multiplied by the *ſecond* average, makes the inhabitants in 1780 to be 37039. In order, however, to allow for the increaſe of *Birmingham*, and to be more ſure of finding a number not leſs than the truth, let the burials in 1782 be reckoned 1500, and the proportion dying annually 1 in 28; and it will follow that the inhabitants were then 42,000, and the number of perſons in a houſe $5\frac{1}{7}$, including about 700 in the workhouſe and hoſpital. ——I am ſenſible that this falls below the common eſtimates; but I pay no regard, in caſes of this kind, to any eſtimates which are not derived from careful ſurveys.

houſes in the kingdom from the proportion (aſcertained by ſurveys) of the houſes taxed to

The annual average of births at *Birmingham* was (according to the regiſter) 1408 for 10 years to 1780. The exceſs of the births above the deaths is plainly owing to that over-proportion of people in the firſt ſtages of mature life, which always takes place in towns, in conſequence of their being kept up or increaſed by an influx of people from other places. See the Firſt of the following Eſſays. That this is the cauſe of the increaſe of *Birmingham* is undoubted, for the exceſs of the births cannot account for a 40th part of the increaſe; and before it became ſo rapid as it has been for ſome time, the burials *exceeded* the births, the annual average of the former having been, if the regiſter deſerves any regard, 708; and of the latter, 619.——The ſame regiſter makes the annual medium of burials for 10 years to 1697 to have been 156, and of births, 150. But this only confirms an obſervation before made, that the regiſters in former times were very deficient; for it is not probable, that *Birmingham* was then ſo ſmall a town; and an old account which I have ſeen of a ſurvey in 1700 makes it to conſiſt in that year of 2504 houſes, and 15032 inhabitants. The regiſter, therefore, did not then give above a third of the births and burials.

In Vol. I. p. 301, I have alſo given the number of houſes and inhabitants at *Maidſtone* in *Kent*, from a ſurvey in 1781. I have ſince learnt, that another ſurvey was made at *Maidſtone* in *September* 1782; and as ſome inſtruction may be derived from it, I will here give the reſults juſt as I find them in a pamphlet publiſhed in this town by Mr. *Howlett*, and entitled, *Obſervations on the increaſed Population, Healthineſs*, &c. of the town of *Maidſtone*.

	Families.	Houſes.	Inhabitants.	Males.	Females.	Male ſervants.
In the town —	1037	982	5028	2306	2722	145
In the country —	139	133	727	357	370	41
In the *whole* pariſh	1176	1115	5755	2663	3092	186

In

to the totals of houſes in country towns and pariſhes, muſt be too favourable; becauſe this

	Female ſervants.	Women above 70.	Men above 70.	Girls under 15.	Boy. under 15
In the town -	325	161	96	847	776
In the country -	40	9	10	165	144
In the *whole* pariſh	365	170	106	1012	920

Perſons to a houſe *in* the town	5 1/16
In the pariſh *out* of the town	5 2/3
Perſons to a family *in* the town	4 5/6
In the pariſh *out* of the town	5 1/4
Proportion of children under 15 to the total of inhabitants *in* the town -	as 100 to 309
In the pariſh *out* of the town	as 100 to 235

In the town one in 17 of the women exceeds 70 years of age, and one in 24 of the men; but in the country only one in 41 of the women exceeds this age, and one in 36 of the men.

Annual average (according to the regiſter) in the whole pariſh for 20 years—

Of births to 1702	130	Of marriages	29	Of burials	132
to 1722	120	—	30	—	118
to 1742	129	—	40	—	144
to 1762	143	—	46	—	140
to 1782	160	—	50	—	148

By a ſurvey in 1695, the inhabitants were 3676.

From theſe particulars it ſeems to appear, that *Maidſtone*, at the beginning of this century, was a *de*creaſing town; but that lately it has been *in*creaſing, not by an exceſs of births, but, like other towns, by drawing ſupplies from other places. The ratio of the births to the burials, (if it can be depended on) and the great overproportion of perſons in mature life in the town, prove this.

The number of females in it turned of 70 is greater than the number of males, partly, becauſe males are more

this proportion in *London*, *Southwark*, and all *Middleſex* (containing at leaſt an 8th or more ſhort-lived, but chiefly in this inſtance becauſe the males, after removing to the town, are taken off again to the navy, army, &c. And the proportion of both males and females turned of 70 in the country is ſmaller than in the town, becauſe removals from thence are chiefly to the town; and theſe being alſo chiefly removals of females, the town is rendered, at every age, much fuller of females than of males.

It is farther obſervable, that the town, when compared with the country round it, appears to be particularly unfavourable to population, the proportion of children under 15 being much leſs there than in the country.——The ſame is remarkable in the country round *Mancheſter*. See the Firſt of the following Additional Eſſays.

It ſeems, indeed, that the conſumption of towns tends to promote the population of the country near them; and were they fed with people only from hence, they would not probably be ſo prejudicial as they are to population. But the fact is, that there are few towns which would not ſoon come to nothing, did they draw their ſupplies of people only from the adjacent country. So true is this of *London* in particular, that, notwithſtanding this natural tendency of its conſumption, there is ſcarcely a village or pariſh within ten or twelve miles of it, in which, if we may believe Mr. *Howlett*'s extracts from the regiſters, the births do not fall conſiderably ſhort of the burials. See his *Examination*, &c. p. 96, 97, &c.

In a note at the beginning of the Firſt of the following Eſſays, it appears that the number of houſes at MANCHESTER, excluſive of *Salford*, in 1773, was 3446, including 44 empty houſes. My friend Dr. *Percival* has juſt informed me, that at the end of laſt year (1782) a new and very accurate enumeration of this town (excluſive of *Salford*) was completed, which made the houſes then to be 4606. An addition, therefore, has been made to MANCHESTER of 1160 houſes within the laſt ten years.

9th

9th of the kingdom) is, and, for obvious reaſons, muſt be much higher than it is in the other diſtricts of the kingdom. The returns in 1777 make the houſes taxed in *London*, *Southwark*, and all *Middleſex* to be 77,008, and the total of houſes 90,570; whereas the ſame returns for the whole kingdom make the former to be 701,473, and the latter 952,734,——I think it worth adding, that from a return for *London* and *Middleſex*, in 1780, and laid before parliament, it appears that the number of empty houſes in this part of the kindom had increaſed, between 1777 and 1780, from 3,381 to 6,810.

The evidence now inſiſted on, taken from the returns of the ſurveyors and aſſeſſors of the houſe and window-duties, is the only direct evidence comprehending the whole kingdom with which we are furniſhed on this ſubject; and it is ſo diſcouraging, that I do not wonder that the advocates for the increaſe of our population endeavour to diſcredit it; and I ſhould certainly join them in this, were I leſs deſirous to know things as they are, than to prove them what I wiſh them.—The care and attention of Mr. *Roſe* (now one of the ſecretaries to the treaſury, but lately the ſecretary of the tax-office), in collecting theſe returns, cannot, I believe, be doubted; and he who conſiders that they are founded upon old taxes, and made upon oath, will not be able eaſily to perſuade

ſuade himſelf that they can be very groſsly deficient.

Mr. *Wales*, a writer whoſe abilities I reſpect and whoſe accounts I am not inclined to diſtruſt, has collected ſeveral accounts of enumerations of houſes *in* or *about* 1750 and 1780, which he thinks afford a preſumptive proof of a general increaſe during that period. I will tranſcribe his ſummary of them, p. 48 (*a*).

	Houſes in 1750.	Houſes in 1780.
North Riding in *Yorkſhire* —	1716	1985
Eight villages in the *Weſt-Riding*	784	943
Seventeen villages in *Derbyſhire* —	1001	1348
Twenty-ſeven villages in *Northamptonſhire* —	1036	1024
Fourteen pariſhes in *Suffolk* (families)	653	704
Four pariſhes in *Suſſex* —	144	223
Four villages in *Somerſetſhire* —	428	388

Mr. *Wales* has added an account taken from the returns (which in this inſtance he is willing to truſt) of the ſurveyors for *Agbridge* and *Morley* diviſions in the *Weſt-Riding* of *Yorkſhire*. From theſe returns it appears, that in 1761 the houſes in theſe

(*a*) In p. 67, there is a compariſon of enumerations at different periods of *Mancheſter*, *Liverpool*, *Birmingham*, *Leeds*, *Nottingham*, *Norwich*, and *Farnham*, which ſhews, what is well-known concerning the four firſt of theſe towns, that they have greatly increaſed.

diviſions

diviſions were 17,764; that in 1767, they were 20,526; and in 1779, 21,929.

I will add a ſimilar account of a diſtrict in the county of *Suffolk*, where

In 1761	the houſes *charged* were	5584
	the houſes *excuſed* were	1391
		6975
In 1777	the houſes *charged* were	6118
	the houſes *excuſed* were	1521
		7639

There has undoubtedly been an increaſe in *Yorkſhire*, and perhaps alſo in *Derbyſhire*; but he that will judge of it from the numbers in theſe accounts will be in danger of being miſled: For I underſtand, that it is in part an *apparent* increaſe only, owing to the converſion of houſes holding two or more families, and *formerly* charged as *ſingle* houſes, into apartments having no communication, and therefore *now* charged as ſo many ſeparate houſes.——The inducements to ſuch converſions among the lower ranks of people have been ſo great ſince 1761, as to be irreſiſtible. For firſt, their poverty has increaſed, and therefore they have found it more neceſſary to ſave every needleſs expence.——And ſecondly, in 1761 the window-duties were nearly doubled; and houſes having 8 or 9 windows, *before* excuſed, were ſubjected to the payment of 1*s*. *per ann*

for

for every window. In 1766 theſe duties were again increaſed, and houſes having only *ſeven* windows were ſubjected to them. By dividing, therefore, ſingle houſes holding more than one family into ſeveral tenements having each of them few windows, the tax upon them might be either leſſened or entirely avoided (*a*). The decreaſe of ſmall farms has likewiſe contributed to this change, by cauſing many farm-houſes to be turned into cottages for day-labourers.

Perhaps, theſe have been the only cauſes of the increaſe of the diſtrict in *Suffolk* juſt mentioned; and there is reaſon to believe that they have been the principal cauſes of the increaſe in *Agbridge* and *Morley* diviſions in *Yorkſhire*. For the returns ſhew an increaſe in theſe diviſions equal to above a 6th of the whole number of houſes in ſo ſhort a time as ſix years, or from 1761 to 1767; but afterwards, or from 1767 to 1779, they do not ſhew *half* this increaſe in *double* the time. The firſt increaſe, therefore, was probably occaſioned, as I have obſerved, by the alteration in the window-duties in 1761; nor, indeed, could it have any other cauſe than either this, or the de-

(*a*) In Mr. *Wales*'s accounts of the increaſe of houſes in the *North-Riding* of *Yorkſhire*, and in *Derbyſhire*, it appears that a great part of it proceeded from alterations in old houſes; that is, perhaps, from ſuch alterations as thoſe here meant.

ſertion

ſertion of other parts of the kingdom; for it was too great and too ſudden to be accounted for by an exceſs of the births above the deaths, which is the only cauſe that can produce a general and permanent increaſe.

There is one more ſource of information on the ſubject of our population which is of particular importance; I mean, a compariſon of the births and burials and marriages at different periods. Such a compariſon for the whole kingdom would decide the queſtion I am diſcuſſing. But we are far from being furniſhed with the means of making it. It is, however, the evidence on which the advocates for a progreſſive increaſe in our population principally rely; and I ſhall here give a fair repreſentation of it, with ſuch remarks as a regard to truth will render neceſſary.

	Baptiſms.	Burials.
Annual average of baptiſms and burials about or ſoon after the Revolution, in 33 pariſhes in ten counties, taken indiſcriminately in different parts of *England*.—See Mr. *Wales*'s *Enquiry*, p. 49. *(a)* —	1460	1518

(*a*) In Mr. *Wales*'s liſt the average of burials correſponding to the births is not given for *Liverpool* and *Bowden* in *Lancaſhire*, and for *Lamborn*, *Shefford*, and *Wilford* in *Berkſhire*; and, therefore, theſe places are not included in this account.

All

	Baptiſms.	Burials.
Annual average in the ſame pariſhes for ſome years before 1780.—*Ib.* p. 50	4064	3537
Annual average of baptiſms and burials about the year 1745 in 142 pariſhes in 21 counties taken indiſcriminately.—*Ib.* p. 53.	4712	4067
Annual average in the ſame pariſhes between 1770 and 1780.—*Ib.* 57.	7179	5689
Annual average of births and burials in the Deaneries of *Melineth*, *Elvel*, *Buillt*, *Hay*, and *Brecon* in the dioceſe of St. *David*'s.—*Ibid.* p. 65.		
From 1700 to 1730 —	341	325
From 1730 to 1760 —	715	587
From 1760 to 1763 or 1764	727	580
Annual average in the other parts of the dioceſe		
From 1700 to 1730 —	888	753
From 1730 to 1760 —	1111	921
From 1760 to 1763 or 1764	1302	1183
Annual average in the whole dioceſe of St. *David*'s		
From 1700 to 1730 —	1229	1078
From 1730 to 1760 —	1826	1508
From 1760 to 1763 or 1764	2029	1663

All

All theſe accounts have been extracted from the pariſh regiſters. The deficiencies in theſe regiſters, and the careleſſneſs with which they are kept, have been often complained of. I wiſh, therefore, ſomething had been ſaid to eſtabliſh their credit; or at leaſt to ſhew, that they have been preſerved entire, and that they were not *more* deficient formerly than they are now *(a)*. Suppoſing them

(*a*) May it not be doubted whether at the *Revolution* the pariſh regiſters had recovered from the confuſion into which all church affairs had been thrown in the times of the civil war and commonwealth?——The number of popiſh and proteſtant diſſenters was then probably much greater than it is now.——But the obſervation moſt to the preſent purpoſe may be, that regiſters of mortality are of late origin, and have been for a courſe of years growing more and more into uſe and eſtimation. Among the Diſſenters in *London* the regiſtration of births was, ſome years ago, much neglected. At preſent it is more practiſed in conſequence of notifications of the eſtabliſhment of a public regiſter, which have been read annually from the pulpit. And in the country I ſuſpect, that people of all denominations are got ſo much more into the habit of reckoning it important, as ſometimes to regiſter in more than one place.

§ In 1538 *Henry the Eighth* gave orders that the incumbent of every pariſh ſhould keep true and exact regiſters of all chriſtenings, weddings, and funerals in his diſtrict. But this order, in many places, was little regarded till Queen *Elizabeth*, in 1558, gave another order for keeping them more exactly. Yet after all they were but remiſsly kept in many pariſhes, and often committed only to looſe papers, by which means ſome were loſt, ſome rotted away, and others were devoured. To remedy theſe evils, orders were given

them correct, they take in but a very inconſiderable part of the kingdom, and chiefly that very part which, it is well known, has increaſed, but the increaſe of which muſt have been, in ſome meaſure, occaſioned by removals from other parts of the kingdom. The *ſecond* of theſe accounts is the principal; and, if from the numbers in it are deducted the births and burials in *Mancheſter*, *Rochdale*, and *Warrington* in *Lancaſhire*; and in *Shef-*

" in 1559, that all regiſters ſhould be kept in parch-
" ment-books only, and that all preceding ones which
" could be found, ſhould be tranſcribed into new books.
" But no place in *England* ſlighted theſe orders ſo much
" as *London*; for, except in two or three years of great
" plagues, we find no bills in *London* till 1604.——But
" neither country nor city regiſters, where there has been,
" or ſtill is any conſiderable body of diſſenters, popiſh or
" proteſtant, are to be much relied on after 1644, when
" the diviſion in the church firſt broke out. And even
" in places where there are no diſſenters, regiſters are
" little to be regarded on account of ſeveral unhappy
" concurring circumſtances, as the negligence or fre-
" quent abſence of the regiſter-keeper, and the igno-
" rance, poverty, miſtakes, and *prejudices* of ſeveral of
" the people."—— See the preface to the *New Obſervations on Town and Country Bills of Mortality*, by Dr. *Short*, p. 9, &c.

In *London* the bills did not include the diſtempers till 1629; nor the ages till 1728; and ſtill it is well known that they are very defective.

Concluſions drawn from regiſters of burials, be they ever ſo exact, are rendered more uncertain than is commonly imagined, by epidemics, and the different degrees of healthineſs or ſicklineſs of different years. This may be learnt in ſome meaſure from what is related of SWEDEN in p. 146.

field,

ſield, Wakefield, Halliſax, &c. in *Yorkſhire,* the remainder will be, in the firſt period, 1630 births *per ann.* and 1408 burials; and, in the ſecond 2010 births *per ann.* and 1502 burials, which makes a ſmall increaſe.

The *firſt* account overthrows itſelf by making the burials at the *Revolution* in eleven counties to exceed the births. Theſe counties, therefore, if we are to judge from theſe extracts, muſt have been *then* decreaſing. The increaſe which appears at preſent is almoſt entirely the increaſe of the towns juſt mentioned; and if they are ſtruck out, the remainder in this *firſt* account, as well as the *ſecond,* will be little; and that little will ſhew a decreaſe in *Somerſetſhire,* no increaſe in *Nottinghamſhire,* and only a ſmall increaſe even in *Yorkſhire.*

Mr. *Wales's third* liſt ſhews an increaſe at the beginning of this century ſo rapid in the dioceſe of St. *David's* as in 30 years to double the inhabitants of five deaneries; but, in the other parts of the dioceſe, ſo much ſlower, as in the ſame time not to add a *quarter* to the inhabitants.——It deſerves notice farther, that they repreſent the increaſe which took place in the *firſt* period as changed into a decreaſe in the *ſecond* and *third* periods. This will appear upon conſidering, that had the increaſe in the *firſt* period been continued to the end of the *ſecond,* the annual averages at the end of this

ſecond period, (or which is nearly the ſame) the annual averages from 1760 to 1763, muſt have been much greater than they are; for they muſt have borne the ſame proportion to the averages of the *ſecond* period that the mean between theſe averages and the averages of the *firſt* period bear to theſe laſt averages. That is, in the five deaneries, the average of burials about 1760 ſhould have been to 587 as the mean betwen 587 and 325 (or as 456) is to 325. It ſhould have been, therefore, 823 (or ſome number not very diſtant from this) inſtead of 580; which laſt number is ſo much too little as to be nearly equal to the annual burials about the middle of the ſecond period; and, therefore, if not very wrong, proves a decreaſe muſt have taken place.

By the ſame reaſoning it will appear, that in the whole dioceſe, if the increaſe in the *firſt* period had continued, the burials at the end of the *ſecond*, or the beginning of the *third* period ſhould have been nearly 1808, inſtead of 1663. The ſame concluſions may be deduced by computing from the births.

Theſe are circumſtances which give a ſuſpicious appearance to this regiſter evidence (*a*); but there is a third circumſtance which deſtroys its credit.

At

(*a*) One plain reaſon of the inconſiſtencies in theſe accounts has been intimated, namely, that the births and

At the ſame time that, in the five deaneries, they ſhew an extravagant increaſe in the *firſt* period, they give the births and burials nearly equal, and therefore make it impoſſible there ſhould have been any increaſe (*b*).—The like will be obſerved preſently of the whole dioceſe.

That part of the kingdom where the pariſh regiſters give the ſtrongeſt proofs of an increaſe is the dioceſe of *Cheſter*.——The following is a ſummary of the extracts from them as I have received it from a friend in the dioceſe.

		Births.	Burials.
In the archdeaconry of *Cheſter* —	in 1717	7703	6380
	in 1779	16791	12573.
In the whole dioceſe	in 1717	10604	8755
	in 1779	21463	16080

There appears here an increaſe which has *doubled* the inhabitants in 62 years; and

and burials in former periods are given by the extracts much *more* below the truth than in the latter periods. And as far as this is the caſe, they prove nothing.

(*b*) The births in the firſt period, in order to produce (in conformity to the extracts) a double number in 30 years, ſhould have been more than double the burials; that is, ſuppoſing the burials not too high, the births ſhould have been about 700; and both the births and burials in the *ſecond* period, inſtead of being 715 and 587, ſhould have been double theſe numbers.

there is no reaſon to doubt out that this part of the kingdom (including in it ſome of the chief manufacturing towns in *Lancaſhire*, *Cheſhire*, and *Yorkſhire*) has conſiderably increaſed. I cannot, however, truſt my belief of this merely to theſe extracts *(a)*; for they deſtroy their own authority by giving a proportion of the births to the burials, which is inconſiſtent with any ſuch increaſe, as will appear from the following obſervations.

If the annual average of burials about 1717 is multiplied by 35 (a multiplier which, in the caſe of a large *country* diſtrict cannot be much too high), it will appear that the whole number of inhabitants in the dioceſe was then 306,000. The exceſs of the births above the burials was 1849, or the 166th part of the inhabitants; and this is an exceſs which, ſuppoſing the increaſe produced by it uniformly accelerated, without being once checked by ſickly ſeaſons

(a) The author of the pamphlet entitled, *The Uncertainty of the Population of the Kingdom*, mentions a very material circumſtance relating to the regiſters of births kept in *Lancaſhire*, and ſome other northern counties.— "I am aſſured," ſays he, "by the moſt authentic information, that, in conſequence of the late multiplication of chapels, it is no uncommon thing for baptiſms (and ſometimes burials) to be entered, in ſome pariſhes in theſe counties, twice over; firſt in the chapel regiſter, and afterwards, for greater ſecurity, in that of the mother church, p. 28."

and emigrations (that is, ſuppoſing it a much greater increaſe from a given ſurplus of births than there is reaſon to expect), could not have doubled the inhabitants in leſs time than 115 years, as may be found by computing in the manner directed in the Note, Vol. I. p. 279. If, therefore, agreeably to the pariſh extracts, they were doubled in 62 years, it muſt have been the effect, not of the exceſs of the births above the burials (the only general cauſe of the increaſe of countries), but of an influx of people from other parts of the kingdom; and, therefore, proves no more than that one part of the kingdom has gained by taking away from other parts. And this may probably have happened in this dioceſe. The truth, however, more probably is, that the pariſh regiſters do not give us true information in conſequence either of having been more deficient formerly, or not having been duly preſerved. See the Notes in p. 321, &c.

This obſervation is applicable to all the other accounts which I have met with taken from pariſh regiſters.——In the dioceſe of St. *David*'s there appears, by the extracts, to have been an addition (between 1715 and 1760) of *three fifths* to the inhabitants. But the exceſs of the births above the deaths will not account for more than a *third* of this increaſe; and as very probably more people leave WALES than flock into it, either (in conformity to the exceſs of the births) there

may have been no increaſe, or the regiſter in the firſt period muſt have been ſo deficient as to give the births near a third leſs than the truth *(a)*.

This argument holds equally with reſpect to the ſecond of the accounts taken from Mr. *Wales*. And his firſt account carries, as before obſerved, impoſſibility on the face of it.

The following is a ſummary of Mr. *Howlett*'s accounts, taken from p. 128 of his *Examination*, &c.

Annual average of births and burials for 20 years about the Revolution, compared with the annual average for the laſt 20 years, in 68 pariſhes in *Kent*, 43 in *Eſſex*, and 17 in *Surry*.

	Births.	Burials.
About the *Revolution* —	2993	3054
For the laſt 20 years —	3947	3983

In the ſame pariſhes, with the addition of 18 in *Suſſex*, 15 in five ſouthern counties, 29 in *Suffolk*, the city of *Norwich*, and five pariſhes in *Wales*.

	Births.	Burials.
About the *Revolution* —	7553	7740
For the laſt 20 years —	10023 *(b)*	10175

To

(a) If the burials are ſuppoſed deficient, as certainly they ought, the births muſt have been proportionably more deficient than the third here reckoned.

(b) There are many errors in Mr. *Howlett*'s numbers, but I have not diſcovered any that will materially affect the proportion of the totals here given

In

To theſe accounts Mr. *Howlett* has added (in p. 13.) a compariſon of the births and burials for two periods of *five* years in 62 pariſhes in 26 counties; the firſt period beginning with 1758, 1760, or 1761; and the ſecond with 1773, 1775, or 1776.

	Annual average of births.	Annual average of burials.
In the firſt period	9527	9910
In the ſecond period	1191	1060

This is all the regiſter evidence which Mr. *Howlett* has produced, excluſive of Mr. *Wales*'s, and that taken from the pariſh regiſters in the dioceſe of *Cheſter* already noticed. This evidence he has diſplayed with great pomp, and inſiſted upon as a full proof of an *aſtoniſhing* increaſe in our population. But never before was an evidence offered ſo abſurd and ſelf-deſtructive. For it ſhould be obſerved, that, according to theſe accounts, the deaths in the kingdom from the Revolution to the preſent time have exceeded

In a poſtſcript he has added to the pariſhes abovementioned the births and burials in 17 others; and all together make the annual averages.

	Births.	Burials.
At the Revolution — — —	8375	8493
At preſent — — —	11195	11382

the

the births *(a)*. Mr. *Howlett*, therefore, will, I hope, ſome time or other, inform us how the increaſe in which he triumphs has been produced.——But to be ſerious. An exceſs of deaths cannot exiſt long in any kingdom. The appearance of it, therefore, in

(a) It may be ſaid, that the exceſs of burials in this and the other accounts before noticed, is occaſioned by a great over-proportion of omiſſions in the regiſtration of births. But what confidence can be placed in regiſters which admit of ſuch defects? or how is it to be known that they were not much greater formerly, agreeably to the obſervations in the Note p. 321?

The omiſſion of ſtill-born and unbaptized infants ſcarcely deſerves notice, becauſe they contribute nothing to population, and are probably, in moſt places, omitted in the burials as well as the births. And with reſpect to other omiſſions, were we to reckon them a *tenth* of the births, and only *half* as much of the burials, ſtill an exceſs of births would be left, which would be almoſt equally inadequate to the increaſe.

In ſhort; let the regiſters of births be ever ſo deficient, the increaſe they ſhew muſt have taken place if they were not more deficient formerly than they have been lately: And yet, this increaſe could not take place unleſs they were deficient to a degree which is incredible, and which, were it credible, would render them unworthy of much notice.——The increaſe, for inſtance, which on this ſuppoſition muſt have taken place in the dioceſe of *Cheſter*, cannot be accounted for from the exceſs of births without reckoning the omiſſions in the regiſters of births equal in both periods to at leaſt a *third* of the regiſtered births, even though the regiſters of burials are reckoned correct and complete. This will appear to any one who will calculate in the manner explained in p. 326, &c. The ſuppoſition, therefore, muſt be wrong that the regiſters of births were not more deficient formerly than they have been lately.

The

in theſe extracts muſt be owing either to their being miſerably erroneous; or to their being taken moſtly from *towns*; for in theſe it ſeldom happens that an exceſs of deaths does not take place; nor is there any worſe cauſe or ſymptom of depopulation than their increaſe.

All the evidence taken from the pariſh regiſters has been now laid before the reader, as far as I am acquainted with it. I am informed that Mr *Wales* and Mr. *Howlett* are proceeding with their enquiries *(a)*; and I hope they will be able hereafter to offer to the public ſome more conſiſtent and probable accounts. When, however, I conſider the reaſon there is for believing that the

The effect which the omiſſion only of baptiſms among Diſſenters may have, will appear from the following fact.——The number of baptiſms at *Sandwich* in *Kent*, among Proteſtant Diſſenters (excluſive of *Baptiſts*) was

From 1690 to 1699	—	120
From 1730 to 1739	—	58
From 1770 to 1779	—	13

The number of baptiſms in the ſame town for the ſame periods reſpectively was, excluſive of Diſſenters, 755, 744, and 758

(a) I have not ſought for any accounts of this kind, not chuſing to give trouble to obtain ſo indeciſive and precarious an evidence. The following are all I can add from my own information to thoſe already given.

Lincoln.

the parish registers were in former periods particularly defective, I cannot help doubting

	Annual births	Annual burials	Annual marriages
Lincolnshire—*Swinderby* parish 10 years to 1690	7.3	7.5	2.5
to 1720	5.8	5.0	2.0
to 1770	7.1	5.0	1.4
Durham—*Staindrop* parish 10 years to 1745	37.6	28.5	7.0
to 1771	49.3	44.8	12.9
Kent——*Tenterden* parish 20 years to 1729	29.8	33.6	9.1
to 1769	34.5	34.0	11.9
Sandwich parish 10 years to 1629	148.3	159.6	41.3
to 1689	103.2	95.8	31.7
to 1739	74.4	70.4	16.3
to 1779	75.8	68.8	21.3
Eastry parish 10 years to 1629	20.1	12.1	6.4
to 1689	13.7	12.2	2.6
to 1739	17.3	13.0	4.2
to 1779	20.7	13.4	5.2
Word parish 10 years to 1739	7.6	4.9	1.2
to 1779	6.7	4.8	2.0
Woodnesborough parish 10 years to 1719	15.5	10.9	7.3
to 1779	14.8	12.4	4.1
Ash parish 20 years to 1578	27.7	25.7	6.6
to 1777	50.0	39.7	13.9
Cornwall——*Liskeard* parish 20 years to 1719	51.7	45.3	13.0
to 1769	48.3	45.3	12.8
Devonshire—*Okeford* parish 20 years to 1719	12.2	8.0	
to 1769	12.2	7.5	
Staffordshire—*Biddulph* 20 years to 1719	20.3	15.6	4.3
to 1739	27.8	21.1	4.4
to 1769	38.9	21.1	6.1

whether

whether any examination of them is capable of furniſhing with ſufficient evidence to prove that our population has not decreaſed ſince the *Revolution*, I queſtion even whether it can inform us properly of the proportion of births to deaths in the kingdom. This alone, could it be aſcertained, would enable us to form ſome judgment of the preſent ſtate of our population, and to determine, with ſome probability, whether it is increaſing or decreaſing. If we unite all the extracts before given, rejecting Mr. *Howlett*'s, this proportion will come out $\frac{128}{100}$. Were theſe extracts to be depended on, they would probably give this proportion too high for the kingdom at large, becauſe taken chiefly from the regiſter of the dioceſe of *Cheſter*, the moſt populous and flouriſhing part of the kingdom (*a*). We may, however, argue upon it, and reckon it the juſt proportion for

(*a*) Dr. *Short* has employed much time and pains in collecting extracts from the regiſters of a great variety of market-towns and country pariſhes and villages in different parts of the kingdom for two periods, the firſt extending from the reign of Queen *Elizabeth* to the middle of the laſt century; and the ſecond from different years at the end of the laſt century to the middle of the preſent century: and from a compariſon of theſe extracts it appears, that in the former period the births exceeded the burials in the proportion of 124 to 100: but that in the latter they exceeded them only in the proportion of 111 to 100

This,

for *England* and *Wales*, excluſive of *London* and its environs; on which ſuppoſition, if we reckon the annual burials ſuch as, in conſequence of multiplying by 35, will make the inhabitants of *England*, excluſive of *London*, four millions and a half, the annual burials will be nearly 128,000, and the births 164,000, leaving an annual exceſs of 36,000; and this is an exceſs which would produce an increaſe in moſt other countries, notwithſtanding the waſte in their capitals, and all the other cauſes which uſually check the increaſe of countries (*a*) But

This, were there ſufficient evidence for it, would manifeſt too plainly an encumbered and declining population. It appears (as Dr. *Short* ſpeaks) with no leſs evidence from the regiſters *than that the ſun ſhines in a cloudleſs day at noon*; and he concludes from it, that in conſequence of the irregularities and debauchery occaſioned ſince the Revolution, by increaſing opulence and luxury, the kingdom has been for many years growing leſs healthy. But the truth is, that the regiſters (having certainly been more defective formerly than they are at preſent) cannot be truſted as a juſt foundation for any concluſions.—— See Dr. *Short's New Obſervations*, Tables 1ſt, 2d, and 3d, and p. 80.—See likewiſe the Preface to his *Hiſtory of the Comparative Increaſe and Decreaſe of Mankind*; and the Tables at the end.

(*a*) The proportion of births to deaths in all SWEDEN for 9 years to 1763 was } 130 to 100
In the kingdom of NAPLES for 5 years to 1777 — — — } 144 to 100
In all FRANCE for 5 years to 1774 as 928,918 to 793,931, or — — } 117 to 100

Annual

But perhaps there are few kingdoms now exiſting in which moſt of theſe cauſes operate ſo

Annual average of births, deaths, and marriages in *Breſlaw*, *Glogaw*, and the other *towns* of SILESIA for four years to 1778.

Births.	Deaths.	Marriages.	Proportion of births to marriages.	Proportion of births to deaths.
10900	10935	2409	45 to 10	996 to 1000

Annual average of births, deaths, and marriages in the *country pariſhes and villages* of SILESIA for the ſame period.

Births.	Deaths.	Marriages.	Proportion of births to marriages.	Proportion of births to deaths.
53694	42894	11848	45 to 10	125 to 100

SILESIA appears from hence to conſiſt of near two millions of inhabitants; of whom the inhabitants of towns are about a *ſixth* part.

The following accounts (copied from the Tables at the end of the Firſt Volume of Mr. *Suſmilch's Gottliche Ordnung*, 3d Edition) will ſhew, in ſome meaſure, the uſual progreſs of population in a country. They will alſo ſerve for a contraſt to the inconſiſtent extracts which I have given from our pariſh regiſters; for it will appear that inſtead of ſhewing an increaſe too great for the ſurplus of births, they always (in conſequence of ſickly years and other cauſes) ſhew a much ſmaller increaſe than it was capable of producing.

In the old PRUSSIAN dominions and the provinces of *Brandenburg*.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
4 years to 1701	66247	41680	18145	36 to 10	148 to 100
7 years to 1728	82934	60821	20726	40 to 10	136 to 100
6 years to 1756	102935	78863	24487	40 to 10	136 to 100

In

ſo much as in this. Few kingdoms have been engaged within ſo ſhort a period in ſo many deſolating wars. Few kingdoms have had ſuch armies and garriſons and ſettlements to maintain in ſo many diſtant regions, and

In the kingdom of *Pruſſia* and dukedom of *Lithuania*.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
10 years to 1702	21963	14718	5908	37 to 10	150 to 100
5 years to 1716	21602	11984	4968	39 to 10	180 to 100
5 years to 1756	28392	19154	5599	50 to 10	148 to 100

N. B. In 1709 and 1710 a peſtilence carried off 247,733 of the inhabitants of this country; and in 1736 and 1737 epidemics prevailed, which again checked its increaſe.

In the *Churmark* of BRANDENBURGH.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1702	13433	7605	3597	37 to 10	176 to 100
4 years to 1756	23486	18840	6646	38 to 10	124 to 100

Duchy of POMERANIA.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
6 years to 1702	6540	4647	1810	36 to 10	140 to 100
6 years to 1708	7455	4208	1875	39 to 10	177 to 100
6 years to 1726	8432	5627	2131	39 to 10	150 to 100
4 years to 1756	12767	9281	2957	43 to 10	137 to 100

In this inſtance the inhabitants appear to have been almoſt doubled in 56 years, no very bad epidemic having once interrupted the increaſe; but the three years immediately following the laſt period (to 1759) were years ſo ſickly that the births were ſunk to 10,229, and the burials raiſed to 15,068.

Neumark

and in ſuch unhealthful climates. No kingdom ever ſupported ſuch a navy, or carried on ſo extenſive a foreign commerce, or wanted, on theſe accounts, ſuch a ſupply of *men* for the ſea-ſervice: Nor was there ever a king-

Neumark of BRANDENBURG.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1701	5433	3483	1436	37 to 10	155 to 100
5 years to 1726	7012	4254	1713	40 to 10	164 to 100
5 years to 1756	7978	5567	1891	42 to 10	143 to 100

Epidemics prevailed for 6 years from 1736 to 1741, which checked the increaſe.

Dukedom of MAGDEBURG.

Annual average.	Births.	Burials.	Marriages.	Proportion of births to marriages.	Proportion of births to burials.
5 years to 1702	6431	4103	1681	38 to 10	156 to 100
5 years to 1717	7590	5335	2076	36 to 10	142 to 100
5 years to 1756	8850	8069	2193	40 to 10	109 to 100

The years 1738, 1739, 1740, 1741, 1750, and 1751 were particularly ſickly.

Duchy of HALBERSTADT.

Annual average.	Births.	Burials.	Marriages.	Births to marriages.	Births to burials.
4 years to 1692	2366	1478	604	39 to 10	160 to 100
5 years to 1746	2803	2052	712	39 to 10	136 to 100
6 years to 1756	2917	2621	778	37 to 10	111 to 100

Duchy of RAVENSBERG.

Annual average.	Births	Burials.	Marriages.	Births to marriages.	Births to burials.
5 years to 1692	3899	2552	964	40 to 10	152 to 100
4 years to 1756	5041	3814	1371	36 to 10	132 to 100

a kingdom which conſiſted ſo much of people employed in trades and manufactures, which

Dukedom of CLEVE and County of *Mark*.

Annual average	Births.	Burials.	Marriages.	Births to marriages.	Births to deaths.
4 years to 1701	6249	4132	1729	36 to 10	151 to 100
5 years to 1739	7358	5535	1741	42 to 10	134 to 100
4 years to 1756	7612	5567	1966	38 to 10	136 to 100

AUSTRIAN MILANESE;

Conſiſting in 1774, of 211,479 families, and 1,116,859 inhabitants; and in 1769, of 1,101,723 inhabitants, of whom 9638 were prieſts, 5616 friars, and 7140 monks and nuns.

Annual average of	Births.	Burials	Marriages.	Births to marriages.	Births to deaths.
1769, 1773 and 1774	44030	40030	9619	45 to 10	110 to 100

N. B. The laſt of theſe years appears to have been particularly ſickly; for the burials exceeded the births, and were 9156 higher than the average of the years 1769 and 1773.

DENMARK.

Annual average of	Births.	Burials.	Births to burials.
5 years to 1747	22996	18864	121 to 100
5 years to 1756	24298	21706	112 to 100

Epidemics prevailed in 1755, and 1756, which made the burials in thoſe years nearly equal to the births.

The medium of theſe ten years is nearly 20,000; and, multiplying it by 35, will make the number of inhabitants then in *Denmark* 700,000.

NORWAY.

Annual average of	Births.	Burials.	Births to burials.
5 years to 1747	17522	10955	160 to 100
14 years to 1756	19947	14661	136 to 100

Multiplying 16000 (the average of burials in *Norway* for four years to 1756) by 35, will make the number of inhabitants 560,000 in 1756.

In

which ſhorten life, or whoſe metropolis was ſo large, or *half* ſo large, in compariſon with the number of its inhabitants.——If we include in LONDON all the pariſhes and little towns near LONDON, where, almoſt univerſally, the burials exceed the births, it is moderate to reckon that the former exceeds the latter in this part of the kingdom about 10,000 annually; and that, conſequently, LONDON demands a recruit of people every year equal to this number. Forty years ago there was this exceſs of burials within the bills only. This will make the annual ſurplus for the whole kingdom 26,000 which may probably be ſufficient, or perhaps more than ſufficient, to ſupply all the waſte occaſioned by ſickly ſeaſons, emigrations to the colonies, and the other cauſes I have mentioned.——But the truth is, that it cannot be reckoned with any degree of

In 1056 country pariſhes and villages in the *Churmark* of *Brandenburgh*, conſiſting (in 1748) of 106,204 males and 107,540 females.

Annual average of	Births.	Burials.	Marriages.	Births to marriages.	Births to burials
10 years to 1748	7099	5561	1966	36 to 10	127 to 100

In ſeven market-towns and 54 country-pariſhes in *England*, conſiſting (in 1740) of 10434 families and 46,650 inhabitants, according to Dr. *Short's New Obſervations*, p. 133.

Annual average.	Births.	Burials.	Marriages.	Births to marriages.	Births to burials.
In 1748	1575	1360	399	40 to 10	115 to 100

confidence, that there exiſts any ſuch ſurplus.

Mr. *King*, in 1693, ſtated the births of the kingdom, excluſive of thoſe in *London*, at 170,000, and the burials at 148,000, which makes the proportion of the former to the latter as 115 to 100. See Dr. *Davenant*'s Works, Vol. II. p. 180. Mr. *King* deduced this from the aſſeſſments then impoſed on births, marriages and burials; and he has ſhewn ſuch ſagacity in his other eſtimates, that I cannot help paying ſome regard to him in this. If he was right, the kingdom has probably been decreaſing, ſuch a ſurplus being incapable of ſupporting a population ſo encumbered as ours, and which ever ſince Mr. *King*'s time has had ſuch increaſing demands upon it.

I cannot help taking this opportunity to obſerve, that there is reaſon to believe that poor countries (provided the ground ſupplies them with plenty of food, and the poverty of the inhabitants conſiſts only in their wanting *conveniencies* and *elegancies*, in other countries deemed *neceſſaries*) increaſe faſter than *rich* countries. The reaſon is obvious. The greateſt enemies of population are the artificial wants, the accumulation of property, and the luxury and vices which are the conſtant attendants of opulence, and which prevent a regular and early union between the ſexes. The inhabitants of poor countries are more ſimple, more

more healthy, and more virtuous; and, wanting little beſides food, families are no burdens, and the prolific powers of nature have free ſcope to diſplay themſelves.—Perhaps IRELAND is one inſtance of this. If we may depend on an account in the Philoſophical Tranſactions (Abridgement, Vol. III. p. 666.) the number of people in *Ireland*, in 1695, did not much exceed a million. At preſent they are, I ſuppoſe, about two millions.——According to an account publiſhed annually at *Dublin*, in *Watſon*'s Almanack, the houſes in *Ireland*, in 1754, were 395,439. In 1767 they were increaſed to 424,046; and in 1777 to 448,426. But I have been informed that this account is of no authority, and deſerves little credit. Nor can I learn that there are in *Ireland* any *documents* from which a judgment tolerably correct can be formed of the progreſs and preſent ſtate of its population. It might have been expected, that the hearth-tax would have furniſhed ſuch *documents*: But this is not the caſe; and all that is known with certainty is the yearly produce of the tax; the average of which for the laſt five years to 1781, having been 60,648*l.* makes the number of hearths that pay the tax (at 2*s.* per hearth) to be 606,480. It is ſuppoſed that a houſe may be allowed for every *two* hearths, and that a *third* of the houſes are excuſed on account of inability

and, on theſe ſuppoſitions, the number of houſes will exceed 400,000 (*a*); and, conſequently, the inhabitants will be (as juſt reckoned) about two millions (*b*).

(*a*) In the year 1787 the following account was returned to the Houſe of Commons of Ireland, of the number of houſes in that kingdom paying hearth-money.

No. of Houſes containing Hearths.		No. of Houſes containing Hearths.		No. of Houſes containing Hearths.		No. of Houſes containing Hearths.	
1	397,644	15	99	29	4	45	4
2	24,031	16	127	30	16	46	1
3	7,562	17	46	31	4	50	3
4	5,542	18	42	32	4	55	1
5	4,062	19	23	33	6	56	1
6	3,556	20	61	34	3	67	1
7	3,330	21	13	35	3	92	1
8	2,209	22	10	36	6	112	1
9	985	23	9	37	1	Houſes exempted by law	23,075
10	772	24	20	39	1		
11	316	25	20	40	7		
12	295	26	10	41	3		
13	147	27	5	42	3		
14	139	28	8	44	2		

From this table it appears that the number of hearths (excluſive of thoſe exempted by law) is 612,577; and therefore, on the ſuppoſition adopted in this poſtſcript, the whole number of houſes in Ireland will be 408,384.—But if the preceding accounts be accurate, their real number amounts to 474,234, and conſequently the inhabitants will rather exceed two millions and a quarter. ED.

(*b*) A ſurvey of BELFAST was made in *Jan.* 1782, from which it appeared, that it conſiſted of 2026 houſes, containing 13,105 inhabitants, 6133 of whom were males, and 6972 females.——Looms 388; and houſes for ſelling beer and ſpirits 119, or a 17 part of all the houſes. ——On *Jan.* 1, 1757, the number of looms was 399, and the houſes 1779, containing 8549 inhabitants, of whom 7993 were *Proteſtants*, and 556 *Papiſts*.

Sweden,

Sweden, *Norway* (*a*), and the kingdom of *Naples*, are increaſing faſt; and alſo RUSSIA, if we may judge from the following facts.

In the viceroyalty of *Tweer* (in 1780) there died 4315 males; 3646 females; but there were born 11948 males, and 9013 females. The marriages were 6074.

In the eparchy of *Vologda* the deaths in the ſame year were 2688 males, and 2377 females, The births were 6517 males, and 5366 females. The marriages 3232.

In both theſe provinces, therefore, the births were conſiderably more than *double* the deaths; and the increaſe muſt be rapid.

At the beginning of the ſame year (1780) there were found in the diſtrict of Moscow 137,698 males, and 134,918 females; of whom died in the courſe of the year 2101 males, and 1601 females, or the 65th part of the males, and 84th part of the females. But there were born in the courſe of the year 4546 males, and 4075 females, which added 5919 (or a 46th part) to the inhabitants; and the number of inhabitants actually counted at the end of the year was 140,143 males, and 137,392 females (*b*)

(*a*) See the Preliminary Obſervations to Table XLII. p. 146; and the Eſſay on the *Population of England*, p. 14.

(*b*) Theſe accounts have been given by authority in RUSSIA; and were communicated to me by Mr. *Howard*; who with views of unparalleled humanity, travelled through that country in 1781——To Mr. *Howard*'s enquiries I likewiſe owe the account in the note p. 335 of SILESIA.

But there exiſts probably among mankind no ſuch increaſe as that among the United States of NORTH-AMERICA; according to the account of it in Vol I. p. 276, &c.

The reflection on theſe facts muſt be mortifying to this country (the richeſt upon earth) if it be indeed true that our population is declining. But we muſt comfort ourſelves by conſidering that in this caſe, *value* is of more conſequence than *number*. Commerce, arts, and liberty, once placed the little ſtate of *Athens* at the head of the world; and the ſame cauſes once raiſed this iſland to the ſame eminence.

To the direct evidence already ſtated of a decreaſe in our population, it is proper to add the following facts.

1ſt. The decreaſe of LONDON. This I muſt reckon certain, till ſome other ſatisfactory reaſon (*a*) can be given for a diminution ſince 1727, of more than 7000 *per ann.* in the regiſtered burials, and near 2000 in the regiſtered births.

(*a*) The new burying grounds (taken notice of in the Notes p. 255 and p. 260, Vol. I.) have been opened but lately; and therefore cannot account for this diminution; nor do the burials in them amount to a number equal to it.

Annual medium of regiſtered burials in LONDON.

For five years	to 1722 inclusive	26,443
	to 1727	26,747
	to 1732	26,582

Annual

Secondly. The decreaſe in the produce of the hereditary and temporary exciſe upon beer. This was almoſt the only exciſe that exiſted before the Revolution; and though the country was then poorer, it produced a *quarter* more than it has lately. This fact, together with the objections to the inference I have drawn from it, may be found diſtinctly ſtated in the Eſſay on the *Population of England*, &c. p. 18, &c. and p. 45, &c.

Thirdly. The growing diſtreſs among the lower orders of people, who are the majority of the nation, deſerves to be parti-

For five years to	1737	26,848
	to 1742	28,344
	to 1748	23,884
	to 1753	22,006
	to 1758	20,875
	to 1763	22,593
	to 1768	23,319
	to 1773	22,754
For four years to	1777	20,945
For three years to	1780	20,438
For two years to	1782	19,313

Annual medium of regiſtered births in LONDON.

For five years to	1727	18,898
	to 1768	16,291
	to 1782	16,966

The decreaſe which this Table ſhews to have taken place lately in the exceſs of burials above the births, has been aſcribed to an improved ſtate of LONDON with reſpect to its influence on the health of its inhabitants; but the true reaſon is the fact referred to at the beginning of this note.

cularly

cularly attended to on this ſubject. The increaſe of the poor rates proves this fact; and it ſeems to be univerſally acknowledged. A people at their eaſe will increaſe; but increaſing difficulties in procuring the means of ſubſiſtence, producing a forced induſtry, and an averſion to marriage, muſt depopulate.

The increaſed produce of the taxes on candles, leather, &c. the incloſures of waſte lands, and the improvements in agriculture which have taken place lately, have been urged in oppoſition to theſe facts. But I am afraid they only prove that luxury has increaſed conſumption more than it has leſſened the number of our people.

Upon the whole. I beg it may be remembered, that my opinion, in this inſtance, is by no means a clear and decided conviction. I may probably be influenced too much by a deſire to maintain an aſſertion once delivered.——Some time or other, perhaps, the Legiſlature will think this a point worth its attention. Much light may be thrown upon it, and the ſtate of our population kept conſtantly in view, by only ordering exact regiſters to be kept of the births, burials, and marriages in the kingdom. This is done in other kingdoms. It has lately been done in *France*; and the reſult has been a diſcovery that the population of FRANCE exceeds all that had been conjectured

jectured concerning it *. Should a like diſcovery be the conſequence of carrying ſuch an order into execution here, it will give the kingdom an encouragement which at preſent it greatly wants; and I ſhall rejoice in my own confutation.

* See the Appendix to a Diſcourſe on the Love of our Country, delivered by the Author on November 4th, 1789, to the Society for commemorating the Revolution in Great Britain.—In this Appendix it is obſerved, that the medium of annual deaths, births, and marriages, in the kingdom of France, was

Of births	for four years, to		1774	914,710
Of deaths	—	—	—	793,931
Of marriages	—	—	—	192,180
Of births,	for ſix years, to		1780	958,419
Of deaths	—	—	—	834,865
Of marriages	—	—	—	228,170

If 834,865, the number of deaths to 1780, be multiplied by 35, agreeable to the rule in p. 326, it will appear that the whole number of inhabitants in this kingdom exceeds *twenty-nine millions*. ED.

THREE

ADDITIONAL ESSAYS,

AND

NOTES.

ADDITIONAL ESSAYS.

FIRST ADDITIONAL ESSAY.

Obſervations on the Difference between the Duration of Human Life in Towns and in Country Pariſhes and Villages.

Read to the Royal Society, June 22, 1775, *and publiſhed in the* 65th *Volume of the* Philoſophical Tranſactions, Part II.

THIS Society has lately been much obliged to Dr. *Percival*, for the accounts he has communicated of the ſtate of population at *Mancheſter* and other adjacent places (*a*). Theſe accounts contain ſome facts, which appear to me curious and

(*a*) See *Philoſophical Tranſactions*, vol. 65, p. 322, and vol. 64, p. 57.

The particulars of the ſurveys here referred to are the following.——According to a ſurvey executed with great care

and important. From the laſt in particular, there appears to be reaſon for concluding, that whereas a 28th part of the inhabitants die annually in the town of *Mancheſter*, not more than a 56th part die annually in the adjacent country. This implies a difference ſo great between the rates of human mortality in theſe different ſituations, that ſome, whoſe judgements I reverence, have thought it incredible. I will, there-

care there were, in the ſummer of 1773, in the town of

Mancheſter,		*Salford*,
3402	Houſes	866
5317	Families	1099
10548	Males	2248
11933	Females	2517
7724	Married	1775
432	Widowers	89
1064	Widows	149
7782	Under 15	1793
3252	Above 50	640
342	Male Lodgers	18
150	Female Lodgers	13
44	Empty Houſes	26

According to a ſurvey in 1774 there were in the pariſh of *Mancheſter*, containing thirty-one townſhips, excluſive of the towns of *Mancheſter* and *Salford*.

Tenanted Houſes	2371	Under 15	5545
Families	2525	Above 50	1762
Inhabitants	13786	Above 60	470
Males	6942	Above 70	261
Females	6844	Above 80	87
Married	4319	Male Lodgers	68
Widowers	232	Female Lodgers	51
Widows	315	Empty Houſes	41

fore,

therefore, beg leave to offer the following obſervations on this ſubject.

In the firſt place, the evidence in this inſtance is ſuch as ſeems to leave little room for doubt. From an accurate ſurvey it appears, that the number of inhabitants in the town was 27,246 in the year 1773. The number of deaths the ſame year (and alſo the average for 1772, 1773, and 1774), was 973 (*a*); that is, a 28th part of the number of inhabitants. From an equally careful ſurvey it appears, that the number of inhabitants in that part of the pariſh of *Mancheſter* which lies in the country, was 13,786. The number of deaths in 1772 was 246; that is, a 56th part of the number of inhabitants. The chief objection to this evidence is, that the number of deaths in that part of the pariſh which lies in the country is given only for one year; whereas the average of ſeveral years ought to be given.

(*a*) The numbers of burials in the town, including the addition of 50 every year for Diſſenters, was, in

1772, —— 954
1773, —— 973
1774, —— 1008

Within the pariſh, but out of the town, there are 13 epiſcopal and diſſenting chapels; and the number of burials in all theſe chapels, in 1772, was 246 The chriſtenings were 401. The number of burials brought from the country into the town is not conſiderable; and it is, I am informed, pretty exactly balanced by the burials carried out of the town into the country.

But firſt, the number of deaths in 1772, in the town, was nearly the ſame with the medium for ſeven years; and from hence there ariſes a probability, that in the adjacent country, the number of deaths, in the ſame year, could not have been much lower than the medium. Secondly, ſuppoſing it lower, there is the higheſt probability, that it was not more than a 4th or 5th lower. Suppoſe then the true annual medium to be 300, inſtead of 246, and it will follow, that whereas a 28th part of the inhabitants die in the town annually, a 46th part die in the country; and this is a difference very conſiderable. But farther, I would obſerve, that the difference which this ſurvey gives between the rate of mortality in the town of *Mancheſter* and the adjacent country, is confirmed by a variety of other accounts. It may be ſtated in general, that whereas in great towns, the proportion of inhabitants dying annually is from 1 in 19 to 1 in 22 or 23, and in moderate towns from 1 in 24 to 1 in 28 (*a*); in country pariſhes and villages, on the contrary, this proportion ſeldom exceeds 1 in 40 or 50. The

(*a*) The number dying annually in towns is ſeldom ſo low as 1 in 28, except in conſequence of a rapid increaſe produced by an influx of people, at thoſe periods of life when the feweſt die. This is the caſe at *Mancheſter*. It is alſo the caſe at *Liverpool* and at *Berlin*; in the former of which towns, 1 in 27 dies annually; and in the latter, 1 in 26½ died from 1755 to 1759. See Vol. I. of this Treatiſe, Eſſay I. page 250—295.

proofs

proofs of this are numerous and unexceptionable; and I have elſewhere given a particular account of them. I will here only mention the following facts.

The number of inhabitants at *Stockholm* in 1763 was 72,979. The average of deaths for the ſix preceding years had been 3802 (*a*). One, therefore, in nineteen died there annually.

At *Rome*, an account is taken every year of the number of inhabitants; and, in the year 1771, it was 159,675. The average of deaths for ten years had been 7367. One, therefore, in $21\frac{1}{2}$ died annually.

(*a*) See a Memoir by M. *Wargentin*, in the 15th volume of the *Collection Academique*, printed at *Paris*, 1772. From this memoir I learn, that in 1757, and 1760, and 1763, a ſurvey was made of the inhabitants of *Sweden*, diſtinguiſhing, particularly, the numbers of both ſexes living at every age; and that alſo, for nine years (or from 1755 to 1763), an exact Regiſter was kept of the number of births and burials in each year, diſtinguiſhing the age and ſex of every one that died. The reſult, as given by M. *Wargentin* in this Memoir, contains indeed a moſt curious account of the ſtate of population in *Sweden*; and it is particularly to my preſent purpoſe to mention, that it ſhews, that though a 19th part of the inhabitants of *Stockholm* die every year, yet in the whole kingdom, taking all the towns and country together, not more than a 35th part die every year. In 1757, *Sweden* conſiſted of 1,101,595 males, and 1,221,600 females; in 1760, of 1,121,053 males, and 1,246,445 females; and in 1763, of 1,165,489 males, and 1,280,905 females. The annual average of births, from 1755 to 1763, was 46,223 males, and 44,017 females; of marriages, 21,219; of deaths, 34,088 males, and 35,037 females.

In *London* I have ſhewn, with an evidence which I think little ſhort of demonſtration, that at leaſt 1 in $20\frac{3}{4}$ of the inhabitants die annually (*a*). And, from a particular ſurvey and a very accurate regiſter of mortality at *Northampton*, it appears, that 1 in $26\frac{1}{2}$ die there annually.

Let theſe facts be compared with the following. In 1767, a ſurvey was made of the inhabitants of the iſland of *Madeira*, under the direction of Dr. *Thomas Heberden*, and their number was found to be 64,614. The average of burials for eight preceding years had been 1293. Only 1 in 50, therefore, of the inhabitants died annually (ſee *Philoſophical Tranſactions*, vol. lvii. p. 461.)

The diſtrict of *Vaud*, in *Switzerland*, in 1766, contained 112,951 inhabitants. The average of deaths for ten preceding years had been 2504. Only 1 in 45, therefore, died annually (*b*).

The number of inhabitants in the pariſh of *Ackworth*, in the county of *York*, in 1757, was 603; and the average of deaths for ten years had been $10\frac{7}{10}$, or a 56th part. In 1767, the inhabitants were increaſed to

(*a*) See Volume I. of this work, Eſſay IV. page 267, &c.

(*b*) See M. *Muret*'s Memoir on the State of Population, in the *Pays de Vaud*, printed at *Bern*, in 1766.

728;

728; and the annual average of deaths was $15\frac{3}{10}$, or nearly a 47th part. (*a*)

The reaſon of this ſtriking difference between the rate of human mortality in towns and in country pariſhes and villages muſt be, firſt, the luxury and the irregular modes of life which prevail in towns; and, ſecondly, the foulneſs of the air. But it has been inquired, whether the migrations of people from the country to towns may not produce this difference, by leſſening the proportion of inhabitants that die in the country, and increaſing the ſame proportion in towns? In anſwer to this enquiry I would obſerve; firſt, that this difference being a difference of near a half, it is apparently much greater than can be accounted for by any ſuch cauſe. But, ſecondly, it ſhould be conſidered, that if migrations leſſen the number of deaths, they alſo leſſen the number of inhabitants; and that it depends intirely on the ages at which the inhabitants remove from a place, whether the effect of their removal ſhall be lowering or raiſing the proportion of the annual deaths to the number of inhabitants. In the preſent caſe, the truth appears to be, that the moſt common age of migration

(*a*) I owe this information concerning the pariſh of *Ackworth* to a curious Regiſter kept there by Dr. *Lee*. I have taken the liberty to inſert this regiſter in the Poſtſcript, together with the annual regiſter and ſurvey of *Rome* from 1762 to 1771.

from the country is ſuch as raiſes this proportion in the country. This will be evident from the following conſiderations. The period of life in which perſons remove from the country to ſettle in towns is chiefly the beginning of mature life, or from the age of 10 or 15 to 25 or 30. Towns, therefore, will be inhabited more by people in the firmeſt parts of life; and, on the other hand, the country will be inhabited more by people in the weakeſt parts of life; and the conſequence of this is, that in the country, the inhabitants muſt die faſter in proportion to their number than they otherwiſe would, and that in towns they muſt die more ſlowly. In particular, the number of children is always much greater in the country than in towns; and this is a circumſtance which muſt be extremely unfavourable to the former: for it is well known, that there are no years of life, in which ſo many die as the firſt three or four years. Till the age of five, human life, like a fire beginning to burn, is very feeble; and in ſome ſituations more than half, and in others, a third or fourth of all that are born die before that age. After this, life grows leſs and leſs precarious till it acquires its utmoſt vigour at 10 or 12; and of the living at this age, not above 1 in 70 or 80 dies annually in the worſt ſituations; and in the beſt ſituations, not above

above 1 in 150 or 160. After 15, life declines, and continues to do ſo more and more, till it becomes quite extinct in old age. If therefore, in any ſituation, the inhabitants conſiſt more of perſons in mature life, and yet die faſter, it muſt be owing to ſome particular cauſes of mortality that operate there. This is the caſe in all towns where any obſervations have been made. *Mancheſter*, in particular, is not only kept up, but increaſes faſt, by removals to it of perſons in the prime of life. The country round it increaſes likewiſe; but it is by an exceſs of the births above the deaths; that is, by acceſſions to it of children in the very feebleſt part of life. This ought to raiſe the proportion of annual deaths to inhabitants in the country, much above the ſame proportion in the town; but, inſtead of this, it is near one-half lower.

It may be needleſs to add any thing to theſe obſervations.

In order, however, to put this matter out of all doubt, I will obſerve farther, that it appears in fact, from the accounts furniſhed by Dr. *Percival*, that the number of inhabitants in the period of life when mankind die faſteſt (*a*) (that is in the firſt and laſt ſtages

(a) In towns, about a fourth of the inhabitants die commonly between 14 and 51; a fifth or ſixth die at 51 and upwards; and the remainder die under 15. In coun-

ſtages of life), is conſiderably leſs in the town of *Mancheſter* than in the adjacent country. The number of inhabitants in the town under 15 and above 50, is 13,467; in the country, 7305. And the whole number is, in the town, 27,246; in the country, 13,786. In the town, therefore, the inhabitants, in the firſt and laſt ſtages of life, do not make half the whole number; but in the country they make conſiderably more than half. At *Ackworth*, likewiſe, in *Yorkſhire*, the inhabitants under 15 and above 50 are more than half the whole number; and the ſame is true at *Hale* near *Altringham*; at *Horwich*; at *Darwen*, near *Blackburn*, in *Lancaſhire*; and at *Cockey Moor* (*a*), near *Bolton*,

try pariſhes and villages about a fifth die between 14 and 51; about two-fifths at 51 and upwards; and the remainder under 15.

(*a*) I am much indebted to Dr. *Percival* for the following account of theſe places. The ſociety belonging to the chapel at *Hale* is compoſed of 140 males, 136 females, 92 married perſons, 8 widowers, 12 widows, 105 under 15, and 41 above 50. The deaths, during ſeven years, have been 28, and the births 68. Mr. *Evans*'s congregation at *Horwich*, conſiſts of 305 individuals; *viz.* 149 males, 156 females, 94 married perſons, 9 widowers, 8 widows, 127 under 15 years of age, and 50 above 50. The births, for ſeven years, 101; the deaths 32. A 66th part, therefore, die annually in both theſe places. The Rev. Mr. *Smalley*'s congregation at *Darwent*, conſiſts of 1850 individuals; viz. 900 males, 950 females, 640 married perſons, 30 widowers, 48 wi-

Bolton, in the ſame county; and yet in ſome of theſe places it appears, that not a 60th part of the inhabitants die annually.

At

48 widows, 737 perſons under the age of 15, and 218 above 50. During the laſt ſeven years the births have amounted to 508, the deaths to 233. A 56th part, therefore die annually. Mr. *Barnes*'s congregation at *Cockey Moor*, conſiſts of 154 families and 711 individuals; namely, 320 males, 391 females, 248 married perſons, 10 widowers, 27 widows, 252 perſons under the age of 15, and 99 above 50. Deaths in ſeven years 114; in which period the deaths were conſiderably increaſed by an uncommon fatality of the ſmall pox. One perſon in 44 died annually. The Rev. Mr. *Mercer's* congregation at *Chowbent* in *Lancaſhire*, conſiſts of 1160 perſons; *viz.* 554 males, 606 females, 173 males and 150 females under the age of ten, 83 males and 91 females above 50, 398 married perſons, 26 widowers, and 43 widows. The baptiſms during ſix years, wanting ſix weeks, have amounted to 293, and the deaths to 169. One perſon, therefore, in 41 died annually. Theſe ſurveys were made in the year 1773.——In Auguſt 1774 the inhabitants of *Tattenhall* and *Waverton* (two pariſhes in the neighbourhood of *Cheſter*) were ſurveyed. The former conſiſted of 382 males and 399 females, of whom 462 were above 14 years of age. The latter contained 310 males and 322 females, of whom 406 were above 14 years of age.——At *Tattenhall* the annual average of chriſtenings, for 10 years ending in 1773, had been 28; of burials, 13.——At *Waverton* the ſame average had been 19.3 and 8$\frac{4}{10}$.—In the former pariſh, therefore, a 60th part of the inhabitants, and in the latter a 75th part had died annually.——In 1775 the town and pariſh of *Aſhton under Line* (diſtant 8 miles from *Manchester*, and conſiſting of manufacturers and farmers) were ſurveyed. The number of inhabitants was 5097, of whom 2534 were males, and 2513 females; 1679 were married; and their ages were, under five, 896—from 5 to 10,

At *Stockholm*, in 1763, the inhabitants under the age of 5 were only a 12th; above 70, only a 46th part of the whole number, But in all *Sweden*, the number under 5 was a 7th; and above 70, near the 32d part of all the inhabitants: and yet 35 die in the town to 19 in the whole kingdom. This may be eaſily deduced from Table I. in the Poſtſcript.

To the accounts which give the proportion of inhabitants to annual deaths ſo high as 50 or 60 to 1, it has been farther objected, that if true, it muſt follow, that in ſuch ſituations half the inhabitants muſt live to 50 or 60 years of age. But were this a right inferrence, there would be nothing in it incredible. For though in moſt cities one-half die in the firſt two or three years after birth; yet, in many country ſi-

10, 764—from 10 to 20, 1011—from 20 to 50, 1882—from 50 to 70, 471—from 70 to 90, 73. Of theſe 2709 at leaſt, or more than half, muſt have been under 15, and above 50.—See a communication of Dr. *Percival*'s in the Philoſophical Tranſactions, vol. 66, p. 160.

I will add here that, according to an accurate ſurvey communicated to me by one of the gentlemen concerned in making it, of the townſhip of *Leeds*, in *Yorkſhire*, it conſiſted (in 1775) of 15,216 inhabitants in the town, and 1905 inhabitants in the villages and country near the town. The number of males was 8112; of females 9009; of whom 6309 were married; 724 were widows, and 417 widowers; 1333 were females, and 861 males above 20 who had never married; and 3765 were girls, and 3712 boys under 20.

tuations.

tuations, the greater part live to marry: and in the pariſh of *Ackworth*, particularly, it appears with undeniable evidence from the Regiſter, that one-half of all born there live to the age of 46. It appears alſo, with equal evidence, from M. *Muret*'s Tables in the *Bern* Memoirs for 1766, that in 43 pariſhes in the diſtrict of *Vaud*, one-half of all born there live beyond the age of 41. In truth, did all mankind lead natural and virtuous lives, that waſte of the ſpecies which happens in infancy and childhood would not take place, and few would die except in old age. The inference, however, which I have mentioned, cannot be made with reaſon. It is juſt only in the particular caſe of an uniform decreaſe in the probabilities of living from birth to old age; and this is a caſe that has never exiſted. In all other caſes, there is not any neceſſary connexion between the proportion of inhabitants dying annually, and the age to which the greater part live. In moſt cities one-half, as I have juſt obſerved, of all that are born die before two or three years of age. But it cannot be imagined, that there is any place where ſo many as one-half or a third of the inhabitants die every year.

But to return to Dr. *Percival*'s account of the town and pariſh of *Mancheſter*. It appears from this account, that the number of children under 15 compared with the number

number of inhabitants between 14 and 51, is greater in the country than in the town of *Mancheſter*, in the proportion of no leſs than 5 to 4 (*a*). It follows, therefore, that though in conſequence of a conſtant influx of people to the town, it is more filled than the country with inhabitants in the moſt vigorous periods of life; yet one child in four leſs is born in the town than in the country. This is a remarkable circumſtance, and the reaſons of it muſt be the two following. Firſt, the town inhabitants being leſs healthy, and dying faſter, have not the ſame ſtrength of conſtitution with the country inhabitants. Secondly, in the town a ſmaller proportion of the adult inhabitants marry; and they marry later than in the country. The ſurvey fully proves this; for it appears, that though the number of inhabitants at the moſt common marrying ages, compared with the whole number of the living above the age of 14, is ſmaller in the country than the town; yet the proportion of the married to the living above 14, is very nearly the ſame in both ſituations.

(*a*) In the town the number of inhabitants between 14 and 51 is 13,779; and 9575 under 15. In the country the former number is 6481; and the latter, 5545. But the laſt number would have been only 4503, had the proportion of the inhabitants between 14 and 51 to the inhabitants under 15 been the ſame in both ſituations. It is owing to this, that the number of perſons in a family in the country is $5\frac{1}{4}$; but in the town only $4\frac{3}{4}$.

And

And there are more widows and widowers in the town than in the country in the proportion of near 16 to 11. We learn from hence, I think, clearly, in what manner towns operate in checking population, and preventing the increase of mankind.

Dr. *Percival* informs us, that the reverend and learned Dr. *Tucker* has been led, by some observations he has made at *Bristol*, to doubt whether the common opinion is right, with respect to the disproportion between the number of male and female births; and that he, therefore, wishes a farther inquiry may be made into this subject. This has induced me to collect the following facts, which, I think, will abundantly settle this point.

	Born Males.	Females.	Proportion.
In *London* for the last 110 years, or from 1664 to 1773	862293	817072	20 to 19
Paris, for 8 years (*a*),	79693	76481	25 to 24
Leyden, for 50 years (*b*),	46773	44933	26 to 25
Vienna, for 27 years, ending 1746 (*c*),	67060	64893	31 to 30
Berlin, for 10 years, ending 1761 (*d*),	71188	67431	20 to 19
Kurmark of *Brandenburgh*, for 9 years, ending 1759 (*e*),	102425	96521	18 to 17

(*a*) See *Susm. Gottlicke* Ordnung Tables, p. 16.
(*b*) Ibid, p. 17. (*c*) Ibid. p. 13.
(*d*) Ibid, p. 12. (*e*) Ibid, p. 3.

Dukedom

	Born Males.	Females.	Proportion.
Dukedom of *Magdeburgh*, for 38 years, ending 1759 (*a*),	153227	145985	21 to 20
All the *Pruſſian* towns, for a courſe of years, (*b*),	691826	659072	21 to 20
In a great number of country pariſhes, for a courſe of years (*c*),	59067	56282	21 to 20
In the ſame country pariſhes, for another period of years (*d*),	89530	84954	19 to 18
Leeds, *Mancheſter*, *Coventry*, &c. for a period of years (*e*),	108784	103449	20 to 19
In the ſame towns, for another period (*f*),	57084	54128	20 to 19
Total	2388950	2271201	20 to 19
Sweden, for 9 years, ending 1763,	416007	396124	20 to 19

Mr. *Derham*, in his Phyſico-Theology, p. 175, has ſtated the proportion of male to female births at 14 to 13, and this proportion has ever ſince been generally received as the true one; but it appears from this Table, that it ought to have been ſtated at 20 to 19. But though it appears that the number of males born is in this proportion greater than the number of females born, yet, in moſt places, the number of

(*a*) See *Suſm. Gottlicke* Ordnung Tables, p. 5.
(*b*) Ibid. p. 9.
(*c*) See Dr. *Short*'s New Obſervations, p. 27. 31.
(*d*) Ibid. p. 30. (*e*) Ibid. p. 49. (*f*) Ibid.

males

males living has been found to be leſs than the number of females. The reaſon is, without doubt, that males are more ſhort-lived than females; and this owing partly to the peculiar hazards to which males are ſubject, and their more irregular modes of life; but it is owing principally to ſome particular delicacy in the male conſtitution which renders it leſs durable: For there are many obſervations which prove, that the greater mortality of males takes place chiefly in the firſt and laſt ſtages of life. A few facts of this kind I will beg leave to mention, becauſe I have juſt met with them.

In the pariſh of St. *Sulpice*, at *Paris*, during 30 years, 5 males under a year old died to 4 females. But under 10, only 13 males died to 12 females (ſee *Suſmilch.* Tables, vol. II. p. 30.)

In *Stockholm*, during 9 years ending in 1763, the number of ſtill-borns amounted to 666; of whom 390 were males, and 276 females; that is, 10 to 7. The number of the living in that city above the age of 80 was, in 1760, 332; of whom 248 were females, and 84 males, or near 3 to 1. In the whole kingdom of *Sweden*, including all town and country inhabitants, the number of ſtill-borns, during the 9 years juſt mentioned, was 19,845; of whom 11,424 were males, and 8421 females, or near 4 to 3. The number of the living in the whole king-

kingdom conſiſted of more females than males, in the proportion of 10 to 9. It conſiſted of more females turned of 80 than males, in the proportion of 33 to 19; and of more females turned of 90 than males in the proportion of near 2 to 1. See a Memoir of M. *Wargentin*'s in the *Memoires abreges de l'Academie Royale des Sciences de Stockholm*, printed at *Paris* in 1772, p. 21. Having now had occaſion to refer again to this Memoir, I will juſt add, that it appears, that by the exceſs of the births above the deaths, *Sweden* gains every year an addition of above 20,000 inhabitants; and that in ſix years they increaſed from 2,323,195 to 2,446,394. I am afraid, were regulations eſtabliſhed for a ſimilar inquiry in this kingdom, we ſhould be far from finding our ſtate ſo encouraging. *London* alone is a gulph which probably ſwallows up an increaſe equal to almoſt the whole increaſe (*a*) of *Sweden*.

RICHARD PRICE.

POSTSCRIPT.

THE following Tables have been ſelected from ſeveral more of the ſame kind in M.

(*a*) This is meant on a ſuppoſition which, I think, not extravagant, that the annual ſupply of people in mature life from the country, to keep up *London* and its environs, is 10,000. In order to provide this ſupply there muſt be about double that number born in the country.

Wargen-

Wargentin's Memoir on the ſtate of population in *Sweden*. I have inſerted them here, becauſe they fully verify moſt of the obſervations in the preceding paper, and contain more diſtinct and authentic information on the ſubject of human mortality than I have ever before met with.

TABLE I.

Shewing the Rate of human Mortality in *Sweden*.

	Annual deaths, being the average of three years, 1761, 1762, & 1763.		Number of the living in 1763.		
	Males.	Femal.		Males.	Females.
Still-born,	1324	988	Born,	47216	44892
Died under 1	11172	9850	Living under 1	36094	35453
Died betwn. 1 & 3	4393	4336	Living betn. 1 & 3	66059	67234
3— 5	2206	2249	3— 5	66454	67711
5—10	2151	2057	5—10	130019	130758
10—15	933	834	10—15	126696	128021
15—20	711	658	15—20	108312	109985
20—25	834	756	20—25	92299	105115
25—30	883	863	25—30	88056	101003
30—35	1020	1146	30—35	85936	95811
35—40	955	923	35—40	74826	81453
40—45	1180	1170	40—45	67448	74854
45—50	1099	938	45—50	52398	59551
50—55	1280	1113	50—55	47298	56646
55—60	1177	1097	55—60	37086	45537
60—65	1586	1721	60—65	34892	44925
65—70	1237	1566	65—70	26649	28964
70—75	1322	2041	70—75	15454	23159
75—80	1092	1695	75—80	8858	13556
80—85	917	1446	80—85	4620	7487
85—90	414	650	85—90	1508	2694
Above 90	215	379	Above 90	527	988
Total of annual deaths,	36777	37488	Total of living at allages,	1165489	1280905

In this Table it is obſervable, that the number of the living, in every equal diviſion of life from birth, decreaſes continually till all become extinct; and that though the males born are more than the females born, in the proportion of 20 to 19; yet the males living of all ages are leſs in number, in the proportion of 1,165,489 to 1,280,905, or nearly of 10 to 11; notwithſtanding which, the males that die annually are to the females as 52 to 53.

TABLE II.

Shewing the Rate of human Mortality at *Stockholm*.

	Annual deaths, being the average of three years, 1761, 1762, & 1763.		Number of the living in 1763.		
	Males.	Femal.		Males.	Females.
Still-born,	54	43	Born,	1406	1340
Died under 1	567	489	Living under 1	684	733
Died betwn. 1 & 3	161	170	Living betn. 1 & 3	1173	1348
3— 5	80	79	3— 5	1022	1106
5—10	71	72	5—10	2630	2774
10—15	49	24	10—15	3151	2918
15—20	53	30	15—20	3018	2865
20—25	91	64	20—25	3070	4056
25—30	121	78	25—30	3380	4251
30—35	141	102	30—35	3705	4234
35—40	118	96	35—40	3019	3288
40—45	140	115	40—45	2846	3130
45—50	101	84	45—50	1775	1984
50—55	105	91	50—55	1581	2129
55—60	61	54	55—60	853	1329
60—65	79	88	60—65	826	1383
65—70	41	54	65—70	370	778
70—75	33	77	70—75	260	574
75—80	28	59	75—80	128	324
80—85	18	45	80—85	58	127
85—90	7	20	85—90	16	51
Above 90	3	11	Above 90	10	22
Total of annual deaths,	2068	1902	Total of living at all ages,	33575	39404

In this Table it may be obſerved, that the number living at every age from birth decreaſes only till five. Between 5 and 10 *Stockholm* begins to receive recruits from the country, and they come in faſter and faſter till 35; after which age it appears, that more die than come in; and that the living in every ſubſequent period goes on decreaſing continually till the end of life. It is farther obſervable, that this Table exhibits a greater difference than the former, between the mortality of males and females.

A compariſon of theſe Tables will ſhew a ſtriking contraſt in other reſpects between the ſtate of human mortality in the whole kingdom of *Sweden* and in its capital. In order to make this more obvious and unexceptionable, I will add the following Table, deduced from all M. *Wargentin*'s Tables taken together.

TABLE III.

	In all *Sweden* for nine years.		In *Stockholm* for 9 years	
	Males.	Females.	Males.	Females.
Still born,	1 in 36	1 in 47	1 in 32	1 in 43½
Died under 1 of all born,	1 in 4⅓	1 in 4⅘	1 in 2⅓	1 in 2 3/10
Died annually of the living betwn. 1 and 3	1 in 17⅓	1 in 17¾	1 in 7	1 in 7⅓
Between 3 — 5	1 in 34½	1 in 36	1 in 13½	1 in 16
5 — 10	1 in 71	1 in 76	1 in 34½	1 in 39
10 — 15	1 in 149	1 in 161	1 in 79	1 in 114
15 — 20	1 in 149	1 in 164	1 in 59	1 in 99
20 — 25	1 in 108	1 in 139	1 in 44	1 in 79
25 — 30	1 in 98	1 in 113	1 in 33	1 in 58
30 — 35	1 in 85	1 in 84	1 in 31	1 in 43
35 — 40	1 in 78	1 in 91	1 in 26½	1 in 39
40 — 45	1 in 56	1 in 63	1 in 23	1 in 31
45 — 50	1 in 49	1 in 65	1 in 19½	1 in 28
50 — 55	1 in 37	1 in 50	1 in 16½	1 in 25½
55 — 60	1 in 31	1 in 40	1 in 14	1 in 24
60 — 65	1 in 23	1 in 26	1 in 11	1 in 16
65 — 70	1 in 17	1 in 18½	1 in 9½	1 in 13⅓
70 — 75	1 in 11⅓	1 in 11½	1 in 7 3/10	1 in 8
75 — 80	1 in 8	1 in 8⅓	1 in 4½	1 in 5
80 — 85	1 in 5¼	1 in 5⅓	1 in 3½	1 in 3½
85 — 90	1 in 3⅖	1 in [illegible]	1 in 2	1 in 2⅔
Above 90	1 in 2½	1 in 2⅛	1 in 2 3/10	1 in 2⅓
Died of all living at all ages	1 in 33½	1 in 36	1 in 17 1/10	1 in 21¼

A general

A general Bill of all the Chriſtenings and Burials in the Pariſh of *Ackworth*, in the County of *York*, extracted from the Pariſh Regiſter, for ten Years, from March 25, 1747, to March 25, 1757.

In ten years chriſtened, Males 62. Females 65, Total, 127.
In ten years buried, Males 58. Females 49. Total, 107.

	Males	Fem.	Tot.
Whereof have died			
Under 2 years old,	6	11	17
Between 2 and 5	1	2	3
5 —— 10	2	2	4
10 —— 20	1	2	3
20 —— 30	6	2	8
30 —— 40	2	3	5
40 —— 50	11	3	14
50 —— 60	9	2	11
60 —— 70	9	7	16
70 —— 80	9	8	17
80 —— 90	1	6	7
90 —— 100	1	1	2
Of all, in 10 years,	58	49	107

	Males	Fem.	Tot.
And there have died of			
Apoplexy,	0	1	1
Cancer,	1	0	1
Cholic,	1	0	1
Conſumptions,	10	13	23
Dropſy,	4	1	5
Fevers,	23	12	35
Infants,	6	7	13
Lunacy,	0	1	1
Old age	9	15	24
Palſey	1	0	1
Quinſey,	0	1	1
Small-pox,	1	0	1
Of the above diſtempers, in 10 yrs.	56	51	107

In this pariſh there are { 160 Houſes, 12 of which are uninhabited.
603 Souls of the following ages, viz.

	Males	Fem.	Tot.
Under 2 years old,	12	19	31
Between 2 and 5	25	19	44
5 —— 10	30	38	68
10 —— 20	59	58	117
20 —— 30		41	96
30 —— 40	26	33	59

	Males	Fem	Tot.
Between 40 and 50	40	22	62
50 —— 60	38	33	71
60 —— 70	25	14	39
70 —— 80	4	8	12
80 —— 90	4	0	4
90 —— 100	0	0	0
Total of all ages,	318	285	603

A general Bill of all the Chriſtenings and Burials in the Pariſh of *Ackworth*, in the County of *York*, for ten years, from March 25, 1757, to March 25, 1767.

In ten years chriſtened, Males 104. Females 108. Total, 212.
In ten years buried, Males 79. Females, 77. Total, 156.

	Males	Fem.	Tot.
Whereof have died			
Under 2 years old,	18	13	31
Between 2 and 5	9	7	16
5 —— 10	4	1	5
10 —— 20	2	2	4
20 —— 30	7	5	12
30 —— 40	3	8	11
40 —— 50	2	4	6
50 —— 60	11	3	14
60 —— 70	13	13	26
70 —— 80	7	14	21
80 —— 90	3	6	9
90 —— 100	0	1	1
Of all ages in 10 yrs.	79	77	156

	Males	Fem.	Tot.
And there have died of			
Apoplexy,	2	1	3
Aſthma,	2	1	3
Cancer,	0	1	1
Caſualties,	5	1	6
Childbed,	0	2	2
Chincough,	0	2	2
Conſumptions,	23	15	38
Convulſions,	4	2	6
Diabetes.	1	0	1
Dropſy,	0	3	3
Dyſentery	1	1	2
Fever,	12	11	23
Jaundice,	1	0	1
Infants,	7	6	13
Lunacy,	0	1	1
Meaſles,	0	2	2
Mortification,	2	1	3
Old age,	11	19	30
Palſey,	1	0	1
Quinſey,	1	0	1
Small-pox,	7	6	13
Teeth,	0	1	1
Of all the above orders, in 10 years	80	76	156

In this pariſh there are { 184 Houſes, 11 of which are uninhabited.
728 Souls of the following ages, viz.

	Males	Fem.	Tot.
Under 2 years old,	31	25	56
Between 2 and 5	32	36	68
5 —— 10	34	38	72
10 —— 20	50	51	101
20 —— 30	44	63	107
30 —— 40	61	62	123
Between 40 and 50	31	38	69
50 —— 60	28	32	60
60 —— 70	20	28	48
70 —— 80	7	10	17
80 —— 90	2	4	6
90 —— 100	0	1	1
Total of all ages.	339	389	728

In 1702 there were only eleven children baptiſed, ſix of whom are now living in the pariſh, and have reſided here almoſt all the time.

Account of the Inhabitants of Rome, from 1762 to 1771.

	1762	1763	1764	1765	1766	1767	1768	1769	1770	1771
Parish churches,	81	81	81	81	82	82	82	82	82	82
Families,	35739	35696	35453	35771	35894	36375	36409	36521	37449	37285
Bishops,	42	62	45	45	51	52	54	47	52	62
Priests,	2742	2699	2718	2617	2531	2652	2676	2819	3031	2925
Religious of sundry orders	4381	4291	3588	4509	4258	4105	4310	4088	3792	3739
Nuns,	1725	1892	1661	1759	1684	1738	1709	1695	1692	1594
Collegians and scholars,	868	970	763	888	734	1153	907	1197	939	491
Cardinals courts or attendants,	812	791	765	544	827	588	491	592	72	665
Poor pensioners of the hospital,	1050	858	1271	1725	1903	2839	2010	1970	1426	1386
Prisoners,	339	240	336	402	370	390	251	405	446	402
Males of all ages,	90239	87396	88618	87205	88280	88577	88865	88415	86610	87547
Females of all ages,	67219	71423	73286	70890	69588	71183	69982	70491	71833	72128
Above 14 years of age,	120696	123211	125391	120300	119661	122150	120820	121455	120385	119984
Under 14,	36762	35608	36508	37795	38207	37610	38027	37451	38058	39691
Nonconformists to the church of *Rome*,	37	61	75	86	120	49	63	77	84	91
Blacks,	9	11	8	8	12	5	10	9	5	5
Devotees,	18	30	28	31	23	22	20	25	20	20
Births,	4989	5336	5420	4828	4962	4310	4595	4891	4967	4216
Deaths,	7149	6493	7361	8375	7722	7528	9574	6972	6646	5850
Total of inhabitants,	157458	158819	161899	158095	157868	159760	158847	158906	158443	159675

ESSAY II.

Proofs of the Inſalubrity of marſhy Situations. In a Letter to the Rev. Dr. Horſley, *read to the Royal Society Jan.* 13, 1774, *and publiſhed in the* Philoſophical Tranſactions. *Vol.* 64, *P.* 96.

DEAR SIR,

DR. *Prieſtley*'s paper on the noxious effects of ſtagnant waters, read laſt Thurſday to the Royal Society, brought to my remembrance a Table exhibiting the rate of mortality in a pariſh ſituated among marſhes, which I had ſeen in Mr. MURET's Obſervations, publiſhed in the Memoirs for 1766 of the Oeconomical Society at *Bern*. I have ſince examined this Table, and found that it contains a full confirmation of Dr. *Prieſtley*'s aſſertions. This pariſh is a part of the diſtrict of *Vaud*, belonging to the canton of *Bern*, in *Switzerland*, and contained 169 families, and 696 inhabitants. Mr. MURET's Table of the rate of mortality in it is formed from a regiſter of the ages at which all died in it for 15 years. With this Table he has alſo given Tables from

from like regiſters of the rates of mortality in ſeven ſmall towns; in 36 country pariſhes and villages; in 16 pariſhes ſituated in the *Alps*; in 12 corn pariſhes; and in 18 vintage pariſhes.——From comparing theſe Tables it appears that the probabilities of living are higheſt in the moſt hilly parts of the province, and loweſt in the marſhy pariſh juſt mentioned. The difference is indeed remarkable, as will appear from the following particulars. One half of all born in the mountains live to the age of 47. In the marſhy pariſh, one half live only to the age of 25. In the hills one in 20 of all that are born live to 80. In the marſhy pariſh, only one in 52 reaches this age. In the hills, a perſon aged 40 has a chance of 80 to 1, for living a year. In the marſhy pariſh, his chance for living a year is not 30 to 1.—In the hills, perſons aged 20, 30, and 40, have an even chance for living 41, 33, and 25 years reſpectively. In the fenny pariſh, perſons, at theſe ages, have an even chance of living only 30, 23, and 15 years.

I am ſenſible that obſervations for only 15 years, in one ſmall pariſh, do not afford as deciſive and ample an authority, in the preſent caſe, as there is reaſon to wiſh for; and that, therefore, the perfect exactneſs of the particulars I have recited, cannot be depended on.—They are, however, ſufficiently

ciently near the truth to demonſtrate, in general, the unhealthfulneſs of a marſhy ſituation, and as the regiſter from whence they are derived is the only one, in ſuch a ſituation, which I have ever met with, and Dr. *Alexander*'s experiments may lead ſome to very wrong concluſions on this ſubject; I could not help thinking, that there would be no impropriety in ſending you the account I have now given. If you think it of any importance, I ſhall be obliged to you for reading it to the Royal Society.

I cannot help taking this opportunity to add my wiſhes, that ſuch regiſters of mortality as thoſe publiſhed by Mr. *Muret*, were eſtabliſhed in every part of this kingdom. We might then determine immediately every ſuch queſtion as that which has occaſioned this letter; and know certainly what influence different airs and different ſituations have on the duration of life. Two ingenious phyſicians, Dr. *Percival* at *Mancheſter* (*a*), and Dr. *Haygarth* at *Cheſter*, have lately, with much zeal, promoted inſtitutions of this kind; and a great deal of uſeful information may be expected from the accurate and compre-

(*a*) Dr. PERCIVAL has not ſucceeded at *Mancheſter*. But it has been ſeen, in the courſe of this work, that I have derived a great deal of information from Dr. *Haygarth*'s regiſter. Dec. 1781.

henſive

henſive regiſters of mortality, which, under their direction, have been eſtabliſhed in theſe towns. But the inſtruction ariſing from theſe eſtabliſhments cannot be complete, till they become univerſal.

I am, Sir,

Your moſt obedient

and humble Servant,

Newington-Green,
Dec. 21, 1773.

RICHARD PRICE.

ESSAY

ESSAY III.

Short and eaſy Theorems for finding, in all Caſes, the Differences between the Values of Annuities payable Yearly, and of the ſame Annuities payable Half-yearly, Quarterly, or Momently. Communicated in a Letter to Sir John Pringle, *Bart. P. R. S. and read to the Royal Society, Nov.* 9, 1775, *and publiſhed in the* Philoſophical Tranſactions, *Vol.* 66, *Part* I.

THE values of annuities, as given in all the common Tables, ſuppoſe them paid yearly. But it is well known, that generally they are paid half-yearly, and ſometimes quarterly: and that this is a circumſtance which always adds to their value. The difference between the values of annuities, according as they are paid in theſe different ways, I have ſeen no where ſtated with accuracy; and therefore, I have thought that the following attempt to do this may be of ſome uſe.

Annuities

Annuities are of two ſorts. They are either payable certainly or conditionally. Of the former ſort are all annuities which are payable at fixed times, without depending on any contingency. Of the latter ſort are all annuities on lives. I will firſt conſider the firſt ſort of annuities.

Let r denote the intereſt of 1 *l.* for a year; and n the term or number of years during which any annuity is to be paid. Let P denote the value of the perpetuity, or the quotient ariſing from dividing 1 *l.* by its intereſt for a year. Let y denote the value of an annuity for n years, ſuppoſing it to be paid yearly; b its value, payable half-yearly; q its value, payable quarterly; and m its value, payable momently.

THEOREM I.

$$y = \mathrm{P} - \frac{1}{r \times \overline{1+r}\,|^{n}}$$

THEOREM II.

$$b = \mathrm{P} - \frac{1}{r \times \overline{1+\frac{r}{2}}\,|^{2n}}$$

THEOREM

THEOREM III.

$$q = P - \frac{1}{\sqrt[4n]{r \times 1 + \frac{r}{4}}}.$$

THEOREM IV.

$M = P - \frac{1}{rN}$. where N denotes the number which hath $r\,n$ for its hyperbolic logarithm, and $r\,n \times 0.43429448$ for its logarithm in *Brigg*'s ſyſtem.

EXAMPLE.

Let the rate of intereſt be 4 *per cent.* and the term 5 years, and conſequently $r = 0{,}04$. $n = 5$. $P = 25$.

Then, $y = 4.4518$
$b = 4.4913$
$q = 4.5120$
$m = 4.5415$

EXAMPLE II.

Let the rate of intereſt be the ſame, and the term for which the annuity is payable 25 years.

Then,

Then, $y = 15.6220$
$h = 15.7118$
$q = 15.7694$
$m = 15.801$

Example III.

Intereſt being the ſame, let the term be 50 years.

Then, $y = 21.4822$
$h = 21.5491$
$q = 21.582$
$m = 21.616$

Example IV.

Intereſt being the ſame, let the term be 100 years.

Then, $y = 24.505$
$h = 24.523$
$q = 24.532$
$m = 24.542$

In the foregoing Theorems it may be obſerved, that the *ratio* to one another of the values of annuities payable yearly, half-yearly, quarterly, and momently, is greateſt when n is leaſt; that it decreaſes continually as n increaſes, till at laſt it vaniſhes when n becomes infinite or the annuity is a

perpetuity. Agreeably to this it appears, in the examples I have given, that the values in the firſt example differ more from one another in proportion than the values in the ſecond example; and that theſe alſo differ more than the values in the third; and that in the laſt example all the values are nearly the ſame.

Theſe values computed by Mr. *De Moivre*'s rules in his Treatiſe on Life-annuities, p. 86 and 124, &c. come out greater when n exceeds and leſs when n falls ſhort of 15 or 20 years. But thoſe rules ſuppoſe the half-yearly and quarterly intereſts of money to be leſs than half or a quarter of the yearly intereſt. For inſtance; the value of an annuity of 1 *l.* payable half-yearly and quarterly for 50 years is, according to Mr. *De Moivre*'s rules, 21,699 and 21,772, or a 99th part and 74th part more than the value of the ſame annuity payable yearly, ſuppoſing money improved at 4 *per cent.* when the annuity is paid yearly; and at 1,98 *l. per cent.* when it is paid half-yearly; and at 0,985 *l. per cent.* when it is paid quarterly: That is, ſuppoſing money improved at a rate of half-yearly or quarterly intereſt, which, inſtead of being a half or a quarter of the yearly intereſt, is only that half-yearly or quarterly payment which, in conſequence of being laid up and improved at compound

compound intereſt, will in a year amount to the ſum that makes the yearly intereſt. It is obvious that this cannot be the proper method of computing theſe values. But not to inſiſt on this; I will next ſtate the different values of the ſecond ſort of annuities; or of *life-annuities*, according as they are ſuppoſed to be payable yearly, half-yearly, quarterly, or momently.

Let r as before be the intereſt of 1 *l.* for a year; n the complement of a given life (*a*); y, h, q, and m, the values reſpectively of an annuity certain for n years payable yearly, half-yearly, quarterly, or momently; P the perpetuity; Y the preſent value of an annuity on a life whoſe complement is n, payable yearly; H the value of the ſame annuity payable half-yearly; and Q and M the values of the ſame annuity payable quarterly and momently.

(*a*) The complement of a life is, in Mr. *De Moivre*'s hypotheſis, the number of years it wants of 86. In all other caſes, it is double the expectation of a life; that is, it is double the quotient (diminiſhed by $\frac{1}{2}$ unity) ariſing from dividing the ſum of all the living in a Table of Obſervations from the age (incluſive) of the given life to the extremity of life, by the number of the living at that age. See Eſſay I. in the preceding volume.

Then, $Y = P - \frac{1+r}{rn} \times y.$

$$H = P - \frac{1+\frac{r}{2}}{nr} \times h.$$

$$Q = P - \frac{1+\frac{r}{4}}{nr} \times q.$$

$$M = P - \frac{m}{nr}.$$

EXAMPLE I.

Let the life be ſuppoſed of the age of 36. The complement of ſuch a life is 50, according to Mr. *De Moivre*'s hypotheſis; and alſo very nearly, according to the *Breſlaw* and the *Northampton* Tables of obſervations. Therefore, n will be 50. Let the rate of intereſt be 4 *per cent.* or $r = 0{,}04$. $P = 25$. $y = 21{,}482$. $h = 21{,}549$. $q = 21{,}582$. $m = 21{,}616$. See p. 385.

Therefore, $Y = 25 - \frac{1{,}04}{50 \times 0{,}04} \times 21{,}482 = 13{,}829$

$$H = 25 - \frac{1{,}02}{50 \times 0{,}04} \times 21{,}549 = 14{,}010$$

$$Q = 25 - \frac{1{,}01}{50 \times 0{,}04} \times 21{,}582 = 14{,}101$$

$$M = 25 - \frac{21{,}616}{50 \times 0{,}04} = 14{,}191$$

EXAMPLE

EXAMPLE II.

Let the life be ſuppoſed of the age of 61. The complement of this life is 25 by Mr. *De Moivre*'s hypotheſis and the *Northampton* Table of obſervations. Therefore, intereſt ſuppoſed at 4 *per cent.*

$$Y = 25 - \frac{1,04}{25 \times 0,04} \times 15,622 = 8,753$$

$$H = 25 - \frac{1,02}{25 \times 0,04} \times 15,712 = 8,973$$

$$Q = 25 - \frac{1,01}{25 \times 0,04} \times 15,769 = 9,072$$

$$M = 25 - \frac{15,801}{25 \times 0,04} = 9,199$$

The different values, given by theſe theorems, (*a*) of life-annuities payable yearly, half-yearly, and quarterly, ſuppoſe nothing to be due to an annuitant for that year, half-year, or quarter, in which he ſhall happen to die. If, on the contrary, he is to be

(*a*) It is of no conſequence that theſe theorems are founded on the *hypotheſis* of an equal decrement of life; for taking equal *yearly* values, (or values nearly equal) the differences between them and *half* yearly and *quarterly* values are almoſt exactly the ſame, whether they are deduced from real obſervations, or from this hypotheſis. ——Even in the hypotheſis itſelf it requires a conſiderable difference in the yearly value, to produce any material difference in the exceſs of the half-yearly and quarterly values.

entitled to ſuch part of the annuity as ſhall be proportioned to the time which ſhall happen to intervene between his death and the time when the payment immediately preceding his death became due; or in other words, if the annuity is an annuity ſecured by land, $\frac{y}{2n}$ muſt be added to the firſt theorem in order to obtain the value of ſuch an annuity payable yearly. And in like manner, $\frac{b}{4^n}$ muſt be added to the ſecond theorem to obtain the value of the ſame annuity payable half-yearly: and $\frac{q}{8n}$ to the third theorem, to obtain its value payable quarterly.

The value, therefore, in the firſt example, of an annuity payable yearly on a life aged 36 being 13,829; its value, if ſecured by land, or to be enjoyed to the laſt moment of life, will be $13,829 + \frac{21,482}{100} = 14,043$. If ſecured by land and payable half-yearly, its value will be $14,010 + \frac{21,549}{200} = 14,117$. If ſecured by land and payable quarterly, its value will be $14,101 + \frac{21,582}{400} = 14,155$. The like values in the ſecond example are 9,065, 9,130, and 9,151.

Life-annuities payable monthly or weekly may be conſidered as of the ſame value with annuities

annuities payable momently; and it is evident, that they muſt be enjoyed nearly to the laſt moment of life.

From theſe rules and examples it may be gathered, that the difference between the values of annuities on lives payable yearly, half-yearly, quarterly, and momently, increaſes continually with the ages; but, if not ſecured by land, this difference can never be ſo great as a quarter of a year's purchaſe in the caſe of annuities payable yearly and half-yearly; three-eighths of a year's purchaſe in the caſe of annuities payable yearly and quarterly; and half a year's purchaſe in the caſe of annuities payable yearly and momently.

Mr. *Simpſon*, in his Treatiſe on the Doctrine of Life-Annuities, p. 78, and in his Select Exerciſes, p. 283, hath given a quarter of a year's purchaſe as the addition always to be made to the value of a life-annuity payable yearly, in order to obtain its value payable half-yearly; and three-eighths of a year's purchaſe, if its value payable quarterly is required. But it appears, that theſe are too large additions; and, whatever be the rate of intereſt or the number of lives, a fifth of a year's purchaſe will be generally more than a ſufficient addition, if the value of the annuity is deſired payable half-yearly; and three-tenths of a year's purchaſe, if the value of the annuity

is deſired payable quarterly. Mr. *De Moivre*'s rules, in p. 85 of his Book on Life-annuities, for finding the values of life-annuities payable half-yearly and quarterly from their values payable yearly, are ſtill leſs correct; for they ſuppoſe the difference between theſe values the ſame, whether the annuities are life annuities or annuities certain.

Mr. *Dodſon*, in the firſt queſtion in the third volume of his Mathematical Repoſitory, hath given a rule for finding the value of an annuity ſecured by land and payable yearly, which coincides with that here given; and Mr. *De Moivre*, in p. 338 of his Treatiſe on the Doctrine of Chances, hath given a theorem for this purpoſe, which alſo brings out nearly the ſame anſwers. But Mr *Simpſon*, in Prob. I. p. 323 of his Select Exerciſes, makes the exceſs of the value of ſuch an annuity above the value of an annuity payable yearly but not ſecured by land, double to the ſame exceſs derived from Mr. *Dodſon*'s and Mr. *De Moivre*'s rules. The truth is, that Mr. *Dodſon*'s rule gives the exact value; and that Mr. *Simpſon*'s problem gives the value, not of an annuity ſecured by land and payable yearly, but of an annuity ſecured by land and payable momently; and alſo, that his method of ſolution implies a rate of intereſt ſomewhat leſs when the annuity is payable momently than when it is payable yearly.

But

But to prevent all perplexity on this ſubject, I will ſubjoin the following inveſtigations, which will be eaſily underſtood by thoſe who are acquainted with the common methods of calculating the values of life-annuities.

Let r, as before, be the intereſt of 1 *l.* for a year. Then the preſent value of 1 *l.* payable at the end of one year, two years, three years, &c. will be $\frac{1}{1+r}$, $\frac{1}{1+r|^2}$, $\frac{1}{1+r|^3}$, &c. reſpectively. And the preſent value of an annuity certain for n years payable yearly is the ſum of this ſeries continued to n terms (*a*), or $\frac{1}{r} - \frac{1}{r \times 1+r|^n} = P - \frac{1}{r \times 1+r|^n} = y$.

In like manner, the preſent value of half 1 *l.* (that is, of 10*s.* = *l.* 0, 5) payable at the end of half a year, a year, a year and a half, &c. reckoning half-yearly intereſt at half

(*a*) In the poſtſcript it will be proved, that the ſum of n terms of the ſeries $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}$, &c. is $\frac{1}{a-1} - \frac{1}{a^n \times a-1}$. Subſtitute $1+r$ for a, and it will appear, that the ſum of n terms of the ſeries $\frac{1}{1+r} + \frac{1}{1+r|^2} + \frac{1}{1+r|^3}$, &c. is $\frac{1}{r} - \frac{1}{r \times 1+r|^n}$.

the

the annual intereſt, is $\frac{0,5}{1+\frac{r}{2}}, \frac{0,5}{\left|1+\frac{r}{2}\right|^2}, \frac{0,5}{\left|1+\frac{r}{2}\right|^3}$, &c.

And the preſent value of an annuity certain payable half-yearly for n years, each payment to be half the yearly payment, is the ſum of this ſeries continued to $2n$ terms; or,

$$\frac{0,5}{\frac{r}{2}} - \frac{0,5}{\frac{r}{2}\times\left|1+\frac{r}{2}\right|^{2n}} = \frac{1}{r} - \frac{1}{r\times\left|1+\frac{r}{2}\right|^{2n}} = P - \frac{1}{r\times\left|r+\frac{r}{2}\right|^{2n}} = h.$$

By the ſame ſteps it will appear, that the preſent value of an annuity certain for n years to be received in quarterly payments, each a quarter of the annual payment, is $\frac{0,25}{\frac{r}{4}} - \frac{0,25}{\frac{r}{4}\times\left|1+\frac{r}{4}\right|^{4n}} = P - \frac{1}{r\times\left|1+\frac{r}{4}\right|^{4n}} = q$. And alſo, that the preſent value of an annuity certain for n years, to be received in momently payments, each the ſame proportional part of the yearly payment that the moment is of the year, muſt be $P - \frac{1}{r\times\left|1+\frac{r}{1000,\text{ \&c.}}\right|^{1000,\text{ \&c. }n}}$

But, by the binomial theorem,

$$\left|1+\frac{r}{1000,\text{ \&c.}}\right|^{1000,\text{ \&c. }n} = 1 + rn + \frac{r^2n^2}{2} + \frac{r^3n^3}{2\times 3} +$$

$\frac{r^4n^4}{2\times3\times4}$, &c. which series approximates indefinitely to the number of which rn is the hyperbolic logarithm, by Prob. 1. Sect. XI. Vol. II. of Mr. *Simpson*'s Fluxions; or by Prop. 1. p. 40, of his Treatise on Trigonometry. Therefore, $P - \frac{1}{r\times\overline{1+\frac{1}{1000,\ \&c.}}|^{1000,\ \&c.\ n}}$

$= P - \frac{1}{rN} = m$, as explained before. See p. 384.

If the value of an annuity of 1 *l.* for n years is required payable half-yearly, and the half-yearly interest of 1 *l.* instead of being half the yearly interest (or $\frac{r}{2}$), is supposed to be $\overline{1+r}|^{\frac{1}{2}} - 1$; the answer will be $\frac{0,5}{\overline{1+r}|^{\frac{1}{2}}} + \frac{0,5}{1+r} + \frac{0,5}{\overline{1+r}|^{\frac{3}{2}}} + \frac{0,5}{\overline{1+r}|^{2}}$, &c. continued to $2n$ terms $= \frac{0,5}{\overline{1+r}|^{\frac{1}{2}}-1} - \frac{0,5}{\overline{1+r}|^{n\times\overline{1+r}|^{\frac{1}{2}}-1}}$

$= \overline{1 - \frac{1}{\overline{1+r}|^{n}}} \times \frac{1}{2\times\overline{1+r}|^{\frac{1}{2}}-2}$; which value is to $\overline{1 - \frac{1}{\overline{1+r}|^{n}}} \times \frac{1}{r}$ (the value of the same annuity payable yearly supposing the yearly interest

intereſt of 1*l.* to be r) as $\frac{\frac{1}{2}}{\overline{1+r}^{\frac{1}{2}}-1}$ to $\frac{1}{r}$ (*a*), agreeably to Mr. *De Moivre*'s deduction in his Treatiſe on Life-annuities, p. 125, 4th edit.

(*a*) In the ſame manner the value payable quarterly is $1-\frac{1}{\overline{1+r}^{n}} \times \frac{1}{4\times\overline{1+r}^{\frac{1}{4}}-1}$ and the value payable momently $=1-\frac{1}{\overline{1+r}^{n}} \times \frac{1}{1000, \&c. \times \overline{1+r}^{\frac{1}{1000}, \&c.}-1}$

Conſequently the value of an annuity certain, payable quarterly or momently, is to the ſame value, payable yearly, as $\frac{1}{4\times\overline{1+r}^{\frac{1}{4}}-1}$, or $\frac{1}{N}$, to $\frac{1}{r}$ (N being the hyperbolic logarithm of $\overline{1+r}$). Suppoſing, therefore, the intereſt to be 4 *per cent.* the value of an annuity payable yearly muſt be *invariably* increaſed in the ratio of 1.0101, or 1.0152 or 1.01986 to 1, according as it is payable either half-yearly, quarterly or momently. The difference, however, between the values of annuities payable yearly and at ſhorter intervals is known to be continually leſſening in proportion to the length of the term, till at laſt, when the term is extended to a perpetuity, thoſe values become the ſame, whether the payments are made yearly or momently. But ſuch an equality can never take place according to Mr. *De Moivre*'s rules; nay, if the term be extended only to 70 years, and intereſt be 6 *per cent.* an annuity payable quarterly will be worth more than even the *perpetuity* when the payments are made yearly. This appears to be very erroneous, and ſufficient to prove the fallacy of Mr. *De Moivre*'s method of ſolution. En.

This

This implying, in the caſe of annuities payable half-yearly, a ſmaller intereſt than half the yearly intereſt (for $\overline{1+r}|^{\frac{1}{2}} - 1$ is leſs than $\frac{r}{2}$) gives the difference between their value and the value of annuities payable yearly, greater than the truth.

But to return to the inveſtigation of the theorems in the former part of this paper.

Let us again call P the perpetuity, and y the value of an annuity certain for r years and payable yearly; it is well known that the value of 1 *l.* payable yearly on a life whoſe complement is n is (ſuppoſing an equal decrement of life) $\frac{n-1}{n \times 1+r} + \frac{n-2}{n \times 1+r|^2}$ $+ \frac{n-3}{n \times 1+r|^3}$, &c. continued to n terms (*a*) $= P - \frac{1+r}{nr} \times y = Y$.

In

(*a*) See Mr. *De Moivre*'s Treatiſe on Life-annuities, p. 99, 4th edit. Or his Doctrine of Chances, p. 311, 3d edition. Or Mr. *Dodſon*'s Mathematical Repoſitory, Vol. II. p. 137. Or Mr. *Simpſon* on Annuities and Reverſions, p. 14. In conſulting theſe writers, care ſhould be taken to remember, that they uſe r to denote the principal and intereſt of 1 *l.* for a year; whereas it hath been moſt convenient for me in theſe obſervations to make r ſtand only for the intereſt. In theſe writers, therefore, r ſignifies the ſame with $1+r$ in this paper; and $r-1$ the ſame with r.

It

In like manner, ſuppoſing money improved at an half-yearly intereſt equal to half the yearly

It is ſaid above, that the value of an annuity payable yearly on a life whoſe complement is n, is $\frac{n-1}{n\times 1+r} + \frac{n-2}{n\times\overline{1+r}^{2}} + \frac{n-3}{n\times\overline{1+r}^{3}}$, &c. continued to n terms. This expreſſion is equal to $\frac{n}{n\times 1+r} + \frac{n}{n\times\overline{1+r}^{2}} + \frac{n}{n\times\overline{1+r}^{3}}$, &c. $(n) - \frac{1}{n}\times\frac{1}{1+r} + \frac{2}{\overline{1+r}^{2}} + \frac{3}{\overline{1+r}^{3}}$, &c. (n). But $\frac{n}{n\times 1+r} + \frac{n}{n\times\overline{1+r}^{2}} + \frac{n}{n\times\overline{1+r}^{3}}$, &c. $(= \frac{1}{1+r} + \frac{1}{\overline{1+r}^{2}} + \frac{1}{\overline{1+r}^{3}}$, &c.$) = \frac{1}{r} - \frac{1}{r\times\overline{1+r}^{n}} = y$ (ſee p. 393.) Alſo, by a theorem which will be demonſtrated in the poſtſcript, and putting a for any given quantity, $\frac{1}{a} + \frac{2}{a^{2}} + \frac{3}{a^{3}}$, &c. continued to n terms, $= \frac{a}{\overline{a-1}^{2}} - \frac{n}{a^{n}}\times\frac{1}{a-1} - \frac{1}{a^{n}}\times\frac{a}{\overline{a-1}^{2}}$. Therefore, if $1+r$ is ſubſtituted for a, and y for $\frac{1}{r} - \frac{1}{r\times\overline{1+r}^{n}}$, the ſum (multiplied by $\frac{1}{n}$) of n terms of the ſeries $\frac{1}{1+r} + \frac{2}{\overline{1+r}^{2}} + \frac{3}{\overline{1+r}^{3}}$, &c. will come out $\frac{1+r}{nr}\times y - \frac{1}{r}\times\frac{1}{\overline{1+r}^{n}}$; or $\frac{1+r}{nr}\times y + y - \frac{1}{r}$. Therefore, the ſeries $\frac{1}{n}\times\frac{1}{1+r} + \frac{2}{\overline{1+r}^{2}}$

yearly intereſt; or to $\frac{r}{2}$, the value of the ſame annuity payable half-yearly; is $\frac{1}{2} \times \frac{n-\frac{1}{2}}{n \times 1+\frac{r}{2}} + \frac{n-1}{n \times 1+\frac{r}{2}\Big|^2} + \frac{n-\frac{3}{2}}{n \times 1+\frac{r}{2}\Big|^3}$, &c. continued to $2n$ terms $= \frac{1}{2} \times \frac{n}{n \times 1+\frac{r}{2}} + \frac{n}{n \times 1+\frac{r}{2}\Big|^2} + \frac{n}{n \times 1+\frac{r}{2}\Big|^3}$, &c. continued to $2n$ terms $- \frac{1}{2} \times \frac{\frac{1}{2}}{n \times 1+\frac{r}{2}} + \frac{1}{n \times 1+\frac{r}{2}\Big|^2} + \frac{\frac{3}{2}}{n \times 1+\frac{r}{2}\Big|^3}$, &c. continued to $2n$ terms. But the ſum of the firſt of theſe two ſeries, or of $\frac{1}{2} \times \frac{n}{n \times 1+\frac{r}{2}} + \frac{n}{n \times 1+\frac{r}{2}\Big|^2}$, &c. $(= \frac{1}{2} \frac{2}{1+r\Big|^2} + \frac{3}{1+r\Big|^3}$, &c. continued to n terms and ſubtracted from the ſeries $\frac{1}{1+r} + \frac{1}{1+r\Big|^2} + \frac{1}{1+r\Big|^3}$, &c. continued to n terms; that is, the value of the life will be $y - \frac{1+r}{nr} \times y + y - \frac{1}{r} = \frac{1}{r} - \frac{1+r}{nr} \times y = P - \frac{1+r}{nr} \times y = Y$.

×

$\times \frac{1}{1+\frac{r}{2}} + \frac{1}{\overline{1+\frac{r}{2}}^{2}}$, &c.) is h, ſee p. 393, &c. And the ſum of the ſecond ſeries is the ſame with half the ſum of the ſeries $\frac{1}{2n} \times \frac{1}{1+\frac{r}{2}} + \frac{2}{\overline{1+\frac{r}{2}}^{2}} + \frac{3}{\overline{1+\frac{r}{2}}^{3}}$, &c. $(2n)$. But by the theorem mentioned in the laſt note, the ſum of n terms of the ſeries $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}$, &c. is $\frac{a}{\overline{a-1}^{2}} - \frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n} \times \frac{a}{\overline{a-1}^{2}}$. Therefore, if $1 + \frac{r}{2}$ is ſubſtituted for a, $2n$ for n, and h for $\frac{1}{r} - \frac{1}{r \times \overline{1+\frac{r}{2}}^{2n}}$, the ſum of the ſecond ſeries (that is, of $\frac{1}{2} \times \frac{1}{2n} \times \frac{1}{1+\frac{r}{2}} + \frac{2}{\overline{1+\frac{r}{2}}^{2}} + \frac{3}{\overline{1+\frac{r}{2}}^{3}}$, &c. $(2n)$ will come out $\frac{1+\frac{r}{2}}{nr} + h - \frac{1}{r} \times \frac{1}{\overline{1+\frac{r}{2}}^{2n}}$, or $\frac{1+\frac{r}{2}}{nr} \times h + h - \frac{1}{r}$. Therefore,

fore, the ſecond ſeries ſubtracted from the firſt, leaves $\frac{1}{r} - \frac{1+\frac{r}{2}}{nr} \times b = P - \frac{1+\frac{r}{2}}{nr} \times b = H$, agreeably to the ſecond theorem in p. 388.

By reaſoning in the ſame way it may be eaſily found, that $Q = P - \frac{1+\frac{r}{4}}{nr} \times q$; and $M = P - \frac{1+\frac{r}{1000, \&c.}}{nr} \times m = P - \frac{m}{nr}$, agreeably to the third and fourth theorems in p. 388.

Theſe theorems, I have ſaid, ſuppoſe that an annuitant is entitled to no payment for that year, half-year, or quarter, in which he dies. If, on the contrary, he is to be entitled when he dies, to ſuch a part of the yearly, half-yearly, or quarterly payment as ſhall bear the ſame proportion to the ſaid payments reſpectively, as the intermediate time between the laſt payment and his death bears to the whole year, half-year, or quarter; in this caſe, ſuppoſing the annuity payable yearly, it is evident, ſince there is the ſame chance for his dying in one half of any year as in the other, that he will have an expectation of half a year's payment more than he would be otherwiſe entitled to. But the value of half 1*l.* to be paid at the death of a perſon

whoſe complement of life is n, is $\frac{1}{2} \times \frac{1}{n \times 1+r} + \frac{1}{2} \times \frac{1}{n \times 1+r|^2} + \frac{1}{2} \times \frac{1}{n \times 1+r|^3}$, &c. continued to n terms (a) $= \frac{y}{2n}$.

In like manner, a perſon who enjoys an annuity ſecured by land, payable half-yearly, will have an expectation of a quarter of a year's payment more than he could be otherwiſe intitled to; the value of which is $\frac{1}{4n} \times \frac{1}{1+\frac{r}{2}} + \frac{1}{1+\frac{r}{2}|^2} + \frac{1}{1+\frac{r}{2}|^3}$, &c. continued to $2n$ terms $= \frac{b}{4n}$. By the ſame reaſoning it will appear, that $\frac{q}{8n}$ is the addition to be made to the value of an annuity payable quarterly, in order to obtain its value when ſecured by land.

POSTSCRIPT.

IN the note, p. 393, the expreſſion $\frac{1}{a-1} - \frac{1}{a^n} \times \frac{1}{a-1}$ is given as the ſum of n terms of the ſeries $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}$, &c. to $\frac{1}{a^n}$, and the expreſſion $\frac{a}{a-1|^2} - \frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n}$

(a) See page 393, &c.

$\times \frac{a}{\overline{a-1}^2}$, is given, in p. 398, as the ſum of n terms of the ſeries $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}$, &c.

The following inveſtigation of theſe theorems being very eaſy, will not, perhaps, be unacceptable to thoſe who have ſtudied this ſubject.

Put $A = \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \frac{1}{a^4}$, &c. $\frac{1}{a^n}$. $B = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}$, &c. $\frac{n}{a^n}$,

Then $A \times a = 1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}$, &c. to $\frac{1}{a^n-1}$.

and $A \times a - 1 + \frac{1}{a^n} = \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}$, &c. to $\frac{1}{a^n-1}$

$+ \frac{1}{a^n} = A$,

and $A \times a - A \left(= A \times \overline{a-1}\right) = 1 - \frac{1}{a^n}$.

Therefore, $A = \frac{1}{a-1} - \frac{1}{a^n} \times \frac{1}{a-1}$, which is the firſt theorem.

Again, $A \times a = 1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}$, &c. to $\frac{1}{a^n-1}$,

and $B \times a = 1 + \frac{2}{a} + \frac{3}{a^2} + \frac{4}{a^3}$, &c. to $\frac{n}{a^n-1}$.

Therefore, $B \times a - A \times a = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}$, &c. to $\frac{n-1}{a^n-1}$.

To both ſides of the laſt equation add $\frac{n}{a^n}$, and it will appear, that

$B \times a - A \times a + \frac{n}{a^n} = \frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3} + \frac{4}{a^4}$, &c. to $\frac{n-1}{a^{n-1}} + \frac{n}{a^n} = B$.

Therefore, $B \times a - B = B \times \overline{a-1} = A \times a - \frac{n}{a^n}$; and $B = \frac{A \times a}{a-1} - \frac{n}{a^{n+1}-a^n}$.

For A, in this laſt equation, ſubſtitute its equal, or $\frac{1}{a-1} - \frac{1}{a^n} \times \frac{1}{a-1}$, and the reſulting equation will be $\frac{a}{\overline{a-1}} - \frac{n}{a^n} \times \frac{1}{a-1} - \frac{1}{a^n} \times \frac{a}{\overline{a-1}^2} = B$, which is the ſecond theorem.

When n is infinite, all but the firſt terms in both theſe theorems vaniſh; and therefore, $\frac{1}{a-1}$ is the ſum of the ſeries $\frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3}$, &c. continued infinitely; and $\frac{a}{\overline{a-1}^2}$ is the ſum of the ſeries $\frac{1}{a} + \frac{2}{a^2} + \frac{3}{a^3}$, &c. continued infinitely.

By a like deduction, putting

$C = \frac{1}{a} + \frac{2 \times 2}{a^2} + \frac{3 \times 3}{a^3} \times \frac{4 \times 4}{a^4}$, &c. to $\frac{n^2}{a^n}$,

and

and $D = \frac{1}{a} + \frac{2\times2\times2}{a^2} + \frac{3\times3\times3}{a^3} + \frac{4\times4\times4}{a^4}$, &c. to $\frac{n^3}{a^n}$, it may be found that $C = \frac{A+2B+1}{a-1} - \frac{n+1}{a^{n+1}-a^n}$, and $D = \frac{A+3B+3C+1}{a-1} - \frac{n+1}{a^{n+1}-a^n}$. And conſequently, ſubſtituting the values of A and B, that

$$C = \frac{a^2+a}{\overline{a-1}^3} - \frac{n^2}{a^n} \times \frac{1}{a-1} - \frac{2an}{a^n} \times \frac{1}{\overline{a-1}^2} - \frac{a^2+a}{a^n} \times \frac{1}{\overline{a-1}^3}.$$

And, ſubſtituting the values of A, B, C, that

$$D = \frac{a^3+4a^2+a}{\overline{a-1}^4} - \frac{n^3}{a^n} \times \frac{1}{a-1} - \frac{3an^2}{a^n} \times \frac{1}{\overline{a-1}^2} - \frac{3a^2n+3an}{a^n} \times \frac{1}{\overline{a-1}^3} - \frac{a^3+4a^2+a}{a^n} \times \frac{1}{\overline{a-1}^4}.$$ Or, ſince all but the firſt terms in theſe expreſſions vaniſh when n is infinite, that the ſum of the ſeries $\frac{1}{a} + \frac{4}{a^2} + \frac{9}{a^3}$, &c. continued infinitely is $\frac{a^2+a}{\overline{a-1}^3}$; and that the ſum of the ſeries $\frac{1}{a} + \frac{8}{a^2} + \frac{27}{a^3} + \frac{64}{a^4}$, &c. continued infinitely is $\frac{a^3+4a^2+a}{\overline{a-1}^4}$.

Theſe are all the theorems neceſſary for calculating the values of annuities on ſingle lives, and on any two or three joint lives,

upon the hypotheſis of an equal decrement of life.

Suppoſing r the intereſt of 1 *l.* for a year, the ſum of n terms of the ſeries $\frac{1}{1+r} + \frac{1}{\overline{1+r}^2} + \frac{1}{\overline{1+r}^3}$, &c. is the preſent value of an annuity certain for n years; and $\frac{1}{1+r} + \frac{2}{\overline{1+r}^2} + \frac{3}{\overline{1+r}^3} + \frac{4}{\overline{1+r}^4}$, (continued to n terms) is the preſent value of an annuity certain beginning with 1 *l.* and increaſing to 2 *l.* the ſecond year, to 3 *l.* the third year, &c.

If this laſt annuity is not an annuity certain for a given term, but a life-annuity, the value of it (ſuppoſing n the complement of the life, A the value of an annuity certain for n years, G the value of two equal joint lives whoſe common complement is n, P the perpetuity, and p the value of 1 *l.* to be received at the end of n years) will be $\overline{A-G} \times n + n, p ; P - A, P \times \overline{1+r}$.

EXAMPLE 8.

Let the term be forty-one years, and the rate of intereſt 4 per cent.

The value of an annuity of 1 *l.* certain for this term is 20 *l.*

The

The value of an annuity certain for the ſame term, and beginning with 1 *l.* at the end of the firſt year, but increaſing to 2 *l.* at the end of the ſecond year, to 3 *l.* at the end of the third year, and ſo on till it becomes 41 *l.* at the end of the forty-firſt year, is (by the Second Theorem, putting $1+r$, or 1,04 for a) 314 *l.* 10 *s.*

The value of an annuity increaſing at this rate without end is 650 *l.*

If the annuity is a life-annuity which is to increaſe at the rate of 1 *l.* every year during the whole poſſible continuance of a life whoſe complement is forty-one years (or whoſe age, according to Table VI. in the collection of tables at the beginning of this volume, is forty-five), the preſent value of it will be, by the laſt theorem, 135 *l.* But a much ſimpler rule for finding the values of annuities of this ſort will be given in the following notes. See Note I; and alſo Mr. *Morgan* on Aſſurances, p. 119.

APPENDIX I.

THE following tables were computed by Dr. *Price*, at the requeſt of a committee of the Houſe of Commons, and were intended to form the foundation of a plan for enabling the labouring poor to provide ſupport for themſelves in ſickneſs and old age, by ſmall weekly ſavings from their wages.—A bill for eſtabliſhing a plan of this kind was formed and approved by the Commons in the year 1789, but, like Mr. *Dowdeſwell's* bill for the ſame purpoſe in the year 1773, *(a)*, it was rejected by the Lords. The importance, however, of theſe tables is not leſſened by this circumſtance, and it was the author's intention to have publiſhed them, had he lived to complete the preſent edition of this work. In order therefore to fulfil his intentions, as well as to preſerve thoſe valuable fruits of his labour from being loſt, I have inſerted them, together with his own explanations of their uſe and conſtruction, in this Appendix; thinking that they may be rendered of great public ſervice in ſome future time, ſhould the Societies for which they were computed be hereafter eſtabliſhed either by the legiſlature or by voluntary aſſociations. ED.

(a) A copy of this bill and of the tables that were computed for it, has been publiſhed by Mr. Baron *Maſeres*, in the 2d volume of his very valuable Treatiſe on the Doctrine of Life-annuities.

TABLE

TABLE I.

Shewing the Weekly Allowances, during Incapacities of Labour, produced by Sickneſs or Accidents, and the correſponding Weekly Contributions neceſſary to entitle Perſons to thoſe Allowances.

N. B. The Ages in this and the following Tables, are the Ages at Admiſſion, and the Contributions at Admiſſion are reckoned to continue invariable till they ceaſe at Sixty-five.

Ages of Contributors at Admiſſion.	Under 32	From 32 to 42	From 43 to 51	From 52 to 58	From 59 to 64		Bedlying Pay.	Walking Pay.
Weekly Contributions. Claſs	*d.*	*d.*	*d.*	*d.*	*s.* *d.*	Weekly Allowances. Claſs	*l.* *s.*	*s.*
I.	1	1¼	1½	1¾	0 2	I.	0 4	2
II.	1½	1⅞	2¼	2⅝	0 3	II.	0 6	3
III.	2	2½	3	3½	0 4	III.	0 8	4
IV.	2½	3⅛	3¾	4⅜	0 5	IV.	0 10	5
V.	3	3¾	4½	5¼	0 6	V.	0 12	6
VI.	3½	4⅜	5¼	6⅛	0 7	VI.	0 14	7
VII.	4	5	6	7	0 8	VII.	0 16	8
VIII.	4½	5⅝	6¾	7⅞	0 9	VIII.	0 18	9
IX.	5	6¼	7½	8¾	0 10	IX.	1 0	10
X.	5½	6⅞	8¼	9⅝	0 11	X.	1 2	11
XI.	6	7½	9	10½	1 0	XI.	1 4	12

SUPPOSITIONS *on which this* TABLE *is formed.*

First, That in ſocieties conſiſting of perſons under 32 years of age, a 48th part of them will be always in a ſtate of incapacitation by illneſs and accidents; and therefore entitled to allowances proportioned to their contributions. Various reaſons, and particularly the experience of friendly clubs, determine me to believe that the proportion of the ſick to the well in ſuch a ſociety will not be ſo great as this, and conſequently that a weekly allowance during ſickneſs will be more than ſupported by weekly contributions not exceeding a 48th part of that allowance.

Secondly, It is ſuppoſed that from the age of 32 to 42 this proportion increaſes to one quarter more than a 48th part; from 43 to 51 to one half more; from 52 to 58 to three quarters more; and from 59 to 64 to double. The reaſon of aſſuming this rate of increaſe is, that the probability of the duration of human life decreaſes after 30 nearly in this manner, or ſo that a perſon of the age of 60 has but half the probability of living any given time that a perſon at 32 has, and conſequently muſt be then doubly ſubject to the cauſes that produce ſickneſs and mortality.

TABLE

TABLE II.

Shewing the Weekly Allowances to Perſons in Old Age after 65 and 70; and the correſponding Weekly Contributions in early Life neceſſary to ſupport thoſe Allowances.

Weekly Contributions till 65. Ages at Admiſſion.	Claſs I.		Claſs II.		Claſs III.		Claſs IV.	
	s.	d.	s.	d.	s.	d.	s.	d.
Under 21	0	1	0	1½	0	2	0	2½
21 & 22	0	1¼	0	1⅞	0	2½	0	3⅛
23 & 24	0	1½	0	2¼	0	3	0	3¾
25 & 26	0	1¾	0	2⅝	0	3½	0	4⅜
27 & 28	0	2	0	3	0	4	0	5
29 & 30	0	2¼	0	3⅜	0	4½	0	5⅝
31 & 32	0	2½	0	3¾	0	5	0	6¼
33	0	2¾	0	4⅛	0	5½	0	6⅞
34	0	3	0	4½	0	6	0	7½
35	0	3¼	0	4⅞	0	6½	0	8⅛
36	0	3½	0	5¼	0	7	0	8¾
37	0	3¾	0	5⅝	0	7½	0	9⅜
38	0	4	0	6	0	8	0	10
39	0	4¼	0	6⅜	0	8½	0	10⅝
40	0	4½	0	6¾	0	9	0	11¼
41	0	4¾	0	7⅛	0	9½	0	11⅞
42	0	5	0	7½	0	10	1	0½
43	0	5½	0	8¼	0	11	1	1¾
44	0	6	0	9	1	0	1	3
45	0	6½	0	9¾	1	1	1	4¼
46	0	7½	0	11¼	1	3	1	6¾
47	0	8½	1	0¾	1	5	1	9¼
48	0	9½	1	2¼	1	7	1	11¾
49	0	10½	1	3¾	1	9	2	2¼
(*) 50	0	11½	1	5¼	1	11	2	4¾

TABLE II. continued.

Class V.		Class VI.		Class VII.		Class VIII.		Class IX.		Class X.		Class XI.	
s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
0	3	0	3½	0	4	0	4½	0	5	0	5½	0	6
0	3¾	0	4⅜	0	5	0	5⅝	0	6¼	0	6⅞	0	7½
0	4½	0	5¼	0	6	0	6¾	0	7½	0	8¼	0	9
0	5¼	0	6⅛	0	7	0	7⅞	0	8¾	0	9⅝	0	10½
0	6	0	7	0	8	0	9	0	10	0	11	1	0
0	6¾	0	7⅞	0	9	0	10⅛	0	11¼	1	0⅜	1	1½
0	7½	0	8¾	0	10	0	11¼	1	0½	1	1¾	1	3
0	8¼	0	9⅝	0	11	1	0⅜	1	1¾	1	3⅛	1	4½
0	9	0	10½	1	0	1	1½	1	3	1	4½	1	6
0	9¾	0	11⅜	1	1	1	2⅝	1	4¼	1	5⅞	1	7½
0	10½	1	0¼	1	2	1	3¾	1	5½	1	7¼	1	9
0	11¼	1	1⅛	1	3	1	4⅞	1	6¾	1	8⅝	1	10½
1	0	1	2	1	4	1	6	1	8	1	10	2	0
1	0¾	1	2⅞	1	5	1	7⅛	1	9¼	1	11⅜	2	1½
1	1½	1	3¾	1	6	1	8¼	1	10½	2	0¾	2	3
1	2¼	1	4⅝	1	7	1	9⅜	1	11¾	2	2⅛	2	4½
1	3	1	5½	1	8	1	10½	2	1	2	3½	2	6
1	4½	1	7¼	1	10	2	0¾	2	3½	2	6¼	2	9
1	6	1	9	2	0	2	3	2	6	2	9	3	0
1	7½	1	10¾	2	2	2	5¼	2	8½	2	11¾	3	3
1	10½	2	2¼	2	6	2	9¾	3	1½	3	5¼	3	9
2	1½	2	5¾	2	10	3	2¼	3	6½	3	10¾	4	3
2	4½	2	9¼	3	2	3	6¾	3	11½	4	4¼	4	9
2	7½	3	0¾	3	6	3	11¼	4	4½	4	9¼	5	3
2	10½	4	4¼	4	10	5	3¾	5	9¼	6	3¼	6	9

TABLE II. continued.

		After 65.		After 70.		
		s.	d.	£.	s.	d.
Weekly Allowances for Life.	Claſs I.	2	0	0	4	0
	II.	3	0	0	6	0
	III.	4	0	0	8	0
	IV.	5	0	0	10	0
	V.	6	0	0	12	0
	VI.	7	0	0	14	0
	VII.	8	0	0	16	0
	VIII.	9	0	0	18	0
	IX.	10	0	1	0	0
	X.	11	0	1	2	0
	XI.	12	0	1	4	0

(*) The weekly contributions in the firſt claſs, which are equivalent to the weekly allowances after 65 and 70 in the ſame claſs, have been computed by Dr. *Price* for all the intermediate ages between 50 and 65, and are as follow;

Age.	Weekly Contribution.			Age.	Weekly Contribution.			Age.	Weekly Contribution.		
	£.	s.	d.		£.	s.	d.		£.	s.	d.
51	0	1	$0\frac{1}{2}$	56	0	1	$11\frac{3}{4}$	61	0	6	0
52	0	1	2	57	0	2	$4\frac{1}{4}$	62	0	8	$0\frac{3}{4}$
53	0	1	$3\frac{3}{4}$	58	0	2	$9\frac{1}{4}$	63	0	12	6
54	0	1	$5\frac{1}{4}$	59	0	3	4	64	1	5	0
55	0	1	$8\frac{1}{4}$	60	0	4	$2\frac{1}{2}$				

From theſe ſums the weekly contributions in the other ten claſſes may be eaſily obtained. But it will be ſeldom neceſſary to have recourſe to them; for at a period of life ſo far advanced, the weekly contributions become ſo high in thoſe claſſes as to render it almoſt impoſſible for the labouring poor to pay them. ED.

Method

Method of calculating Table II.

The rule for finding the value in a ſingle preſent payment of an annuity payable for life to a perſon of a given age, ſhould he ſurvive any other given age, may be found in Volume I. Queſt. 6. p. 17.

EXAMPLE.

Let the rate of intereſt be $3\frac{1}{2}$ *per cent.* The table of the probabilities of the duration of human life, that for Northampton given in Vol. II. p. 36. and the tables of the values of lives that in Vol. II. p. 54. Alſo, let the given age be 20; and let the enquiry be what ſum ought to be given for an annuity of 1 *l.* payable weekly for life to a perſon of this age, provided he ſhould ſurvive 65.

The value by the table juſt referred to, at $3\frac{1}{2}$ *per cent.* of an annuity payable weekly during a life aged 65, is *(a)* 8.332. The probability that a life at 20 will continue in being till it is 65, is (by the other table juſt referred to) $\frac{1653}{5132}$; that is, it is the fraction whoſe numerator is the number of the living at 65; and whoſe denominator is the number living at 20. The value of 1 *l.* payable at the end of a number of years, equal to the difference between the two ages 20 and 65, or at the end of 45 years, is (reckoning intereſt at $3\frac{1}{2}$ *per cent.)* .2126 by Table I. Vol. II. p. 18.

£8.332 multiplied by $\frac{1653}{5132}$ is = 2.648; and this

(a) The values of lives at $3\frac{1}{2}$ *per cent.* are not given in this table; but the *means* between the two values at 3 and at 4 *per cent.* give them with ſufficient exactneſs.

The value of a life-annuity payable *weekly*, is worth three-tenths of a year's purchaſe more than the value of the ſame annuity payable yearly; and therefore, in all theſe calculations, this addition is made to every tabular value.

product multiplied by .2126 makes £.5629 the value required.

The value being thus found, in a ſingle payment of an annuity of 1*l.* payable *weekly* for the life of a perſon of a given age after another given age; the *equivalent* value, in weekly payments, dependant on the continuance of the given life till it reaches the age it is to ſurvive, is found by dividing the value in a ſingle payment, by the value of an annuity payable weekly on the given life, for a term of years equal to the difference between the age of the given life and the age it is to ſurvive *(a)*; which, in the preſent caſe, is for a term equal to the difference between 20 and 65, or 45 years. The value of a life aged 20 for this term is £17.072. And £.5629 (the value in a ſingle payment juſt found) divided by 17.072 gives £.0329 the *annual* ſum payable *weekly* due from a perſon aged 20, for an annuity of 1*l.* payable weekly during what may happen to remain of his life after 65. The payment *per* week equivalent to this *annual* ſum is, plainly, the ſum divided by the number of weeks in the year; that is, £.0329 divided by 52, which will give £.00063. In like manner, an annuity of 1*l. payable* weekly may be found to be equivalent to a payment *per* week of £.0192. Since, therefore, a weekly allowance of £.0192 after 65 is worth to a perſon aged 20, a payment or contribution *per* week till 65 of £.00063, any other weekly allowance will be worth as much more

(a) The value of a life for a term of years is found by ſubtracting the value of the life after the term from its whole value. Thus the value of an annuity on the whole continuance of a life aged 20, is (adding three-tenths to obtain the value of the annuity payable weekly) 17.635 year's purchaſe. Its value after a term of 45 years (that is, after 65) is (as ſhewn above) .5629 year's purchaſe. The difference (£17.072) is its value for 45 years. See Queſt. 6th. Vol. I.

or leſs than £.00063, as the allowance itſelf is more or leſs. The weekly allowance, therefore, after 65 being reckoned *two ſhillings* (or .01) the weekly contribution due for it, will be £.00328; for as .0192 is to 0.1 ſo is £.00063 to £.00328.

By the very ſame method of calculation it may be found that an allowance to a perſon now in his 21ſt year of *two ſhillings per* week for life after 70 years of age, is worth, in weekly contributions till he reaches 65 and ſubject to his death in the intermediate time, £.00171. Therefore, a weekly allowance of two ſhillings *per* week for life to a perſon in his 21ſt year after 65, and alſo an allowance of two ſhillings more to the ſame perſon after 70, is worth, in weekly contributions till he reaches 65 and ſubject to his death, £.00328 added to £.00171; that is, it is worth £.00499, which is nearly *one penny and* $\frac{2}{7}$ *of a farthing*.

In this manner have all the values in the 2d Table been calculated.

The value of any weekly contribution for a given term of years, dependant on the continuance of any life during that term, is 52 times the weekly contribution multiplied by the value of an annuity payable weekly on that life for the given term.—Thus, ſuppoſing the life 20 years of age, and the weekly contribution *two pence*, 52 multiplied by .00833, and alſo by 17.072 (*a*) (that is, £7.397) will be the value in a ſingle preſent payment of that contribution dependant on the continuance of the life till 65. And this, therefore, is the ſum which, according to Table II. a perſon under 21, if a contributor in the firſt Claſs, ought to pay, in order to be excuſed all ſubſequent payments.

(*a*) See the Note in page 416.

TABLE III.

Shewing the Weekly Allowances during Sickneſs and Old Age, and the correſponding Weekly Contributions for ſupporting thoſe Allowances; *being Tables* I. *and* II. *combined.*

Ages at Admiſſion.	Claſs I.		Claſs II.		Claſs III.		Claſs IV.	
	s.	*d.*	*s.*	*d.*	*s.*	*d.*	*s.*	*d.*
Weekly Contributions till 65. Under 21	0	2	0	3	0	4	0	5
21 & 22	0	2¼	0	3⅜	0	4½	0	5⅝
23 & 24	0	2½	0	3¾	0	5	0	6¼
25 & 26	0	2¾	0	4⅛	0	5½	0	6⅞
27 & 28	0	3	0	4½	0	6	0	7½
29 & 30	0	3¼	0	4⅞	0	6½	0	8⅛
31	0	3½	0	5¼	0	7	0	8¾
32	0	3¾	0	5⅝	0	7½	0	9⅜
33	0	4	0	6	0	8	0	10
34	0	4¼	0	6⅜	0	8½	0	10⅝
35	0	4½	0	6¾	0	9	0	11¼
36	0	4¾	0	7⅛	0	9½	0	11⅞
37	0	5	0	7½	0	10	1	0½
38	0	5¼	0	7⅞	0	10½	1	1⅛
39	0	5½	0	8¼	0	11	1	1¾
40	0	5¾	0	8⅝	0	11½	1	2⅜
41	0	6	0	9	1	0	1	3
42	0	6½	0	9¾	1	1	1	4¼
43	0	7	0	10½	1	2	1	5½
44	0	7½	0	11¼	1	3	1	6¾
45	0	8	1	0	1	4	1	8
46	0	9	1	1½	1	6	1	10½
47	0	10	1	3	1	8	2	1
48	0	11	1	4½	1	10	2	3½
49	1	0	1	6	2	0	2	6
50	0	1	1	7½	2	2	2	8½

TABLE III. continued.

Class V.		Class VI.		Class VII.		Class VIII.		Class IX.		Class X.		Class XI.	
s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.	s.	d.
0	6	0	7	0	8	0	9	0	10	0	11	1	0
0	6¾	0	7⅞	0	9	0	10⅛	0	11¼	1	0⅜	1	1½
0	7½	0	8¾	0	10	0	11¼	1	0½	1	1¾	1	3
0	8¼	0	9⅝	0	11	1	0⅜	1	1¾	1	3⅛	1	4½
0	9	0	10½	1	0	1	1½	1	3	1	4½	1	6
0	9¾	0	11⅜	1	1	1	2⅝	1	4¼	1	5⅞	1	7½
0	10½	1	0¼	1	2	1	3¾	1	5½	1	7¼	1	9
0	11¼	1	1⅛	1	3	1	4⅞	1	6¾	1	8⅝	1	10½
1	0	1	2	1	4	1	6	1	8	1	10	2	0
1	0¾	1	2⅞	1	5	1	7⅛	1	9¼	1	11⅜	2	1½
1	1½	1	3¾	1	6	1	8¼	1	10½	2	0¾	2	3
1	2¼	1	4⅝	1	7	1	9⅜	1	11¾	2	2⅛	2	4½
1	3	1	5½	1	8	1	10½	2	1	2	3½	2	6
1	3¾	1	6⅜	1	9	1	11⅝	2	2¼	2	4⅞	2	7½
1	4½	1	7¼	1	10	2	0¾	2	3½	2	6¼	2	9
1	5¼	1	8⅛	1	11	2	1⅞	2	4¾	2	7⅝	2	10½
1	6	1	9	2	0	2	3	2	6	2	9	3	0
1	7½	1	10¾	2	2	2	5¼	2	8½	2	11¾	3	3
1	9	2	0½	2	4	2	7½	2	11	3	2½	3	6
1	10½	2	2¼	2	6	2	9¾	3	1½	3	5¼	3	9
2	0	2	4	2	8	3	0	3	4	3	8	4	0
2	3	2	7½	3	0	3	4½	3	9	4	1½	4	6
2	6	2	11	3	4	3	9	4	2	4	7	5	0
2	9	3	2½	3	8	4	1½	4	7	5	0½	5	6
3	0	3	6	4	0	4	6	5	0	5	6	6	0
3	3	3	9½	4	4	4	10½	5	5	5	11½	6	6

TABLE III. continued.

Instances of Weekly Allowances during Sickness.	1st Instance Bedlying Pay.		2d Instance Walking Pay.		Weekly Allowance during Old Age.	After 65.		After 70.	
Class	£.	s.	£.	s.		£.	s.	£.	s.
I.	0	4	0	2		0	2	0	4
II.	0	6	0	3		0	3	0	6
III.	0	8	0	4		0	4	0	8
IV.	0	10	0	5		0	5	0	10
V.	0	12	0	6		0	6	0	12
VI.	0	14	0	7		0	7	0	14
VII.	0	16	0	8		0	8	0	16
VIII.	0	18	0	9		0	9	0	18
IX.	1	0	0	10		0	10	1	0
X.	1	2	0	11		0	11	1	2
XI.	1	4	0	12		0	12	1	4

TABLE IV.

Shewing the Fines, or Compoſition Money, payable at Admiſſion by Contributors in the Firſt Claſs who have commenced their Contributions at Ages above 21, and who may prefer the Payment of a Fine to an Increaſe of Weekly Contribution, on Account of the Exceſs of their Ages above 21, as ſpecified in Table III.

N. B. The Sums in the following Table are alſo the Sums payable, at Removals, to Contributors, who, at Admiſſion, paid Fines in lieu of an Increaſe of Weekly Contribution.

Age at Admiſſion or Removal.	Weekly Contribution 2d. Sums payable	Age at Admiſſion or Removal.	Weekly Contribution 2d. Sums payable	Age at Admiſſion or Removal.	Weekly Contribution 2d. Sums payable	Age at Admiſſion or Removal.	Weekly Contribution 2d. Sums payable	Age at Admiſſion or Removal.	Weekly Contribution 2d. Sums payable
Year.	£. s.	Year.	£. s.	Year.	£. s.	Year.	£. s.	Year.	£. s.
In 22d	0 9	In 31ſt	4 5	In 40th	9 17	In 49th	19 16	In 58th	35 10
23d	0 18	32d	4 16	41ſt	10 7	50th	21 0	59th	38 6
24th	1 6	33d	5 12	42d	11 0	51ſt	22 5	60th	42 0
25th	1 15	34th	6 6	43d	11 16	52d	23 14	61ſt	46 0
26th	2 3	35th	7 0	44th	12 12	53d	25 6	62d	50 0
27th	2 12	36th	7 12	45th	13 14	54th	26 18	63d	54 0
28th	3 0	37th	8 4	46th	15 0	55th	28 13	64th	58 0
29th	3 8	58th	8 16	47th	16 10	56th	30 16	65th	62 0
30th	3 16	39th	9 7	48th	18 0	57th	33 1		

EXPLANATION *and* USES *of* TABLE IV.

THIS Table implies that all perſons *under* 21 years of age entitle themſelves to the expectation of their different claſſes, as ſpecified in the two laſt columns of Table III. without paying any fine; and alſo that ſhould they remove before they get into their 22d year, no money is payable by the pariſh they leave on that account.

If advanced into their 22d year when they enter, and do not chuſe the *increaſe* of weekly contribution ſpecified in Table III. under that age, this Table ſhews the *fine* due from them in lieu of that increaſe, if they enter into the 1ſt Claſs. The fines to be paid in the other claſſes are in proportion to the weekly contributions in thoſe claſſes, and are immediately obtained from the fines in this Table. Thus, in the 2d Claſs they will be 13*s.* 6*d.*—in the 3d Claſs 18*s.* —in the 4th Claſs 1*l.* 2*s.* 6*d.* and ſo on. In like manner the fines due from perſons in their 23d, 24th, 25th, 26th, &c. years, when they enter in the firſt Claſs (that is, aged then 22, 23, 24, 25, &c.) in lieu of an increaſed weekly contribution, are the ſums correſponding to their ages as ſpecified in this Table; and the fines in the other claſſes will, as obſerved above, be in proportion to the weekly contributions in thoſe claſſes. The ſums payable at *removal* to perſons who have entered under 21, but do not remove before they are turned of this age, are the ſame with theſe fines. For example:

A contributor who has entered in the firſt Claſs under 21, if he leaves the pariſh in which he entered in his 22d, 23d, 24th, 25th, &c, years, will be entitled, at his removal, to the ſums in the Table oppoſite to theſe ages; that is, to 9*s.*—18*s.*—1*l.* 6*s.*—1*l.* 15*s.* &c. If he has entered in the 2d Claſs it may

be found from thoſe ſums that he will be entitled to 13*s.* 6*d.*—1*l.* 7*s.*—1*l.* 19*s.*—2*l.* 12*s.* 6*d.* &c.

If, in the 3d Claſs to 18*s.*—1*l.* 16*s.*—2*l.* 12*s.*—3*l.* 10*s.* &c. according as he is in his 22d, 23d, 24th, 25th, &c. years reſpectively.

It may be a neceſſary obſervation, that it is of no conſequence to a pariſh how many removals a contributor in any particular Claſs had made before he came to it, provided it receives with him the ſum in the Table correſponding to his age and claſs. For example:

A contributor under 21 has entered in the 1ſt Claſs; that is, he has entitled himſelf, by taking upon him a contribution of 2*d.* *per* week, payable till he is 65, to an allowance, whenever he is ſick or diſabled, of *four ſhillings per* week *bedlying* pay, and two ſhillings *per* week *walking* pay; and alſo to an allowance for life after 65 of *two ſhillings per* week, and after 70 of *four ſhillings per* week. Let this perſon be ſuppoſed to remove to another pariſh in his 28th year. This Table ſhews that the pariſh he leaves ought to remit to the pariſh to which he removes 3*l.* Should he remove again, the ſecond pariſh will be obliged to remit to a third pariſh the ſum oppoſite to his age at that time; and the ſame is true of this third pariſh in caſe of a removal to a fourth pariſh; and ſo on.

Again: A contributor aged 22 (that is, in the 23d year of his age) has entered (let us ſuppoſe) in the 3d Claſs; that is, he has entitled himſelf, either by a weekly contribution, *without* a fine, of *four pence halfpenny* payable till he is 65; (ſee Table III) or *with* a fine and a weekly contribution of *four pence* payable till 65, to an allowance during ſickneſs of *eight* ſhillings *per* week *bedlying* pay, and *four* ſhillings *per* week *walking* pay, and alſo to an allowance of *four* ſhillings *per* week *during life* after 65, and *eight* ſhillings

per week after 70.—Such a contributor, ſhould he remove in his 30th year, will, as appears by the Table, be entitled to twice 3*l.* 16*s.* or 7*l.* 12*s.* for the pariſh into which he removes; and ſhould he remove again in his 40th year, he will be entitled to twice 9*l.* 17*s.* or 19*l.* 14*s.* for a *ſecond* pariſh; and ſhould he remove a third time in his 50th year, he will be entitled to twice 21*l.* or 42*l.* for a *third* pariſh.

METHOD *of computing* TABLE IV.

WHEN a contributor removes to a new pariſh he continues there the weekly contribution with which he firſt entered. But to this pariſh he will be the ſame with a new contributor entering at his age; and, therefore, this pariſh will be entitled either to a weekly contribution ſuitable to that age and claſs, as ſpecified in Table III. or to ſuch a ſum as will be equivalent to the value of the *difference* between his contribution and the higher contribution due from a perſon in that claſs and at that age, ſuppoſing him not to have been before a contributor. If this compenſation is not made, the pariſh left will be a gainer at the expence of the pariſh to which the contributor removes; and, conſequently, while the one is benefited, the other will be injured.—In other words, the pariſh left by a contributor is a gainer by the removal; and having no right to that gain, without being liable to ſuſtain the burden, a ſum equivalent to it ought to be transferred to the pariſh into which the removal is made, in order to place it on the ſame footing with reſpect to ſuch a contributor as if he had never before been a contributor. This equivalent is the value of the *difference* juſt mentioned; and it muſt be calculated by the following rule.

Multiply the difference between the contribution to be received by the pariſh to which a contributor removes, and the contribution due from a perſon in his claſs and at his age, when he removes (as ſpecified in Table III.), by the value of an annuity, payable weekly, on a life at that age, for a number of years equal to the difference between his age at removal and 65 years of age. The product will be the equivalent ſum payable at his removal.

EXAMPLE.

Let a perſon be ſuppoſed to have made himſelf a contributor in the *ſecond* Claſs under 21 years of age, and afterwards at 28 or in his 29th year, to remove. In this caſe the contribution is 3*d. per* week: but in Table III. it appears that in that Claſs the contribution due from one at that age, ſuppoſing him then to commence his contribution, is *four pence halfpenny per* week. The difference is *three halfpence per* week, which is the ſame with *ſix ſhillings and ſix pence per ann.*; and the value of this annuity, payable weekly by a perſon aged 28 (or in his 29th year) till he is 65, and ſubject to the contingency of his dying in the mean time, is (by the rule in Queſt. 6th, p. 19. vol. I. and the Obſervations in vol. II. p. 40 and 41) 15.80 year's purchaſe, reckoning intereſt, at $3\frac{1}{2}$ *per cent.* and the probabilities and values of lives as given in Tables VII. and XVII. vol. II. This value multiplied by £.325 gives £5.135, that is nearly 5*l.* 2*s.* 6*d.* which is in due proportion to the ſum ſpecified in this Table for the 1ſt Claſs. In this manner have all the ſums in this Table been computed; and it is evident that they expreſs not only the ſums payable in all caſes at removals, but alſo the

the fines payable by perſons who begin their contributions at a greater age than 21, ſuppoſing them excuſed an *increaſe* of weekly contribution on that account.

The three firſt Tables are neceſſary *data* for compoſing the *fourth* Table. But ſhould *fines* only be admitted on account of exceſs of age, no other Table would be neceſſary beſides the fourth; and this would give great ſimplicity to the ſcheme. Perhaps, however, it may be adviſeable to give an option to contributors above age at entrance, either to pay the higher weekly contributions in Table III. or to compound by paying the fines in the 4th Table. In this caſe the following Tables will be neceſſary, which exhibit the ſums payable at removals to contributors at any particular ages greater than 24 (*a*).

(*a*) Theſe Tables alſo (like the preceding one) exhibit the ſums payable *by* thoſe perſons who ſhall chuſe on their entrance into the club or ſociety, to begin with ſuch contributions as are firſt paid by members of any particular age leſs than their own, and greater than 21 years.—Thus, if a perſon in his 24th year wiſhes to be admitted into the 1ſt Claſs with contributors of 22 years of age, by beginning with a contribution of $2\frac{1}{4}d.$ he ſhould pay 9*s.* for ſuch admiſſion.—If he is in his 40th year he ſhould pay 9*l.* 3*s.*—if in his 50th year 20*l.* 10*s.* and ſo on. Again: If a perſon in his 29th year ſhould chuſe to be admitted into the 1ſt Claſs with contributors of 23 and 24 years of age, by beginning with a contribution of $2\frac{1}{2}d.$ he ſhould pay 1*l.* 15*s.* for ſuch admiſſion—if he is in 39th year he ſhould pay 8*l.*—if he is in his 49th year 18*l.* 14*s.* and ſo on. The fines payable on admiſſion into the other claſſes at thoſe reſpective ages are in proportion to the weekly contributions, and are eaſily deduced from this Table. (See Note, p. 430.)
ED.

TABLES,

TABLES, ſhewing the Sums payable at Removals, to Contributors who have begun their Contributions in the ſeveral Years of their Age, after the 21ſt, without Fines.

Table V. Claſs 1ſt.				Table VI. Claſs 1ſt.			Table VII. Claſs 1ſt.			Table VIII. Claſs 1ſt.			Table IX. Claſs 1ſt.			Table X. Claſs 1ſt.		
Weekly Contributions 2½d.				2½d.			2½d.			3d.			3½d.			3½d.		
Age at Subſcription 22 & 23.				24 & 25.			26 & 27.			28 & 29.			30 & 31.			32.		
Age at Removal.		Sums payable.		Sums payable.			Sums payable.			Sums payable.			Sums payable.			Sums payable.		
	Year	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
In their	24th	0	9	0															
	25th	0	18	0															
	26th	1	7	0	0	9	0												
	27th	1	15	0	0	18	0												
	28th	2	3	0	1	6	0	0	9	0									
	29th	2	11	0	1	15	0	0	18	0									
	30th	3	0	0	2	3	0	1	6	0	0	9	0						
	31ſt	3	10	0	2	11	0	1	14	0	1	0	0						
	32d	4	3	0	3	6	0	2	9	0	1	12	0	0	10	0			
	33d	4	16	0	4	0	0	3	4	0	2	8	0	1	12	0	0	12	0
	34th	5	10	0	4	14	0	4	0	0	3	3	0	2	7	0	1	11	0
	35th	6	4	0	5	8	0	4	15	0	3	17	0	3	2	0	2	7	0
	36th	6	17	0	6	2	0	5	7	0	4	10	0	3	15	0	3	0	0
	37th	7	10	0	6	15	0	6	0	0	5	3	0	4	8	0	3	13	0
	38th	8	2	0	7	7	0	6	13	0	5	17	0	5	2	0	4	7	0
	39th	8	13	0	8	0	0	7	5	0	6	10	0	5	16	0	5	1	0
	40th	9	3	0	8	10	0	7	16	0	7	4	0	6	10	0	5	16	0
	41ſt	9	14	0	9	0	0	8	6	0	7	17	0	7	3	0	6	9	0
	42d	10	5	0	9	10	0	8	17	0	8	10	0	7	16	0	7	2	0
	43d	11	2	0	10	10	0	9	17	0	9	4	0	8	11	0	7	18	0
	44th	12	0	0	11	10	0	10	17	0	10	4	0	9	11	0	8	18	0
	45th	13	0	0	12	10	0	11	17	0	11	4	0	10	11	0	9	18	0
	46th	14	8	0	13	10	0	12	17	0	12	10	0	11	18	0	11	5	0
	47th	15	16	0	15	5	0	14	13	0	14	2	0	13	10	0	12	18	0
	48th	17	10	0	17	0	0	16	8	0	16	0	0	15	9	0	14	18	0
	49th	19	5	0	18	15	0	18	2	0	17	12	0	17	1	0	16	10	0
	50th	20	10	0	20	0	0	19	10	0	19	0	0	18	9	0	17	18	0
	51ſt	21	15	0	21	5	0	20	15	0	20	5	0	19	15	0	19	5	0
	52d	23	6	0	22	16	0	22	6	0	21	10	0	21	0	0	20	11	0
	53d	24	17	0	24	8	0	23	18	0	23	10	0	23	1	0	22	12	0
	54th	26	10	0	26	0	0	25	11	0	25	5	0	24	16	0	24	8	0
	56th	28	5	0	27	17	0	27	8	0	27	0	0	26	12	0	26	6	0
	56th	30	9	0	30	0	0	29	12	0	29	10	0	29	2	0	28	15	0
	57th	32	14	0	32	7	0	32	0	0	31	14	0	31	7	0	31	0	0
	58th	35	6	0	35	0	0	34	13	0	34	10	0	34	3	0	33	17	0
	59th	38	0	0	37	14	0	37	8	0	37	0	0	36	14	0	36	10	0
	60th	41	0	0	40	14	0	40	9	0	40	10	0	40	4	0	39	15	0
	61ſt	45	16	0	45	0	0	45	5	0	45	0	0	44	15	0	44	14	0

TABLES, shewing the Sums payable at Removals, to Contributors who have begun their Contributions in the several Years of their Age, after the 21st, without Fines.

Table XI. Class 1st.				Table XII. Class 1st.			Table XIII. Class 1st.			Table XIV. Class 1st.			Table XV. Class 1st.			Table XVI. Class 1st.		
Weekly Contributions			3½d.	4d.			4½d.			4½d.			4½d.			5d.		
Age at Subscription			33.	34.			35.			36.			37.			38.		
Age at Removal.		Sums payable.		Sums payable.			Sums payable.			Sums payable.			Sums payable.			Sums payable.		
	Year	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
In their	34th	0	15	0															
	35th	1	10	0	0	15	0												
	36th	2	5	0	1	10	0	0	15	0									
	37th	3	0	0	2	5	0	1	10	0	0	15	0						
	38th	3	15	0	3	0	0	2	5	0	1	10	0	0	15	0			
	39th	4	7	0	3	13	0	2	18	0	2	3	0	1	10	0	0	15	0
	40th	5	0	0	4	6	0	3	12	0	2	16	0	2	5	0	1	10	0
	41st	5	14	0	5	0	0	4	6	0	3	10	0	2	16	0	2	2	0
	42d	6	6	0	5	14	0	5	0	0	4	6	0	3	12	0	2	14	0
	43d	7	4	0	6	11	0	5	18	0	5	5	0	4	12	0	3	18	0
	44th	8	7	0	7	14	0	7	2	0	6	9	0	5	16	0	5	3	0
	45th	9	7	0	8	14	0	8	2	0	7	9	0	6	16	0	6	3	0
	46th	10	15	0	10	3	0	9	11	0	9	0	0	8	8	0	7	16	0
	47th	12	7	0	11	15	0	11	3	0	10	11	0	10	0	0	9	8	0
	48th	14	5	0	13	14	0	13	3	0	12	11	0	12	0	0	11	8	0
	49th	16	0	0	15	9	0	14	18	0	14	7	0	13	16	0	13	5	0
	50th	17	10	0	17	0	0	16	10	0	16	0	0	15	10	0	15	0	0
	51st	18	15	0	18	5	0	17	15	0	17	5	0	16	15	0	16	5	0
	52d	20	0	0	19	10	0	19	0	0	18	10	0	18	0	0	17	10	0
	53d	22	0	0	21	11	0	21	0	0	20	15	0	20	5	0	19	16	0
	54th	24	0	0	23	11	0	23	0	0	22	11	0	22	3	0	21	14	0
	55th	26	0	0	25	12	0	25	4	0	24	15	0	24	6	0	23	17	0
	56th	28	10	0	28	2	0	27	14	0	27	7	0	27	0	0	26	12	0
	57th	30	10	0	30	8	0	30	1	0	29	14	0	29	7	0	29	0	0
	58th	33	10	0	33	4	0	32	18	0	32	12	0	32	5	0	32	0	0
	59th	36	5	0	36	0	0	35	14	0	35	8	0	35	0	0	34	14	0
	60th	39	10	0	39	5	0	39	0	0	38	15	0	38	10	0	38	5	0
	61st	44	9	0	44	5	0	44	0	0	43	15	0	43	10	0	43	5	0

TABLES, ſhewing the Sums payable at Removals, to Contributors who have begun their Contributions in the ſeveral Years of their Age, after the 21ſt, without Fines.

Table XVII. Claſs 1ſt.			XVIII. Claſs 1ſt.		Table XIX. Claſs 1ſt.		Table XX. Claſs 1ſt.		Table XXI. Claſs 1ſt.		Table XXII Claſs 1ſt.	
Weekly Contribution 5¼d.			5½d.		5¾d.		6d.		6½d.		7d.	
Age at Subſcription 39.			40.		41.		42.		43.		44.	
Age at Removal.	Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.		Sums payable.	
	Year	£. s. d.	£. s. d.		£. s. d.		£. s. d.		£. s. d.		£. s. d.	
In their	40th	0 16 0										
	41ſt	1 12 0	0 18 0									
	42d	2 6 0	1 16 0		1 0 0							
	43d	3 6 0	2 15 0		2 2 0		1 3 0					
	44th	4 10 0	3 17 0		3 2 0		2 10 0		1 5 0			
	45th	5 12 0	4 17 0		4 4 0		3 15 0		2 10 0		1 7 0	
	46th	7 0 0	6 12 0		6 0 0		5 7 0		4 0 0		3 0 0	
	47th	8 16 0	8 4 0		7 12 0		7 0 0		5 18 0		4 14 0	
	48th	10 14 0	10 6 0		9 15 0		9 0 0		8 0 0		6 15 0	
	49th	12 14 0	12 3 0		11 12 0		11 0 0		10 0 0		8 16 0	
	50th	14 10 0	14 0 0		13 10 0		13 0 0		11 12 0		10 10 0	
	51ſt	15 15 0	15 5 0		14 15 0		14 4 0		13 4 0		12 3 0	
	52d	17 0 0	16 10 0		16 0 0		15 10 0		15 0 0		14 0 0	
	53d	19 6 0	18 18 0		18 8 0		17 19 0		17 0 0		16 0 0	
	54th	21 6 0	20 17 0		20 8 0		20 0 0		19 0 0		18 0 0	
	55th	23 7 0	23 0 0		22 12 0		22 4 0		21 5 0		20 10 0	
	56th	26 5 0	25 18 0		25 10 0		25 4 0		24 0 0		23 5 0	
	57th	28 10 0	28 3 0		27 16 0		27 9 0		26 10 0		26 0 0	
	58th	31 14 0	31 8 0		31 2 0		31 0 0		30 0 0		29 8 0	
	59th	34 8 0	34 2 0		33 16 0		33 10 0		33 0 0		32 8 0	
	60th	38 0 0	37 15 0		37 10 0		37 5 0		36 15 0		36 5 0	
	61ſt	43 0 0	42 15 0		42 10 0		42 5 0		41 15 0		41 5 0	

TABLES, ſhewing the Sums payable at Removals, to Contributors who have begun their Contributions in the ſeveral Years of their Age, after the 21ſt, without Fines.

Table XXIII. Claſs 1ſt.		Tab. XXIV Claſs 1ſt.	Tab. XXV. Claſs 1ſt.	Tab. XXVI. Claſs 1ſt.	Ta. XXVII. Claſs 1ſt.	XXVIII. Claſs 1ſt.
Weekly Contribution 7½d.		8d.	9d.	10d.	11d.	1s.
Age at Subſcription 45.		46.	47.	48.	49.	50.
Age at Removal.	Sums payable.	Sums payable.	Sums payable.	Sums payable.	Sums payable.	Sums payable.
	Year £. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.	£. s. d.
In their	46th 1 10 0					
	47th 3 10 0	1 14 0				
	48th 5 12 0	4 10 0	1 18 0			
	49th 7 14 0	6 12 0	4 10 0	2 2 0		
	50th 9 10 0	8 10 0	6 10 0	4 6 0	2 6 0	
	51ſt 11 3 0	10 3 0	8 5 0	6 4 0	4 4 0	2 10 0
	52d 13 2 0	12 3 0	10 5 0	8 5 0	6 6 0	4 4 0
	53d 15 4 0	14 5 0	12 10 0	10 10 0	8 15 0	7 0 0
	54th 17 8 0	16 10 0	15 0 0	13 0 0	11 8 0	9 12 0
	55th 19 12 0	18 15 0	17 10 0	15 15 0	14 0 0	12 6 0
	56th 22 10 0	21 14 0	20 5 0	19 0 0	17 5 0	16 0 0
	57th 25 7 0	24 12 0	23 5 0	22 0 0	20 10 0	19 0 0
	58th 28 15 0	28 0 0	26 15 0	25 10 0	24 5 0	23 0 0
	59th 31 16 0	31 4 0	30 0 0	29 0 0	27 16 0	26 12 0
	60th 36 0 0	35 10 0	34 15 0	33 18 0	33 0 0	32 0 0
	61ſt 41 0 0	40 10 0	39 15 0	38 18 0	38 0 0	37 0 0

*NOTE.

In the original Tables the ſums to be paid at removal have been computed for all the Eleven Claſſes at every age from 22 to 50; but I have only inſerted the Firſt Claſs for each age in theſe Tables, becauſe the inſertion of the other Ten Claſſes would have ſwelled the work without anſwering any eſſential purpoſe. If the ſums payable at removal be known when the weekly contributions are $2\frac{1}{4}d.$ in the 1ſt Column, $2\frac{1}{2}d.$ in the 2d Column, and ſo on: the ſums to be paid in thoſe reſpective caſes when the weekly contributions are $3\frac{1}{8}d.$, $3\frac{3}{4}d.$, &c. are eaſily obtained by the common rule of proportion. Thus, if inſtead of $2\frac{1}{4}d.$ in the 1ſt Column, the weekly contribution had been $3\frac{1}{8}d.$ the ſum to be paid on removal would have been a fourth proportional to $2\frac{1}{4}d.$ $9s.$ and $3\frac{1}{8}d.$; that is, expreſſing theſe numbers in decimals, it would have been $= \frac{.45 \times .014062}{.009375} = .67497 = 13s.\ 6d.$ or more ſimply $= .45 \times \frac{3}{2}$. If the weekly contributions had been $7\frac{1}{8}d.$ the ſum to be paid on removal would have been $\frac{.45 \times .032812}{.009375} = 1.575 = 1l.\ 11s.\ 6d.$ or $.45 \times \frac{7}{2}$. But if the contributions had been $6\frac{3}{4}d.$ $11\frac{1}{4}d.$ or any other multiple of $2\frac{1}{4}d.$ the ſum to be paid would have been the ſame multiple of $9s.$ and therefore immediately aſcertained. ED.

APPENDIX II.

CONTAINING

NOTES.

Note (A). See Queſtion III. Page 11.

LET E be any given expectation of Life; and $\frac{4E-x}{4E} \times px$ will be the number of perſons alive at the end of x years, ariſing from p perſons left annually as widows (or added annually to a town or ſociety) at the age whoſe *expectation* is E. The *maximum*, therefore, is always pE—. In Mr. *De Moivre's Hypotheſis*, E is always $\frac{1}{2}$ the difference between the given age and 86. See the Note, page 2, and the latter end of the Note in page 37. Vol. I. See likewiſe the beginning of the Firſt Eſſay, in Vol. I.; and Note (K), in the following Notes, where the inveſtigation of this rule will be given.

It will not be amiſs to give the following example of the application of this rule.

At the time of the commencement of the ſcheme among the miniſters and profeſſors in SCOTLAND for making proviſion for their widows, it was neceſſary, that a calculation ſhould be made of the number of widows that would be upon the ſcheme at the end of every year till they came to

a *maximum*,

a *maximum*, on the ſuppoſition that, (agreeably to what particular enquiry had ſhewn to have happened for many preceding years,) 20 new widows would be left every year (*a*). In order to make this calculation, let 4 of the 20 widows be ſuppoſed to be under 32 years of age when left; and let 28 be ſuppoſed their mean age. Let the ſame number be left between 32 and 39, and let 35 be their mean age; between 39 and 47, and 43 their mean age; between 47 and 57, and 52 their mean age; between 57 and the extremity of life, and 63 their mean age. The number in life together to which, in 10 years, 4 widows left annually at the age of 28 will grow, is, by the rule, (E being 29) $\frac{116-10}{116} \times 40$, or 36.55.——The number alive at the end of 20 years, will be $\frac{116-20}{116} \times 80$, or 66.2. At the end of 30 years, the number alive will be 89; of 40 years, 104.82; of 58 years 116.—— Theſe numbers, found in the ſame way, for the 2d claſs, (E being 25.5,) at the end of 10, 20, 30, 40, and 51 years, will be 36.7—64.31—84.7—97.25—102——For the 3d Claſs, (E being 21.5) at the end of 10, 20, 30, 40, and 43 years, 35.34—61.4—78.13—85.6—86——For the 4th claſs, (E being 17) at the end of 10, 20, 30, and 34 years, 34.11—56,47—67—68——For the 5th claſs, (E being 11.5) at the end of 10, 20, and 23 years, 31.3—45.2—46——The whole number, therefore, conſiſting of all the claſſes, will come to a *maximum* nearly in 58 years; and the totals in life, at the end of 10, 20, 30, 40, 50, and 58 years, will be 173.37—293.58—364.83—401.67—418.

(*a*) For a term of 35 years and eight months, being from the commencement of the ſcheme to the year 1783, this number was $19\frac{1}{16}$.

Theſe

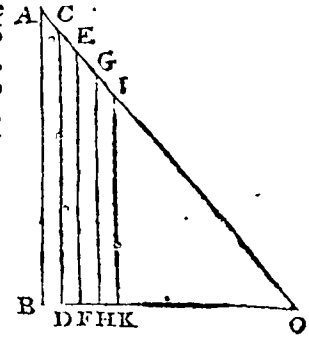
Theſe determinations ſuppoſe none to marry. In 10 years, from 1757 to 1767, I have been informed, that but 9 widows married. Let us then ſuppoſe, that one widow of the firſt claſs marries every year; and let all that marry, be ſuppoſed to continue, one with another, 5 years in widowhood before they marry. On theſe ſuppoſitions, the foregoing totals will, at the end of the ſame periods of years, be 169.23—282—347.5—380.47—394.

Theſe calculations are made from Mr. *De Moivre*'s Hypotheſis. Had they been made exactly from Dr. *Halley*'s or the *Northampton* Table, the reſults would have been very nearly the ſame.

See more on this ſubject in note F (α).

LET

(α) This theorem is deduced from a fluxional computation in note (K); but it may be demonſtrated without having recourſe to fluxions in the following manner:—Suppoſe at the time of admiſſion there were 50 perſons aged 36 years, or a number equal to their common complement. Suppoſe alſo the ſame number were added annually at the ſame age of 36. In the rectangled iſoſceles triangle ABO let AB (= BO) be = 50, or the complement of a life at 36 —CD (= DO) be = 49, EF (= FO) be = 48, and ſo on. Hence BD will be = 1, BF = 2. BH = 3, &c. By the hypotheſis of an equal decrement of life, it is evident that under the circumſtances of this caſe the area ABCD, or $\frac{2AB-BD}{2} \times BD$ will expreſs the number of annuitants at the end of the firſt year; the area ABEF, or $\frac{2AB-BF}{2} \times BF$, the number of annuitants at the end of the 2d year; the area ABGH, or $\frac{2AB-BH}{2} \times BH$, the number of annuitants at



the end of the 3d year, and ſo on. If the number of years be x, the annuitants living will be $\frac{2AB-x}{2} \times x$, or $\frac{4E-x}{2} \times x$; for AB being conſtantly $= 50$, or the complement, will be twice the expectation, or 2E.——As 50 or 2E, (the numbers of perſons admitted annually) is to $\frac{4E-x}{2} \times x$, (the number of annuitants at the end of x years) ſo is any other number (p) to $\frac{4E-x}{4E} \times px$, the number of annuitants in the ſame time from (p) perſons admitted yearly at the age whoſe expectation is E; and when x becomes equal to BO ($= AB = 2E$), the number of annuitants will arrive at its *maximum*, and be conſtantly expreſſed by the area pE. Q. E. D. Ed.

Note

Note (B). Queſtion VI. Page 21. Vol. I.

LET r ſignify the ſum of 1 *l.* and its intereſt, for one year. The value of a life, whoſe complement is n, being (by Mr. *De Moivre*, on *Annuities*, 4th edition, page 14, and p. 100.) $\frac{n-1}{nr} + \frac{n-2}{nr^2} + \frac{n-3}{nr^3} + \frac{n-4}{nr^4}$, &c. the preſent value of the remainder of it after *two* years muſt be $\frac{n-3}{nr^3} + \frac{n-4}{nr^4}$, &c. which is equal to $\frac{1}{r^2} \times \frac{n-2}{n} \times \frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}$, &c.

Now $\frac{1}{r^2}$ is the preſent value of 1 *l.* due at the end of two years. $\frac{n-2}{n}$ is the probability that a life, whoſe complement is n, ſhall continue two years, and $\frac{n-3}{n-2r} + \frac{n-4}{n-2r^2} + \frac{n-5}{n-2r^3}$, &c. is the value of a life two years older than the life whoſe complement is n. And, therefore, (ſince any number of years leſs than n may be ſubſtituted for two years) the firſt rule given in this Queſtion is right (β).

The

(β) The rules in this and the following Notes are demonſtrated rather more ſatisfactorily, and with equal caſe and perſpicuity, from the real probabilities of life.

Let a repreſent the number of perſons living in the table at the age of A, and b, c, d, e, &c. the number living at the end of the 1ſt, 2d, 3d, 4th, &c. years from the age of A. Now ſince the value of an annuity on the life of A is known to be $= \frac{b}{ar} + \frac{c}{ar^2} + \frac{d}{ar^3}$, &c. the value of this annuity after

The ſame proceſs, applied to joint lives, will demonſtrate what is ſaid in the *Scholium.*

two years on the ſame life will be $= \frac{d}{ar^3} + \frac{e}{ar^4} + \frac{f}{ar^5}$, &c. $= \frac{1}{r^2} \times \frac{c}{a} \times \overline{\frac{d}{cr} + \frac{e}{cr^2} + \frac{f}{cr^3}, +}$ &c. But $\frac{c}{a}$ is the probability that A lives two years, and the ſeries $\frac{d}{cr} + \frac{e}{cr^2} + \frac{f}{cr^3}$, &c. is the value of an annuity on a life two years older than A. The general rule therefore in the 6th Queſtion is right; for the reaſoning applied to this particular caſe will alſo apply to any other interval between the preſent time and the period at which the annuity is to commence. ED.

Note

Note (C). See Queſtion VII. P. 22. Vol. I.

LET the complements of any two aſſigned lives be n and m. The preſent value of the firſt poſſible payment of an annuity to be enjoyed by the life whoſe complement is n, provided *both* lives continue 7 years, and the life, whoſe complement is n, ſurvives the other *after* that term, is the probability, that the life of the expectant ſhall continue 8 years, and the other life 7 years and then fail in the 8th year, multiplied by $\frac{1}{r^8}$, or by 1 *l.* diſcounted for 8 years.—The probability that the life of the *expectant* ſhall continue 8 years is $\frac{n-8}{n}$. The probability that the *other* life ſhall continue 7 years is $\frac{m-7}{m}$. The probability that it ſhall continue 7 years, and fail in the 8th year, is $\frac{m-7}{m} \times 1 - \frac{m-8}{m-7} = \frac{1}{m}$. The probability, therefore, that the life of the *expectant* ſhall continue 8 years, and the other life continue 7 years and fail in the 8th, is $\frac{n-8}{n} \times \frac{1}{m}$; and the preſent value of the firſt poſſible payment of the annuity ſuppoſed, is $\frac{n-8}{nr^8} \times \frac{1}{m}$. See *The Doctrine of Annuities*, by Mr. *Simpſon*, p. 6—15, or his *Select Exerciſes*, p. 315, &c.—In like manner, the preſent value of the 2d payment, at the end of the 9th year, may be found to be $\frac{n-9}{nr^9} \times \frac{m-7}{m} \times 1 - \frac{m-9}{m-7}$, or $\frac{n-9}{nr^9} \times \frac{2}{m}$. and the preſent value of all the poſſible payments, $\frac{1}{r^7} \times \frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}$, &c. But this ſeries is equal to $\frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times \frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7} + \frac{n-10}{n-7r^3} \times \frac{3}{m-7}$,

$\frac{3}{m-7}$, &c. Now $\frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7}$, &c. is the value of an annuity for a life seven years older than the expectant, after another life seven years older than the life whose complement is m. $\frac{n-7}{n} \times \frac{m-7}{m}$ is the probability that both the assigned lives shall continue 7 years. And $\frac{1}{r^7}$ is the value of 1*l.* due at the end of 7 years. The rule, therefore, given for solving this question, is right.

This demonstration, as well as that in the last note, is, for the sake of more ease and clearness, applied to the hypothesis of an equal decrement of life. It does not, however, depend upon it, but may be applied to any table of observations (γ).

(γ) Let a, b, c, d, e, f, &c. represent the same quantities as in the preceding Note. Let m represent the number of persons living at the age of B, and n, o, p, q, s, &c. the number living at the end of the 1st, 2d, 3d, &c. years from the age of B.—By reasoning in the same manner with Dr. *Price* in the solution of this question, and supposing the first payment of the annuity to become due at the end of the 4th year, its present value will be $= \frac{e}{a} \times \frac{p-q}{mr^4} = \frac{dp}{amr^3} \times \frac{e}{dr} - \frac{eq}{apr}$ —— the present value of the payment at the end of the 5th year will be $= \frac{f}{a} \times \frac{p-s}{mr^5} = \frac{dp}{amr^3} \times \frac{f}{dr^2} - \frac{sf}{dpr^2}$ ——the present value of the payment at the end of the 6th year will be $= \frac{g}{a} \times \frac{p-t}{mr^6} = \frac{dp}{amr^3} \times \frac{g}{dr^3} - \frac{gt}{dpr^3}$, and so on. Hence the whole value will be $= \frac{dp}{amr^3} \times \frac{e}{dr} + \frac{f}{dr^2} + \frac{g}{dr^3}$, &c. $- \frac{dp}{amr^3} \times \frac{eq}{dpr} + \frac{sf}{dpr^2} + \frac{gt}{dpr^3}$ + &c. Let $\overset{1}{A}$ and $\overset{1\ 1}{A\ B}$ denote the respective values of annuities on the single and joint lives of two persons 3 years older than A and B, and the general value will become $= \frac{dp}{amr^3} \times \overset{1}{A} - \overset{1\ 1}{A\ B}$.——Q.E.D.

ED.

Note

Note (D). Queſtion IX. Page 29. Vol. I.

LET the complement of any two aſſigned lives be n and m, and the given term be *ſeven* years, as in Note (C). The probability that the former life (ſuppoſed to be the life in expectation) ſhall laſt 8 years, is, by Mr. *De Moivre's* Hypotheſis, $\frac{n-8}{n}$; and the probability that the latter life ſhall fail in 8 years, is $\frac{8}{m}$; and the firſt payment of the annuity mentioned in this queſtion, depends on the happening of *both* theſe events, the probability of which is $\frac{n-8}{n} \times \frac{8}{m}$.

The preſent value, therefore, of the firſt poſſible payment of the annuity is $\frac{n-8}{nr^8} \times \frac{8}{m}$.——In like manner, the preſent value of the *ſecond* poſſible payment is $\frac{n-9}{nr^9} \times \frac{9}{m}$; and of all the payments, $\frac{n-8}{nr^8} \times \frac{8}{m} + \frac{n-9}{nr^9} \times \frac{9}{m} + \frac{n-10}{nr^{10}} \times \frac{10}{m}$, &c. But $\frac{n-8}{nr^8} \times \frac{8}{m} = \frac{n-8}{nr^8} \times \frac{1}{m^8} + \frac{n-8}{nr^8} \times \frac{7}{m}$; and $\frac{n-9}{nr^9} \times \frac{9}{m} = \frac{n-9}{nr^9} \times \frac{2}{m} + \frac{n-9}{nr^9} + \frac{7}{m}$. The foregoing ſeries, therefore, is equal to the two ſeries's $\frac{1}{r^7} \times \frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}$, &c. and $\frac{1}{r^7} \times$

$\times \frac{n-8}{nr} \times \frac{7}{m} + \frac{n-9}{nr^2} \times \frac{7}{m} + \frac{n-10}{nr^3} \times \frac{7}{m}$, &c. or to $\frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times \frac{n-8}{n-7r} \times \frac{1}{m-7} + \frac{n-9}{n-7r^2} \times \frac{2}{m-7} + \frac{n-10}{n-7r^3} \times \frac{3}{m-7}$, &c. $+ \frac{1}{r^7} \times \frac{7}{m} \times \frac{n-7}{n} \times \frac{n-8}{n-7r} + \frac{n-9}{n-7r^2} + \frac{n-10}{n-7r^3}$, &c. which is the very rule given for solving this question (δ), as will appear from Notes (B) and (C).

(δ) Retaining the same symbols as in the two foregoing Notes, (β) and (γ) and supposing the first payment of the annuity to become due at the end of the 4th year, the present value of the several payments will be $= \frac{e}{a} \times \frac{m-q}{mr^4} + \frac{f}{a} \times \frac{m-s}{mr^5} + \frac{g}{a} \times \frac{m-t}{mr^6}$ + &c. $= \frac{d}{ar^3} \times \frac{e}{dr} + \frac{f}{dr^2} + \frac{g}{dr^3}$ + &c. $- \frac{dp}{amr^3} \times \frac{eq}{dpr} + \frac{sf}{dpr^2} + \frac{gt}{dpr^3}$ + &c. $= \frac{d}{ar^3} \times \dot{A} - \frac{dp}{amr^3} \times \dot{A}\dot{B}$.—Q. E D.

This rule, though it agrees in its result with the rules given by Dr. *Price*, is rather more concise. and may be thus expressed:—"Find by Quest. VI. the value of the annuity "for the remainder of the life in expectation after the given "time. Find also by the scholium to that question the "value of the annuity for the remainder of the two joint "lives after the given time. The latter subtracted from the "former will be the value required." Ed.

Note

Note (E). See the Scholium to Queſt. X. Page 34. Vol. I.

ACCORDING to the calculations, the time in which the firſt yearly payment of a reverſionary *annuity* becomes due, is the end of the year in which the event happens that entitles to it, however little or much of the year may then happen to be unelapſed. And this, likewiſe, is the time when a reverſionary *ſum* becomes due. Thoſe who know how the calculations of the values of reverſions are inſtituted, muſt know this. But an annuity, the firſt payment of which is to be made at the ſame time with another payment of a ſum in hand, ſufficient to buy an equal annuity, is worth one year's purchaſe more than that ſum. For inſtance. Reckoning intereſt at 4 *per cent.* and r being 1 *l.* increaſed by its intereſt for a year, or 1.04 $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. $= 25l.$ is the preſent value of an eſtate of 1 *l. per annum* for ever. That is, it is the value of it, ſuppoſing the firſt rent of it is to be paid a year hence.——If the firſt rent is to be received immediately, or at the ſame time with another payment of 25*l.* it is worth one year's purchaſe more, or equivalent to 26*l.*——I have not found, that any of the writers on annuities and reverſions, have attended to this obſervation. It ſuggeſts a correction neceſſary to be applied to the common ſolutions of ſeveral important problems: particularly to the 21ſt and 22d in Mr. *Simpſon's Treatiſe on Annuities*, and the 26th, 27th, 32d, 33d, and 40th problems in his *Select Exerciſes*; and to all other problems of the ſame kind in other writers. There

can

can be no great occaſion for being more explicit. It will not, however, be amiſs to add the following demonſtration. $\frac{1}{n}$ is the preſent probability that a life whoſe complement is n will fail in any one aſſignable year of its duration. $S \times \frac{1}{nr} + \frac{1}{nr^2} + \frac{1}{nr^3}$, &c. ($n$), or the preſent value of 1l. per annum for n years, multiplied by $\frac{S}{n}$, is the preſent value of the ſum or legacy denoted by S, payable at the failure of the given life. Therefore, (n being 56; the life 30; intereſt 4 per cent. $r = 1.04$; the ſum 25l.) the value of the expectation, by Mr. *De Moivre*'s hypotheſis, is 9.919.

Further. The value of 1l. to be received at the end of a year, provided the life whoſe complement is n fails, is the probability of the failure of the life multiplied by 1l. diſcounted for a year, or $1 - \frac{n-1}{n} \times \frac{1}{r}$. In like manner; the value of 1l. to be received at the end of two years, if the ſame life fails in 2 years, is $1 - \frac{n-2}{n} \times \frac{1}{r^2}$. And, therefore, the value of all the *poſſible* payments of an eſtate or annuity of 1l. for ever, to be entered upon after the given life, is $1 - \frac{n-1}{n} \times \frac{1}{r} + 1 - \frac{n-2}{n} \times \frac{1}{r^2} + 1 - \frac{n-3}{n} \times \frac{1}{r^3}$, &c. ($n$) $+ \frac{1}{r^n + 1} + \frac{1}{r^n + 2}$

$\frac{1}{r^{n+2}}$, &c. or $\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}$, &c. $-\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}$, &c. that is, the value of the life ſubtracted from the perpetuity; or, in this example, *l.* 14.684, (the value of a life at 30) ſubtracted from 25; that is, *l.* 10.316. But 10.316 is to 9.919, in the ſame ratio with 104 to 100, or 26 to 25, agreeably to the rule in the *Scholium* (ε).

(ε) The difference between the values of reverſionary *ſums* and reverſionary *eſtates* (which was firſt pointed out in this Note) does not depend on the hypotheſis of an equal decrement, but may be as readily demonſtrated from the real probabilities of life. Suppoſing *a*, *b*, *c*, *d*, *e*, &c. to repreſent the ſame quantities as in Note (β), the value of the *ſum* S, to be received on the death of A, will be properly expreſſed by the ſeries $\frac{S}{a}\times\frac{a-b}{r}+\frac{b-c}{r^2}+\frac{c-d}{r^3}+$ &c. $=S\times\frac{1}{r}+\frac{b}{ar^2}+\frac{c}{ar^3}+\frac{d}{ar^4}$ &c. $-S\times\frac{b}{ar}+\frac{c}{ar^2}+\frac{d}{ar^3}+$ &c. $=S\times\frac{A+1}{r}-A.=\frac{S.r-1}{r}\times P-A.$ ——(P denoting the perpetuity, and A the value of an annuity on the life of A).—But in the caſe of an *annuity* or *eſtate*, the value of the reverſion of £ 1 *per ann.* after the death of A will be $=\frac{a-b}{ar}+\frac{a-c}{ar^2}+\frac{a-d}{ar^3}+$ &c ——(*t*) $+\frac{1}{r^{t+1}}+\frac{1}{r^{t+2}}+\frac{1}{r^{t+3}}+$ &c. (*t* denoting the number of years between the age of A and that of the laſt ſurviving life in the table of obſervations). The ſum of theſe two ſeries is eaſily found $=P-A.$——If S repreſent a ſum equal to the perpetuity of £ 1 *per ann.* or, in other words, if S be taken $=\frac{1}{r-1}$, it will appear that the value of the reverſion of an *eſtate* is to the value of the reverſion of an equivalent *ſum* as P—A to $\frac{P-A}{r}$, or as *r* to 1, agreeable to what has been obſerved above.

ED.

Note

Note (F). Queſt. XIII. Page 44. Vol. I.

WHEN I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to ſuppoſe, that there are as many married perſons who exceed theſe ages, as there are who fall ſhort of them. It is likely that the latter are moſt numerous; and it is neceſſary that this ſhould be the caſe, to render the ſuppoſition I make juſt.—If all marriages commenced at 33 for the man, and 25 for the woman, one half of them would be diſſolved by the time the men were 50, and the women 42; for (by the *Hypotheſis*, and alſo nearly by the *Breſlaw*, *Norwich*, and *Northampton* tables) there is an equal chance for the joint continuance of two lives, whoſe ages are 25 and 33, *ſeventeen* years. Forty-two and fifty then would be properly the mean ages at which widowhood would commence: meaning by theſe "the "ages on each ſide of which equal numbers are "left widows and widowers."—But, though in this caſe half the marriages of every year would be diſſolved in 17 years, they would not be *all* diſſolved in twice that time. So far would this be from happening, that about a 7th part would continue beyond twice 17 years; nor would it be *certain*, that they would be all diſſolved till near the extremity of the poſſible extent of life. Though, therefore, an equal number of marriages would be diſſolved, or an equal number of widows and widowers left *before* 50 and 42 and *afterwards*, yet the ages of the latter would, one with another, much more exceed 50 and 42, than the ages of the former (that is, of the widows and widowers left

before

before 50 and 42) would fall ſhort of them. And the number of marriages alſo in the world, among perſons of greater ages than theſe, would be much fewer than among perſons of leſſer ages.—In other words: The period, at which the marriages that have been contracted are half diſſolved, is not the period at which the number of marriages conſtantly exiſting is equally divided, but this period falls ſome years ſooner; and the period I have in view falls in that part of the interval between theſe two periods, where the greater ages of the marriages on one ſide, are juſt enough to compenſate (in ſuch a calculation as that I have given) their deficiencies in number, compared with the number of marriages on the other ſide.

In ſhort. Suppoſe 35 marriages every year, between perſons 33 and 25 (*a*). In 12 years there would be half as many in the world, as could poſſibly ariſe from ſuch a number of yearly weddings. In 17 years, half every ſet would be extinct. The *expectation* of every marriage would be 19 years, by prob. 21 of Mr. *De Moivre's Treatiſe on Annuities*, or by the note (K) in the following notes: That is, taking them all together, they would exiſt juſt as long as an equal number of *ſingle* perſons, ſuppoſed to be ſure of living juſt 19 years, and no more: or, as long as an equal number of ſingle perſons, all 48 years of age, ſuppoſed to be ſubject to the common laws of mortality. One with another, then, they will be all extinct in 19 years; the marriages which continue beyond this term, though fewer in number, enjoying among them juſt as

(*a*) In the *Pais de Vaud, Switzerland*, the mean age at which women marry, is nearly the very age here mentioned: But it is ſhewn in the *Supplement*, that the expectation of marriage there is no leſs than 23 years and ½; ſo much higher are the probabilities of life in the *country* than in *towns*, or than they ought to be according to Mr. *De Moivre's Hypotheſis*. See p. 254, Vol. II.

much *more* duration, as thoſe that fall ſhort of it enjoy *leſs*. *Widows*, then, at a medium, will commence widowhood at 44 (that is, 25 increaſed by 19) years of age, and *widowers* at 52. The values, therefore of the lives of the *former*, when they commence widowhood, will, one with another, be the ſame with the value of a life at 44; or, (reckoning intereſt at 4 per cent.) 12.5 years purchaſe, in one preſent payment, (the annuity to begin at the end of a year); and their *expectation* of life will be 21 years, or half the difference between 44 and 86. The value of the lives of the *latter* will be 10.92, and their *expectation* 17 years.—The whole number of marriages conſtantly exiſting, which would reſult from 35 ſuppoſed to commence annually, would be 19×35, or 665; and 53 years (the difference between 33 and 86) would be the time in which they would increaſe to this number—The chance of ſurvivorſhip would be the odds of 69 to 53, by prob. 18th, Mr. *De Moivre on Annuities*; that is, in 53 years, 35 relicts of theſe marriages would be left every year, and the number of *widows* would be to the number of *widowers*, as 69 to 53; or 19.8 *widows* would be left annually, and 15.2 *widowers*. The *maximum* of widows in life together, if none married, would be 21×19.8, or 416; and they would increaſe to this number in 114 years (or 61 years after the number of marriages had arrived at a *maximum*)——The *maximum* of *widowers* would be 15.2×17, or 258; and they would increaſe to this number in 106 years.

An eaſy method may be hence deduced of ſolving the queſtion which occaſions this note —— If the number of the members of the eſtabliſhment I have ſuppoſed is 665, and the mean ages at which marriage may be deemed to commence are 25 and 33, 19 8 widows will (it has juſt appeared) be left every year; and the values of their lives,

when they commence widowhood, will be, one with another, 12½ years purchaſe. An annuity of 20*l.* will, therefore, be worth, to each widow, 250*l.* and 19.8 ſuch annuities muſt be worth 4950*l.* which, conſequently, is the annual income neceſſary for the ſupport of the eſtabliſhment, the firſt payment to be received immediately: or *l.* 7.44 from each of the 665 members; which anſwers nearly to the determination in Vol. I. p. 44.

In the laſt Eſſay in Vol. I. p. 364, it has been ſhewn, that obſervations determine the chance of ſurvivorſhip in favour of the wife in marriage, to be really ſo great as 3 to 2; and in ſome circumſtances greater. I have alſo there obſerved, that in order to account for this from the difference of age between men and their wives, this difference muſt be at leaſt 12 years, and the mean ages of all who marry annually muſt be ſuppoſed to be about 23 and 35. In this caſe, 19, as before, will nearly be the *expectation* of all marriages. The mean age at which widows and widowers will commence ſuch will be 42 and 54. The number of annual marriages neceſſary to keep up 665 marriages conſtantly exiſting, will be 35. The number of widows left annually, by ſuch a number of marriages, will be 21; and the values of their lives, at the time they commence widowhood, will be 12.85 years purchaſe by the firſt of the following Tables; and, therefore, the whole annual income neceſſary for the ſupport of the ſuppoſed eſtabliſhment, will be 5397*l.* or an annual payment, beginning immediately, of *l.* 8.11 from each member—The number of widows on ſuch an eſtabliſhment will, in 63 years, grow, if none marry, to 462; and the number of widowers to 224. —It may be depended on, that all this would happen as far as Dr. *Halley*'s Table, or the Tables for *Norwich* and *Northampton*, exhibit the true ſtate of human mortality.

Among

Among the miniſters and profeſſors in SCOTLAND, the number of married men being 667, or nearly that here mentioned, the number of annual weddings has, for many years, been at an average 30, and the number of widows left annually 19¼; and, therefore, the chance of ſurvivorſhip in favour of the wife, as 19.2 to 11.8, or 5 to 3. This is not more different from the reſults I have given, than might have been expected; and the chief reaſon of the difference is, that the *expectations* of *ſingle* and *joint* lives among the miniſters and their wives in SCOTLAND are greater than thoſe given by Dr. *Halley*'s, and the other tables of obſervation.—Theſe tables give the expectations of lives as they are among the bulk of mankind in moderate towns. The expectations of lives among the better ſort of men, living moſtly in country villages and pariſhes, are much greater. The fact is, that among the miniſters in *Scotland*, the expectation of a *ſingle* life, at the age of 27, is near 4 years greater; and, of *joint* lives, about three years greater, than the ſame expectations by Dr. *Halley*'s Table. See the latter end of the laſt Eſſay in the former Volume.

I cannot help juſt mentioning another remark here.—It may be obſerved, that ſuppoſing no ſecond marriages, and, at the ſame time, that the odds for the woman's ſurviving in marriage is 3 to 2, the number of *widows* in the world would be *double* the number of *widowers*. But it has been found, in fact, that the number of widows is, in ſome ſituations, five times the number of widowers. How this is to be accounted for, I have ſhewn in the Eſſay juſt referred to.

Note

Note (G). Queſtion XIV. Page 48. Vol. I.

LET r be 1 *l.* increaſed by its intereſt for one year; t the given time or number of years for which the aſſurance is to be made; a, b, c, &c. the *probabilities* taken out of a table of obſervations, that the perſon whoſe age is given ſhall live 1, 2, 3, &c. years; and P the probability that he ſhall live t years. Then $\frac{1-a}{r}+\frac{1-b}{r^2}+\frac{1-c}{r^3}$, &c. $(t-1)+\frac{1-P}{r^t}+\frac{1-P}{r^{t+1}}+\frac{1-P}{r^{t+2}}$, &c. $=\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}$, &c. $(t)-\frac{a}{r}+\frac{b}{r^2}+\frac{c}{r^3}$, &c. $(t-1)+\frac{P}{r^t}+\frac{1-P}{r^t}\times\frac{1}{r}+\frac{1}{r^2}+\frac{1}{r^3}$, &c. will be the exact value of an annuity to be entered upon at the failure of the given life, provided it happens in t years. And the rule is nothing but this value expreſſed in words. In a ſimilar manner may be demonſtrated the other rule for finding the values of aſſurances for a given time, on two joint lives, or the longeſt of two lives.

Note (H. Queſtion XV. Page 56. Vol. I.

LET r ſignify as before; S the given ſum to be aſſured; t the given time; N and n the number of the living in the table of obſervations, at the age of B and A reſpectively; A, B, C, &c. and a, b, c. &c. the number of the living in the table, at the end of 1, 2, 3, &c. years from the ages of B and A; D, D, D, D, &c. and d, d, d, d, &c. the decrements of life in the table, at the end of 1, 2, 3, &c. years from the ſame ages. Then, by reaſoning in the ſame manner with Mr. *Simpſon*, in p. 316, &c. *Select Exerciſes*, it will appear that

$$S \times \overline{\frac{A \times d}{Nnr} + \frac{B \times d}{N_{\text{I}} nr^2} + \frac{C \times d}{N_{\text{II}} nr^3}, \text{ &c. } (t)} + S \times \overline{\frac{D_{\text{I}} d}{2N_{\text{I}} nr} + \frac{D_{\text{I}} d}{2N_{\text{I}} nr^2} + \frac{D_{\text{II}} d}{2N_{\text{II}} nr^3}, \text{ &c. } (t)} = \frac{S}{n} \times \overline{\frac{A d}{Nr} + \frac{B d}{N_{\text{I}} r^2} + \frac{C d}{N_{\text{II}} r^3}, \text{ &c. } (t)} + \frac{S}{2N} \times \overline{\frac{D d}{nr} + \frac{D_{\text{I}} d}{nr^2}, \text{ &c. } (t)}.$$

This is the exact anſwer to Queſtion XV. and the rule is as near an approximation to it as there is reaſon to deſire.

In the ſame manner, retaining all the ſame ſymbols, it may be found, that the anſwer to Queſtion XVI. is

$$S \times \overline{\frac{D d}{2Nnr} + \frac{D_{\text{I}} d}{Nnr^2} + \frac{D + D_{\text{I}} \times d}{N_{\text{I}} nr^3} + \frac{D + D_{\text{I}} + D_{\text{II}} \times d}{N_{\text{II}} nr^4}} \overline{(t), \text{ &c.}} + S \times \overline{\frac{D_{\text{I}} d}{2N_{\text{I}} nr^2} + \frac{D_{\text{II}} d}{2N_{\text{II}} nr^3} + \frac{D_{\text{III}} d}{2N_{\text{III}} nr^4}, \text{ &c.}}$$

$(t-1)$

$(t-1) = \frac{S}{nr} \times \frac{Dd}{Nr} + \frac{D+D \times d}{Nr^2} + \frac{D+D+D \times d}{Nr^3}$,

&c. $(t-1) + \frac{S}{2N} \times \frac{Dd}{nr} + \frac{Dd}{nr^2} + \frac{Dd}{nr^3}$, &c. (t).

But $\frac{D}{Nr} + \frac{D+D}{Nr^2} + \frac{D+D+D}{Nr^3}$, &c. $(t-1)$ is the ſame with the exceſs of the value of an annuity *certain* for a number of years leſs by one year than the given term, above the value of an annuity on the life of A, for the ſame number of years; from whence the reaſon of the rule for ſolving this queſtion may be eaſily diſcovered (ζ).

(ζ) The ſolution of the 15th queſtion may be deduced in a ſimilar, but rather more accurate, manner from the firſt of the two rules given in Note (O); where the value of the reverſion for t years is expreſſed by the two ſeries $\frac{S}{2ab} \times \frac{ca'}{r} + \frac{da''}{r^2} + \frac{ea'''}{r^3}$ ----$(t) + \frac{S}{2ab} \times \frac{ba'}{r} + \frac{ca''}{r^2} + \frac{da'''}{r^3}$ ----(t). If α denote the ſum of the decrements of life from the age of A for t years divided by t (which may be called the *complement* of A's life for the given term), and B and $\overset{1}{B}$ the values of an annuity on the life of B for t and $t-1$ years reſpectively, the ſum of theſe two ſeries may be found $= \frac{S.\alpha}{2a} \times B + \frac{\overset{1}{B}+1}{r}$.

In like manner, the ſolution of the 16th Queſtion may be derived from the ſecond of the two rules given in Note (O);—the ſeries expreſſing the value of the reverſion in this caſe being $\frac{S}{2ab} \times \frac{b-c.a'}{r} + \frac{c-d.a'+a''}{r^2}$ ---$(t) + \frac{S}{2abr} \times \frac{c-d.a'}{r} + \frac{d-e.a'+a''}{r^2}$ ---$(t-1)$. Let β denote the *complement*

plement of B's life for t years, A and $\overset{1}{\text{A}}$ the values of an annuity on the life of A for t and $\overline{t-1}$ years, and N and $\overset{1}{\text{N}}$ the values of an annuity certain for thoſe reſpective terms; then will the above ſeries be found $= \frac{\text{S}.\beta}{2b} \times \overline{\overline{\text{N}-\text{A}} + \frac{\overset{1}{\text{N}}-\overset{1}{\text{A}}}{r}}$.——It is to be obſerved, when the decrements of A's life for t years in the firſt of theſe rules, and the decrements of B's life in the ſecond are equal, that the exact value of the reverſion is obtained; and if the term do not exceed 10 or 12 years, the values are always ſo nearly true as not to require greater accuracy. This alſo is the caſe in general with regard to Dr. *Price*'s rules; againſt which there can be no objection, excepting the application of Mr. De *Moivre*'s hypotheſis in one part of them, which it is beſt *entirely* to exclude from the doctrine of ſurvivorſhips.

Ed.

Note

Note (*I*). Page 139 and 177. Vol. I.

SUPPOSING r to ſignify as in the laſt notes, and n to be the complement of a given life; the preſent value of 1*l.* 2*l.* 3*l.* &c. payable at the end of 1, 2, 3, &c. years to t years, but ſubject to failure when the life fails, is $\frac{n-1}{nr} + \frac{\overline{n-2} \times 2}{nr^2} + \frac{\overline{n-3} \times 3}{nr^3}$, &c. continued to t years; which expreſſion is equal to $n \times \frac{\overline{n-1}}{nr} + \frac{\overline{n-2}}{nr^2} + \frac{\overline{n-r}}{nr^3}$, &c. (t) $- n \times \frac{\overline{n-1}^2}{n^2r} + \frac{\overline{n-2}^2}{n^2r^2} + \frac{\overline{n-3}^2}{n^2r^3}$, &c. (t).

To find, therefore, the value of an annual payment dependent on a given life, to begin with 1*l.* and to increaſe at the rate of 1*l.* every year after the firſt, for a given term; find the value of an annuity on the given life for the given term; and alſo the value for the given term of an annuity on two joint lives both equal to the given life. The difference between theſe two values multiplied by the complement of the given life, will be the value ſought.——If ſuch a courſe of payment, inſtead of beginning at the end of a year, is to begin immediately, and to be made at the beginning of every year till $t+1$ payments are made in t years; add to the preceding value the value increaſed by unity of an annuity on the given life for t years, found by Queſtion VI, and the ſum will be the value ſought. And this value, divided by the

preſent value of what may happen to remain of the given life after t years, found by the ſame queſtion, will give the annuity to which ſuch a ſeries of increaſing annual payments, beginning immediately, will entitle for the remainder of the given life after t years.

If ſuch a courſe of payment is to begin at the end of a year, and to be continued during life (that is, if $t = n$) it is obvious, that its value will be the complement of the life multiplied by the difference between the value of the life, and the value of two joint lives having the ſame common age with it; and that if it is not to commence till the end of a given number of years, its value will be the value for a life ſo many years (leſſened by one) older than the given life, and multiplied by the value of 1*l.* payable at the end of a number of years leſs by one year than the given number of years, and alſo multiplied by the probability that the given life will exiſt for the ſame number of years. —Suppoſing, for inſtance, the given life 30 years of age, and ſuch a courſe of payment to begin when it has completed its 56th year, the value would be the value of a life aged 55, diminiſhed by the value of two joint lives both 55, and the remainder multiplied by the complement of a life aged 55, and alſo by the product of the probability that a life aged 30 will exiſt 25 years, into the value of 1*l.* payable at the end of 25 years.——The value thus computed will, in this caſe, come out 19*l.* nearly, in a ſingle preſent payment, reckoning intereſt at 4 *per cent.* and taking the probabilities of the duration of life from the *Northampton* Table of Obſervations.

With

With the aſſiſtance of theſe rules, all that is ſaid in Vol. I. p. 139 and p. 177, may be inveſtigated. But more particular directions for computing the values of annuities of this ſort may be found in Mr. *Morgan's Treatiſe on Life-Annuities and Aſſurances*, p. 119, &c.

Note (K). See Eſſay I. P. 230, 231. Vol. I.

THE *ſum* of the probabilities that any given lives will attain to the end of the 1ſt, 2d, 3d, &c. *years* from the preſent time to the utmoſt extremity of life (for inſtance, $\frac{45}{46} + \frac{44}{46} + \frac{43}{46}$, &c. to $\frac{1}{46} = 22\frac{1}{2}$ for lives of 40, by the *hypotheſis*) may be called their *expectation*, or the number of payments due to them, as *yearly annuitants*. The ſum of the probabilities that they will attain to the end of the 1ſt, 2d, 3d, &c. *half years*, (or, in the particular caſe ſpecified, $\frac{91}{92} + \frac{90}{92} + \frac{89}{92} + \frac{88}{92}$, &c. $= \frac{2145}{2}$ *half years*, or $22\frac{3}{4}$ *years*) is their expectation as *half-yearly annuitants*. And the ſums juſt mentioned of the probabilities of their attaining to the end of the 1ſt, 2d, 3d, &c. *moments* (equal in the ſame particular caſe to 23 years) is properly their *expectation of life*, or their *expectation* as annuitants ſecured by land.

Mr. *De Moivre* has omitted the demonſtrations of the rules he has given for finding the *expectations* of lives, and only intimated in general, that he diſcovered them by a calculation deduced from the method of fluxions. See his *Treatiſe on Annuities*, page 66. It will, perhaps, be agreeable to ſome to ſee how eaſily they are deduced in this method, upon the hypotheſis of an equal decrement of life.

Let $\dot{x}$ ſtand for a moment of time, and n the *complement* of any aſſigned life. Then $\frac{n-\dot{x}}{n}$, $\frac{n-2\dot{x}}{n}$, $\frac{n-3\dot{x}}{n}$, &c. will be the *preſent* probabilities of its continuing to the end of the 1ſt, 2d, 3d, &c. moments; and $\frac{n-x}{n}$ the probability of its continuing

to

to the end of x time. $\frac{n-x}{n} \times \dot{x}$ will therefore be the *fluxion* of the ſum of the probabilities, or of an *area* repreſenting this ſum, whoſe *ordinates* are $\frac{n-x}{n}$, and *axis* x.—The *fluent* of this expreſſion, or $x - \frac{x^2}{2n}$, is the ſum itſelf for the time x; and this, when $x = n$, becomes $\frac{1}{2}n$, and gives the *expectation* of the aſſigned life, or the ſum of all the probabilities juſt mentioned, for its whole poſſible duration.—In like manner: ſince $\frac{\overline{n-x}^2}{n^2}$ is the probability that two equal joint lives will continue x time, $\frac{\overline{n-x}^2}{n^2} \times \dot{x}$ will be the *fluxion* of the ſum of the probabilities. The *fluent* is $x - \frac{x^2}{n} + \frac{x^3}{3n^2}$, which, when $x = n$, is $\frac{n}{3}$, or the expectation of two equal joint lives.—Again: ſince $\frac{n-x}{n} \times \frac{2x}{n}$ is the probability that there will be a ſurvivor of two equal joint lives at the end of x time, $\frac{n-x}{n} \times \frac{2x}{n} \times \dot{x}$ will be the *fluxion* of the ſum of the probabilities; and the *fluent*, or $\frac{x^2}{n} - \frac{2x^3}{3n^2}$ is (when $x = n$) $\frac{1}{3}n$, or the *expectation* of ſurvivorſhip between two equal lives; which, therefore, appears to be equal to the *expectation* of their joint continuance. The expectation of two *unequal* joint lives, found in the ſame way, is $\frac{m}{2} - \frac{m^2}{6n}$, m (*n*) being the *complement* of the

(*n*) The expectation of two unequal joint lives is $= \frac{m-x}{m} \times \frac{n-x}{n} \times \dot{x}$, whoſe *fluent* (when $x = m$) is eaſily found $= \frac{m}{2} - \frac{mm}{6n}$. Ed.

oldeſt

oldeſt life, and n the *complement* of the youngeſt. The whole expectation of ſurvivorſhip is $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{3n}$ (9). And the expectation of ſurvivorſhip of the oldeſt will be to the expectation of ſurvivorſhip of the youngeſt, as $\frac{m^2}{6n}$ to $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{6n}$. It is eaſy to apply this inveſtigation to any number of joint lives, and to all caſes of ſurvivorſhip.

It may be obſerved, concerning the firſt of the fluents here given, that it expreſſes not only the expectation of a given life for the time x, and therefore its whole expectation when $x = n$, but likewiſe the number of perſons alive, to which one perſon added annually to a ſociety, at a given age, will increaſe in x time.——Thus: Suppoſe one

(9) The expectation of ſurvivorſhip due to the oldeſt life is expreſſed by the *fluxion* $\frac{m-x}{m} \times \frac{x}{n} \times \dot{x}$, whoſe *fluent* (when $x = m$) is $\frac{mm}{6n}$. The expectation of ſurvivorſhip due to the youngeſt life for m years is the *fluent* of $\frac{n-x}{n} \times \frac{x}{m} \times \dot{x}$, which (when $x = m$) is $\frac{m}{2} - \frac{mm}{3n}$. But this life has a further expectation, after m years, expreſſed by the *fluent* of $\frac{n-m-x}{n-m} \times \frac{n-m}{n} \times \dot{x}$, which (when $x = n-m$) will be $\frac{n}{2} - m + \frac{mm}{2n}$ The ſum of theſe two *fluents*, or $\frac{n-m}{2} + \frac{mm}{6n}$ will therefore be the whole expectation of ſurvivorſhip due to the youngeſt life. And this expreſſion added to $\frac{mm}{6n}$ (which has been found above to be equal to the expectation of ſurvivorſhip due to the oldeſt life) will give $\frac{n}{2} - \frac{m}{2} + \frac{mm}{3n}$ for the whole expectation of ſurvivorſhip due to both lives. ED.

annuitant

annuitant, whoſe age is 28, (and whoſe *complement* of life, therefore, is 58, or *expectation* of life 29) to come upon a ſociety every year; the number of annuitants alive, deduced from hence, will, in x years, be $x - \frac{x^2}{4 \times 29}$, or $\frac{4 \times 29 - x^2}{4 \times 29} \times x$; and, therefore, the number of annuitants alive, deduced in the ſame time from p annuitants left annually at the ſame age, will be $\frac{4 \times 29 - x^2}{4 \times 29} \times px$.——In like manner, the 2d fluent, or $\frac{x^3}{3n^2} - \frac{x^3}{n} + x$, gives the number of marriages in being together, that will, in x years, grow out of *one* yearly marriage, between perſons of *equal* ages, whoſe complement of life is n. If they are of *unequal* ages, and the complement of the oldeſt life is m, and of the youngeſt n, this number will be $\frac{x^3}{3nm} - \frac{\overline{n+m} \times x^2}{2nm} + x$. And if the number of years is required, in which any given number of yearly marriages, between men and women at given ages, will increaſe ſo far as to be in any given proportion to the greateſt number that can poſſibly grow out of ſuch marriages, this expreſſion muſt be made equal to the *expectation* of the joint lives, or of each marriage, multiplied by the fraction expreſſing the given proportion; and the root of the equation will be the anſwer. Thus: it may be found, that one marriage every year, between perſons 33 and 25 years of age, would in 10 years increaſe to 8.35; in 15 years, to 11.38; and in 53 years, to 19, or their greateſt poſſible number; and, conſequently, that 35 ſuch yearly marriages would, in 10 years, increaſe to 292; in 15 years to 398; and in 53 years, to 665.——And if it is enquired in what number of years 35 ſuch yearly marriages would increaſe to half the number in being together, poſſible to

be

be derived from them, the value of x, in the cubic equation $\frac{x^3}{3nm} - \frac{\overline{n+m} \times x^2}{2nm} + x = \frac{\overline{m}}{2} - \frac{\overline{m}}{6n} \times \frac{r}{2}$, muſt be found; which, in the preſent inſtance, is nearly 12.

I have, in ſome parts of this work, had occaſion to make ſuch deductions as theſe. See note (A), p. 431; and note (F), p. 444; and Queſtions III. and XIII. Vol. I.

Note

Note (L). Vol. I. Eſſay II. Page 306.

LET r ſignify 1 *l.* increaſed by its intereſt for one year.

V the PERPETUITY.

n the difference between the age of the youngeſt life, and 86; or its *complement*.

m the complement of the oldeſt life.

P the value (in Table II. at the beginning of this volume) of an annuity certain for m years.

And the exact value of any two given joint lives, according to the hypotheſis of an equal decrement of life, will be $V - \frac{V+1}{u} \times \overline{n-m-2V-1} \times \frac{P}{m} \mp 2V$ (x). Example:

Let

(x) This general rule is taken from Mr. *Simpſon*'s Doctrine of Annuities, and is eaſily demonſtrated by the aſſiſtance of the Poſtſcript to the third additional Eſſay in this work, p. 402. Vol. II.—The ſeries expreſſing the value of an annuity on two joint lives, whoſe complements are n and m, is known to be $= \frac{m-1}{m} \times \frac{n-1}{nr} + \frac{m-2}{m} \times \frac{n-2}{nr^2}$ &c. $\ldots\ldots (m) = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$ $\ldots\ldots (m) - \frac{m+n}{mn} \times \overline{\frac{1}{r} + \frac{2}{r^2} + \frac{3}{r^3} +}$, &c. $\ldots\ldots (m) + \frac{1}{mn} \times \frac{1}{r} + \frac{4}{r^2} + \frac{9}{r^3}$ &c. $\ldots (m)$. By the rule in the Poſtſcript juſt referred to, the firſt of theſe ſeries may be found $= \frac{1}{r-1} - \frac{1}{r^m \overline{r-1}}$ the ſecond $= -\frac{m+n}{mn} \times \frac{r}{\overline{r-1}^2} + \frac{mm+mn}{mn} \times \frac{1}{r^m \overline{r-1}} + \frac{m+n}{mn} \times \frac{r}{r^m \overline{r-1}^2}$, and the third $= \frac{r^2+r}{mn.\overline{r-1}^3} - \frac{mm}{mnr^m} \times \frac{1}{r-1} - \frac{2mr}{mnr^m} \times \frac{1}{\overline{r-1}^2} - \frac{r^2+r}{mnr^m} \times \frac{1}{\overline{r-1}^3}$.—Adding theſe different ſeries together their ſum will

Let the ages be 27 and 38; and the rate of intereſt 4 *per cent.* Then $n = 59$. $m = 48$. $V = 25$ $P =$

will be $= \frac{1}{r-1} - \frac{m+n}{mn} \times \frac{r}{r-1|^2} + \frac{n-m}{mn} \times \frac{r}{r^m-1|^2} + \frac{r^2+r}{mn.r-1|^2} - \frac{r^2+r}{mn.r^m-1|^3}$ = (ſince $\frac{r}{r-1}$ is $= V+1$, and $\frac{r^2+r}{r-1|^3}$ is $= \frac{r}{r-1|^2} + \frac{2r}{r-1|^3}$) $V - \frac{m+n-1}{mn} \times \frac{V+1}{r-1} + \frac{n-m-1}{mn.} \times \frac{V+1}{r.^m r-1} + \frac{V+1.\ 2V}{mn.\ r-1} - \frac{V+1.\ 2V}{mn.r.^m r-1}$. But $\frac{1}{r-1} - \frac{1}{r.^m r-1}$ is $= P$; therefore the above expreſſion becomes $= V - \frac{V+1}{n} \times \overline{n-m-2V-1} \times \frac{P}{m} + 2V$. Q. E. D.

If the annuity be payable *half-yearly*, and $1 + \frac{r}{2}$ denote, as in the 3d additional Eſſay of this work (p. 383) £1 increaſed by its intereſt for half a year, the ſeries expreſſing the value of this annuity will be $= \frac{1}{2} \times \frac{m-\frac{1}{2}.\ n-\frac{1}{2}}{mn.\ 1+\frac{r}{2}} + \frac{1}{2} \times \frac{n-1.\ n-1}{mn.\ 1+\frac{r}{2}|^2} + \frac{1}{2} \times \frac{m-\frac{3}{2}.\ n-\frac{3}{2}}{mn.\ 1+\frac{r}{2}|^3}$ ----(2m). By proceeding in the ſame manner as in the foregoing theorem, and putting H to repreſent the value of an annuity certain, payable half yearly, for m years, the general rule in this caſe may be found $= V - \frac{V+\frac{1}{2}}{n} \times \overline{n-m-\frac{1}{2}-2V} \times \frac{H}{m} + 2V$.

If the annuity be payable *quarterly*, and $1 + \frac{r}{4}$ denote £1 increaſed by its intereſt for three months, the ſeries will be $= \frac{1}{4} \times \frac{m-\frac{1}{4}.\ n-\frac{1}{4}}{mn.\ 1+\frac{r}{4}} + \frac{1}{4} \times \frac{m-\frac{1}{2}.\ n-\frac{1}{2}}{mn.\ 1+\frac{r}{4}|^2} + \frac{1}{4} \times \frac{m-\frac{3}{4}.\ n-\frac{3}{4}}{mn.1+\frac{r}{4}|^3}$ + &c.

$P = 21.195.\ n - m - 2V - 1 = -40.\ \overline{n - m - 2V - 1} \times \frac{P}{m} + 2V = 50 - 17.660 = 32.340$. And $V - \frac{V+1}{n} \times \overline{n - m - 2V - 1} \times \frac{P}{m} + 2V = 25 - \frac{26}{39} \times 32.340 = 10.748$, the value of two joint lives whoſe ages are 27 and 38.

$+$ &c. - - - - - $(4m)$, and its ſum $= V - \frac{V+\frac{1}{4}}{n} \times \overline{n - m - \frac{1}{4} - 2V} \times \frac{Q}{m} + 2V$, Q repreſenting the value of an annuity certain payable *quarterly* for m years.—In like manner, if the annuity be payable *momently*, and $1 + \frac{r}{1000, \&c.}$ denote £ 1 increaſed by its intereſt for a moment, the general rule for determining the value of the annuity will be $= V - \frac{V + \frac{1}{1000, \&c.}}{n} \times \overline{n - m - \frac{1}{1000, \&c.} - 2V} \times \frac{M}{m} + 2V = V - \frac{V}{n} \times \overline{n - m - 2V} \times \frac{M}{m} + 2V$; M repreſenting the value of an annuity certain payable momently for m years.

Suppoſing the ages of two lives to be 20 and 36 years, the value of an annuity at 4 *per cent.* during their joint continuance, and payable either yearly, half yearly, quarterly, or momently will, by Mr. *De Moivre*'s hypotheſis, be 11.227 ...11.427...11.565...or 11.629. If their ages be 36 and 61, the values will be 7.448...7.673...7.793...or 7.901. If both their ages be 36, the values will be 10.394...10.600... 10.703...or 10.808...and if both their ages be 61, the values will be 6.144...6.374...6.517...or 6.602.

By comparing the values of the equal joint lives, given above, with the values of the ſingle lives of the ſame ages, computed in the third additional Eſſay (p. 388 & 389), it appears that the differences in the former between annuities payable yearly and thoſe which are payable half yearly, quarterly, or momently, are greater than the differences in the latter; and therefore that the addition to be made to an annuity on the longeſt of two lives, in conſequence of its being payable at ſhorter intervals than a year, will be rather leſs than the addition to be made on this account, either to the ſingle or the joint lives of the ſame ages. Ed.

Note

Note (M). Vol. I. Eſſay III. Page 324.

IT is plain that the purchaſer of A's right, as ſtated in the firſt of the queſtions to which this note refers, cannot get into poſſeſſion till the year when A and B ſhall be both dead; nor then, unleſs A happens to die *laſt*. Now, ſuppoſing the common complement of life n; the probability that A and B ſhall be *both* dead at the end of the *firſt* year, and A die laſt, is $1 - \frac{n-1}{n} \times 1 - \frac{n-1}{n} \times \frac{1}{2} = \frac{1}{2} - \frac{n-1}{2n} - \frac{n-1}{2n} + \frac{\overline{n-1}^2}{2n^2}$. —In like manner, the probability that they ſhall be *both* dead at the end of the 2d, 3d, &c. years, and A ſurvive is, $\frac{1}{2} - \frac{n-2}{2n} - \frac{n-2}{2n} + \frac{\overline{n-2}^2}{2n^2}$; $\frac{1}{2} - \frac{n-3}{2n} - \frac{n-3}{2n} + \frac{\overline{n-3}^2}{2n^2}$, &c. The *preſent* value, therefore, of the 1ſt, 2d, 3d, &c. rents of the reverſionary eſtate is $\frac{1}{2r} - \frac{n-1}{2nr} - \frac{n-1}{2nr} + \frac{\overline{n-1}^2}{2nr}$, $\frac{1}{2r^2} - \frac{n-2}{2nr^2} - \frac{n-2}{2nr^2} + \frac{\overline{n-2}^2}{2n^2r^2}$, $\frac{1}{2r^3} - \frac{n-3}{2nr^3} - \frac{n-3}{2nr^3} + \frac{\overline{n-3}^2}{2n^2r^3}$, &c. Suppoſing r to ſignify 1*l.* increaſed by its intereſt for a year; and the eſtate to be 1*l. per annum*. And the *ſum* of theſe terms continued *in infinitum* is the value *required*.—But $\frac{1}{2r} + \frac{1}{2r^2} + \frac{1}{2r^3}$, &c, is *half* the perpetuity. And $\frac{n-1}{2nr} + \frac{n-1}{2nr} - \frac{\overline{n-1}^2}{2n^2r}$, &c. $\frac{n-2}{2nr^2} + \frac{n-2}{2nr^2} - \frac{\overline{n-2}^2}{2n^2r^2} + \frac{n-3}{2nr^3} + \frac{n-3}{2nr^3} - \frac{\overline{n-3}^2}{2n^2r^3}$, &c. is half the value of the *joint* lives, ſubtracted from *half* the ſum of the values of the two *ſingle* lives;

that is, *half* the value of the *longeſt* of the two lives.

A ſimilar demonſtration may be applied to the other queſtion (λ).

(λ) The purchaſer of A's right, in the 2d Queſtion, will get into poſſeſſion in that year in which A either ſurvives B, or dies after him. The value of his expectation in the 1ſt year will be $= \frac{n-1}{nr} \times 1 - \frac{n-1}{n} + \frac{1}{2r} \times 1 - \frac{n-1}{n} \times 1 - \frac{n-1}{n} = \frac{1}{2r} - \frac{\overline{n-1}^2}{2nnr}$ ....In the 2d, 3d, 4th, &c. years, his expectation depending on the ſame events will be worth $\frac{1}{2r^2} - \frac{\overline{n-2}^2}{2nnr^2}$, $\frac{1}{2r^3} - \frac{\overline{n-3}^2}{2nnr^3}$, $\frac{1}{2r^4} - \frac{\overline{n-4}^3}{2nnr^4}$, &c. The preſent value therefore of the 1ſt, 2d, 3d, &c. rents of the reverſionary eſtate is $\frac{1}{2r} + \frac{1}{2r^2} + \frac{1}{2r^3} +$ &c. $- \frac{\overline{n-1}^2}{2nnr} - \frac{\overline{n-2}^2}{2nnr^2} - \frac{\overline{n-3}^2}{2nnr^3}$ — &c.——If inſtead of an *eſtate* the value of a given *ſum* were required it would, agreeable to the foregoing demonſtrations, be expreſſed in the firſt caſe by $\frac{S.\overline{r-1}}{2r} \times \overline{V + BB - 2B}$, and in the 2d caſe by $\frac{S.\overline{r-1}}{2r} \times \overline{V - BB}$ (V denoting the perpetuity, B the value of an annuity on the life of B, and BB the value of an annuity on two joint lives whoſe common age is that of B). The latter value therefore according to *De Moivre*'s hypotheſis, and in the particular caſe where the two lives are equal, exceeds the former value by $\frac{S.\overline{r-1}}{r} \times \overline{B-BB}$. ——That this is likewiſe true whatever be the decrements of life, or the ages of A and B, may be proved from the two Theorems in Note (O): For by the 2d of theſe theorems the value of S is $= \frac{S}{2} \times \frac{\beta.\overline{F-AF}}{b} - \frac{c.\overline{P-AP}}{br} - \frac{\overline{r-1}.\overline{B-AB}}{r}$, and by the firſt it is $= \frac{S}{2} \times \frac{\beta.\overline{F-AF}}{b} - \frac{c.\overline{P-AP}}{br} + \frac{\overline{r-1}.\overline{B-AB}}{r}$; from which it appears that the latter reverſion exceeds the former by $\frac{S.\overline{r-1}}{r} \times \overline{B-AB}$, and conſequently that the difference between them will be the ſame in all caſes. Ed.

Note (N). Vol. I. Eſſay II. Page 320.

LET r be 1*l.* increaſed by its intereſt for one year.

Let S repreſent any given interval of time, or number of years, during which the decrements of life in a table of obſervations continue equal.

a the number of the living in the table at the beginning of the firſt year of that interval.

b the number of the living in the table at the beginning of the year immediately following the ſame interval.

P the value of an annuity certain for S years.

p the value of 1*l.* due at the end of S years.

Q the value, in Table I. immediately following this Note, of an annuity for the life of a perſon whoſe age wants S years of 86.

N the value, in ſtrict agreement with the given table of obſervations, of an annuity on the life of a perſon whoſe age is S years greater than the age at which the interval of equal decrements begins. Then,

$Q + \frac{b}{a} \times \overline{P - Q}$ will be the value, according to the table of obſervations, of an annuity for S years, on a life of the ſame age with that at which the interval of equal decrements begins. And

$Q + \frac{b}{a} \times \overline{\overline{P - Q} + pN}$ will be the value of an annuity on the whole duration of that life.

When S repreſents *one year*, Q vaniſhes, and the laſt expreſſion becomes $\frac{b}{ar} \times \overline{1 + N}$; which is the rule for finding, from the value given of any life, the value of a life one year younger (μ).

In

(μ) The value of an annuity payable *half yearly* during any life (A), may be deduced from the value of the ſame

annuity

In like mauner, ſuppoſing G to ſignify the value of two given joint lives by any table of ob-

annuity during a life (B), one year younger than A, with nearly as much eaſe as the values of annuities payable *yearly* are deduced. Let b repreſent the number of perſons living in the Table at the age of B, and c, d, e, f, &c. the number living at the end of the 1ſt, 2d, 3d, &c. years from the age of B. Let r repreſent the intereſt of £1 for a year, and $p = 1 + \frac{r}{2}$; then will the value of the annuity be $= \frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3} + \frac{d}{2bp^4} + \frac{d+e}{4bp^5}$ + &c. which may be found $= \frac{1}{4p} + \frac{c}{4b} \times \frac{1}{p} + \frac{2}{p^2} + \frac{1}{p^3} + \frac{d}{4b} \times \frac{1}{p^3} + \frac{2}{p^4} + \frac{1}{p^5} + \frac{e}{4b} \times \frac{1}{p^5} + \frac{2}{p^6} + \frac{1}{p^7}$, &c. From this ſeries, if the age of B be very old, the value of the life annuity will be obtained with little difficulty; and having this, the value of an annuity on a life one year younger may be derived from it in the following manner:—Let a denote the number of perſons living at the age of (A), who is one year younger than B; then, ſince the ſeries expreſſing the value of an annuity on the life of the latter is found above to be $= \frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3}$ + &c. the ſeries expreſſing the value of an annuity on the life of the former will be $= \frac{a+b}{4ap} + \frac{b}{2ap^2} + \frac{b+c}{4ap^3}$ + &c. $= \frac{a+b}{4ap} + \frac{b}{ap^2} \times \frac{1}{2} + \frac{b+c}{4bp} + \frac{c}{2bp^2} + \frac{c+d}{4bp^3}$ + &c. Therefore if the value of the annuity on the life of B be called M, the value of the annuity on the life of A will be $= \frac{a+b}{4ap} + \frac{b}{ap^2} \times \frac{1}{2} + \mathrm{M}$.

From this Theorem a table may be computed of the values of annuities payable *half yearly* on lives of all ages; and by proceeding in the ſame manner a general Theorem may be obtained for computing a table of the values of annuities payable *quarterly*. But the labour of forming a table of this kind will be rendered unneceſſary, if we are poſſeſſed of the values payable *yearly*: for I have found that the differences between annuities payable half yearly and yearly are the ſame, whether thoſe values be derived from the real probabilities of life and the preceding Theorems, or from M. *De Moivre*'s hypotheſis, and the Theorems in the 3d additional Eſſay in this work (pag. 388, Vol. II.). Ed.

ſervations, a the living at the age of one of them, c the living at the age of the other, and b and d the numbers living at the two next younger ages, $\frac{a \times c}{b \times d \times r} \times \overline{1+G}$ will be the value of two joint lives each one year younger than the former.

The method of calculating the values of lives from any given tables of obſervations, deſcribed at the end of the Second Eſſay in the preceding volume, is founded entirely on theſe Theorems; and a diſtinct explanation of them has been given by Mr. *Morgan*, in the Second Section of the Second Chapter of his book on the Doctrine of Life-Annuities and Aſſurances.

The expreſſions $Q+\frac{b}{a} \times \overline{P-Q}$, and $Q+\frac{b}{a} \times \overline{\overline{P-Q}+pN}$, with their inveſtigation, may be found in p. 341, 3d Edition of Mr. *De Moivre's Treatiſe of the Doctrine of Chances* (v). But it is neceſſary

(v) The Solution of this theorem may be deduced in a manner different from that of M. *De Moivre*. Let α be the number of perſons dying annually in s years, while the decrements of life continue equal, then will the value of the annuity during this term be $= \frac{a-\alpha}{ar} + \frac{a-2\alpha}{ar^2} + \frac{a-3\alpha}{ar^3} \ldots + \frac{a-s\alpha}{ar^s} = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} \ldots (s) - \frac{s\alpha}{a} \times \overline{\frac{1}{sr} + \frac{2}{sr^2} + \frac{3}{sr^3} \ldots (s)}$. But the firſt ſeries is $= P$, and the ſecond ſeries is $= -\frac{s\alpha}{a} \times \overline{P-Q} =$ (ſince $a-b$ is $= s\alpha$) $-\frac{a-b}{a} \times \overline{P-Q}$, and therefore the value of the annuity during the firſt s years will be $= Q+\frac{b}{a} \times \overline{P-Q}$....The value of the annuity after s years (ſuppoſing m, n, o, p, q, &c. to denote the number of perſons living in the table at the end of $\overline{s+1}$, $\overline{s+2}$, $\overline{s+3}$, &c. years

ſary to obſerve, that the direction of Mr. *De Moivre* has given for finding the value of Q is wrong. In conſequence of calculating agreeably to this direction, he gives the value of a life at the age of 42 by Dr. *Halley*'s table greater than the value of the ſame life by his own hypotheſis; whereas, it is evident that the probabilities of living after 42, years is $= \frac{m}{ar^{s+1}} + \frac{n}{ar^{s+2}} + \frac{o}{ar^{s+3}} + \&c. = \frac{b}{ar^s} \times \overline{\frac{m}{br} + \frac{n}{br^2} + \frac{o}{br^3} + \&c.} = \frac{b}{a} \times pN$. If this expreſſion be added to the value of the annuity, found above, for the firſt s years, the whole value will be $= Q + \frac{b}{a} \times \overline{P - Q} + pN$. Q. E. D.

It is neceſſary to obſerve that the ſeries $\frac{a-\alpha}{ar} + \frac{a-2\alpha}{ar^2}$, &c. ſuppoſes the annuity to be payable *yearly*, and therefore that $\frac{s\alpha}{a} \times \overline{\frac{1}{sr} + \frac{2}{sr^2} + \frac{3}{sr^3}}$, &c. expreſſes the difference, multiplied into $\frac{s\alpha}{a}$, between the values of an annuity certain for s years, and of an annuity payable yearly during the continuance of a life whoſe complement is s.---The latter of theſe values, denoted by Q, is given in the 1ſt Table at the end of this volume.—But M. *De Moivre* has deduced the value of Q from the fluxional quantity $\frac{x}{n.r-1} - \frac{x}{nr^2.r-1}$, which, expreſſing the value of an annuity *ſecured upon land*, muſt neceſſarily be always greater than the ſeries $\frac{s-1}{sr} + \frac{s-2}{sr^2}$ + &c.; for the one ſuppoſes the annuity to be payable to the laſt moment of exiſtence, while the other makes no allowance for that part of the year which ſhall have elapſed between its commencement and the extinction of the life. This value of Q therefore is improperly applied to the foregoing Theorem, where the value of N, as well as the whole ſolution, is founded upon the principle of the annuity's being payable only at the concluſion of each year, provided the life ſhall continue ſo long. Ed.

being all along leſs in Dr. *Halley*'s table than in the hypotheſis, the value of the life muſt be alſo leſs.——The mathematical reader may eaſily ſatisfy himſelf, that the value of Q ought to be taken, as I have directed, from Table I. at the end of this volume.

I cannot help adding here, that though the rules for finding from the value given of any ſingle or joint lives, the value of any ſingle or joint lives one year younger, are an obvious corollary from the two expreſſions juſt mentioned, yet it is probable that Mr. *De Moivre* did not attend to them, or conſider the facility which they give to calculations of this kind; for if he had, he would not probably have inſiſted ſo much as he has on his hypotheſis of an equal decrement of life; much leſs would he, in order to obtain an eaſy method of calculation, have had recourſe to that Second Hypotheſis, which, in the Second Eſſay in the preceding volume, has been ſhewn to be ſo very erroneous.

Mr. *Simpſon* is, I believe, the firſt who has given theſe rules, in his Treatiſe on the Doctrine of Annuities and Reverſions; but in his *Select Exerciſes*, p. 275, he has given a rule for approximating to the values of ſingle lives, according to Dr. *Halley*'s table, which muſt not be depended on, for I have found it half a year's purchaſe, and ſometimes three-quarters of a year's purchaſe wrong.

Note

Note (O).

IN a note at the concluſion of the 3d Eſſay *, Dr. *Price* refers to the end of this work for more accurate ſolutions of his 11th and 12th Queſtions, which had been inveſtigated by myſelf, and publiſhed in the 78th vol. of the Philoſophical Tranſactions.—With the view of fulfilling his intentions in this reſpect, I ſhall here, in an abridged manner, inſert the ſolutions to which he refers.

SOLUTION OF QUESTION XI. Let a repreſent the number of perſons living in the Table at the age of A, the younger of the two lives, a', a'', a''', &c. the decrements of life at the end of the 1ſt, 2d, 3d, &c. years from the age of A; b the number of perſons living at the age of B, the older of the two lives, and c, d, e, f, &c. the number of perſons living at the end of the 1ſt, 2d, 3d, &c. years from the age of B. Then will the value of S (the given ſum), depending on the contingency of B's ſurviving A, be expreſſed by $\frac{S}{2ab} \times \overline{\frac{ca'}{r} + \frac{da''}{r^2} + \frac{ea'''}{r^3} + \&c.} + \frac{S}{2ab} \times \overline{\frac{ba'}{r} + \frac{ca''}{r^2} + \frac{da'''}{r^3} + \&c.}$ $= \frac{S}{2r} \times \frac{\beta r.\overline{F-AF} - c.\overline{P-AP}}{b} + \overline{r-1.B-AB}$; F denoting a life one year younger, and P a life one year older than B; AF, AP, AB, the values of the joint lives of A and F, A and P, and A and B; and β the number of perſons living in the Table at the age of F.—Having now the value of the given ſum payable on the contingency of B's

* Vol. I. page 326.

ſurviving A, the value of the ſame ſum payable on the contingency of A's ſurviving B is eaſily obtained; by ſubtracting the value found above from the whole value of the Reverſion after the extinction of the joint lives of A and B.

SOLUTION OF QUESTION XII. Retaining the ſame ſymbols as in the preceding ſolution, the value of the ſum S will in this caſe be $= \frac{S}{2ab} \times$

$$\frac{\overline{b-c}.a'}{r} + \frac{\overline{c-d}.a''}{r^2} + \frac{\overline{d-e}.a'''}{r^3} + \&c. + \frac{S}{abr} \times$$

$$\frac{a'.\overline{c-d}}{r} + \frac{\overline{a'+a''}.\overline{d-e}}{r^2} + \frac{\overline{a'+a''+a'''}.\overline{e-f}}{r^3} + \&c. =$$

$$\frac{S}{2r} \times \frac{\beta r.\overline{F-AF} - c.\overline{P-AP}}{b} - \overline{r-1}.\overline{B-AB}.$$

When the value of the reverſion is required, depending on the contingency of A's having died after B, the foregoing value is to be ſubtracted from the whole value of the Reverſion after the extinction of *both* lives.

The ſolutions which are given of theſe queſtions in the 1ſt Volume of this work, have been taken from Mr. *Simpſon*'s Select Exerciſes, and are in ſome inſtances ſo incorrect as to be unfit for uſe, —more eſpecially when one of the lives is very young and the other very old; in which caſe the reſults are often *one third*, and ſometimes even *one half* wrong.—This inaccuracy ariſes from Mr. *Simpſon*'s having had recourſe to Mr. *De Moivre*'s hypotheſis, by deducing his ſolutions from the *expectations* rather than from the *real probabilities* of life. When the ages of neither of the lives exceed 60, or fall ſhort of 10 years, his rules are tolerably correct; but ſince the *exact* values may be obtained with ſo little difficulty, I think it can ſeldom be adviſeable to have recourſe to them.

The

The general rule derived from both the foregoing Theorems may be expreſſed in nearly the ſame words.—"Let K repreſent a life one year "younger, and C a life one year older than B. "Multiply the difference between the values of "the life of K, and of the joint lives of A and "K into the number of perſons living in the table "at the age of K, and alſo into £ increaſed by "its intereſt for a year. Multiply the difference "between the values of the life of C, and of the "joint lives of A and C, into the number of per-"ſons living in the table at the age of C. Sub-"tract this from the former product; divide the "remainder by the number of perſons living in "the table at the age of B, and reſerve the quo-"tient.—Again; multiply the difference between "the values of the life of B, and of the joint "lives of A and B, into the intereſt of £1 for a "year—then, if the *ſum* of this product and the "reſerved quotient in the 11th queſtion, or their "*difference* in the 12th, be divided by £1 increaſed "by its intereſt for a year, and multiplied into "half the given ſum, this laſt product will be the "value of the Reverſion, when B the *expectant* is "the oldeſt of the two lives"——If B be the youngeſt, the value will be obtained in the ſame manner as in Mr. *Simpſon*'s rules,—by ſubtracting the value of A's expectation, found above, from the whole value of the Reverſion after the *joint lives* of A and B in the former caſe, and after the *longeſt of their two lives* in the latter.

EXAMPLE I.

Let it be required to determine the value of £100 payable on the death of A aged 35 ſhould B aged 75 be then living; computing at 4 *per cent.* and

and from the probabilities of life in the *Northampton* Table of Obſervations.—In this caſe the ages of K and C will be 74 and 76 years.—The value of an annuity on the life of K is 5.230, and on the joint lives of A and K, 4.737.—The difference between theſe two ſums, or .493, multiplied into 912, the number of perſons living at the age of K, and into 1.04, produces 467.6005. —The difference between 4.710 and 4.303, the reſpective values of annuities on the life of C, and the joint lives of A and C, is .407; which being multiplied into 752, the number of the living at the age of C, gives 306.064. This product ſubtracted from 467.6005, and 161.5365, (the remainder) divided by 832, the number of perſons living at the age of B, quotes .185416 to be reſerved.——Again; the values of annuities on the life of B, and the joint lives of A and B, are 4.962 and 4.516 reſpectively. Their difference, or .446, multiplied into .04 gives .01784; which being added to .185416, the reſerved quotient, amounts to .203256. This ſum divided by 1.04, and the quotient multiplied into 50, produces £9.772 for the value of the Reverſion.—If A had been 75 and B 35 years of age, the foregoing value muſt have been deducted from 78.784, the whole value of the Reverſion after the extinction of the joint lives of A and B (a), and the remainder, or £69.012, would have been the anſwer in this caſe.

(a) The *whole* values of the Reverſions in theſe Examples are deduced from Queſt. X. Vol. I. by ſubſtituting the *joint*, or the *longeſt of the two lives*, inſtead of the *ſingle life* in that Rule.

EXAMPLE

EXAMPLE II.

Let it be required to determine the value of £100 payable on the death of B aged 75, ſhould that happen *after* the death of A aged 35, computing at the ſame rate of intereſt, and from the ſame probabilities of life, as in the preceding Example.—This caſe belongs to the 12th Queſtion, and as the ages are the ſame with thoſe above, the reſerved quotient, and the product to be ſubtracted from it will alſo be the ſame.—Theſe having been found to be .185416 and .017784 reſpectively, their *difference* is .167576; which being divided by 1.04, and .161131, the quotient, multiplied into 50, will give £8.05655 for the value of the Reverſion.

Suppoſing A to be 75 and B 35 years of age, the foregoing ſum muſt be ſubtracted from 40.442 (a), the whole value of the reverſion after the longeſt of the two lives of A and B and £32.385, the remainder, will be the value required. ED.

Note

Note (P).

IN the ſame note to which Dr. *Price* refers (*) for more accurate ſolutions of his 11th and 12th queſtions (and which have been given in the preceding pages), a further reference is made to the end of this volume, for rules which give in all caſes correct values of ſums payable on any ſurvivorſhips between any three lives. Theſe rules have been deduced by myſelf; and when the above note was written, it was my intention to have ſubmitted the whole of them to Dr. *Price*, in order that he might uſe his own diſcretion in the manner of inſerting them. But this is no longer poſſible, and I am now induced for many reaſons to withhold for the preſent the greater part of them from the public. Were thoſe rules together with their demonſtrations to be given (and the one would be very unſatisfactory without the other), I am apprehenſive that my additions to this invaluable work would be much too long. I ſhall therefore inſert here only ſuch rules as have been already publiſhed in the 79th and 81ſt volumes of the *Philoſophical Tranſactions*, to which the reader is referred for their demonſtrations.

From the complicated nature of queſtions involving ſurvivorſhips between three lives, it becomes neceſſary in their ſolution to have recourſe to a great variety of ſymbols.—In order however to prevent repetition, the ſame ſymbols are uniformly made to denote the ſame quantities in all the following rules, and it may not be improper to begin with explaining them.

(*) Vol. I. p. 326.

A. B. C. denote the value of an Annuity on the reſpective lives of A, B, and C.

D. denotes the value of S on the contingency of C's ſurviving A (by Queſt. XI. Note O).

E. denotes the ſame value on the contingency of B's ſurviving A, found by the ſame Queſtion.

F. denotes the value of an annuity on a life one year younger than B.

G. denotes the value of the abſolute Reverſion of S after the death of A (by Queſt. X. Vol. I.)

H. denotes the value of an annuity on a life one year younger than A.

K. denotes the ſame value on a life one year younger than C.

L. denotes the value of an annuity on the longeſt of the three lives A, B, and C.

M. denotes the value of S, by the firſt Problem in this Note, on the contingency that A's life ſhall be the *firſt* that fails.

N. denotes the value of an annuity on a life one year older than A.

P. denotes the ſame value on a life one year older than B.

Q. denotes the value of S, by the 8th Problem, on the contingency of A *or* B, being *either* of them the *firſt* that fails.

R. denotes the value of S on the contingency of B's dying after A (by Queſt. XII. Note O).

S. denotes the given ſum.

T. denotes the value of an annuity on a life one year older than C.

V. denotes the perpetuity.

W. denotes the value of S on the contingency of C's dying after A (by Queſt. XII. Note O).

α and a, denote the number of perſons living in

a table of obſervations at the ages of H and A.

β and b, denote the number of perſons living at the ages of F and B.

$\varkappa$ and c, denote the number of perſons living at the ages of K and C.

s, m, and d, denote the number of perſons living at the end of the firſt year from the reſpective ages of A, B, and C.

r, denotes the value of £1 increaſed by its intereſt for a year.

The combinations of two or three of the ſeveral letters, A, B, C, F, H, &c. denote the values of annuities on the *joint* continuance of two or three of thoſe reſpective lives.

PROBLEM I.

To determine the value of a given ſum, payable if A ſhould be the *firſt* that fails of the three lives A, B, and C.

SOLUTION.

When B or C are the oldeſt of the three lives the value of the Reverſion will be $=$ S into $\frac{\varkappa}{3c} \times \frac{\beta.\,\overline{FK-AFK}}{b} + \frac{\overline{BK-ABK}}{2} + \frac{\beta}{6b} \times \overline{FC-AFC} + \frac{r-1.}{3r} \times \overline{BC-ABC} - \frac{m.\,\overline{PC-APC}}{6br} - \frac{d}{3cr} \times \frac{\overline{BT-ABT}}{2} + \frac{m.\,\overline{PT-APT}}{b}$.

When

When A is the oldeſt of the three lives the value will be $= S$ into $\frac{\beta}{3b} \times \frac{a.\overline{HF+\frac{1}{2}HFC}}{a} - \overline{AF+\frac{1}{2}AFC}$ $+ \frac{1}{6} \times \frac{a.\overline{HB+2HBC}}{a} - \overline{AB+2ABC} + \frac{1}{3r}$ $\times \frac{s.\overline{BN-BNC}}{d} - \overline{AB-ABC} + \frac{m}{6br} \times$ $\frac{s.\overline{PN-PNC}}{a} - \overline{AP-APC}.$

When the three lives are equal, the value will be $= \frac{S}{2} \times \frac{r-1.\overline{V-CCC}}{r}$.

PROBLEM II.

To determine the value of a given ſum, payable if A ſhould be the *ſecond* that fails of the three lives A, B, and C.

SOLUTION.

When the ages are *unequal*, the value of the Reverſion will be $= D + E - 2M$.

When the ages are *equal*, its value will be $= \frac{S}{3} \times \frac{r-1}{r} \times \overline{V-3CC-2CCC}$.

PROBLEM III.

To determine the value of a given ſum, payable on the death of A, if his life ſhould be the *laſt* that fails of the three lives A, B, and C.

SOLUTION

SOLUTION.

The value of the Reverſion in this caſe will be either $G + M - \overline{D+E}$, or $\frac{S.\overline{r-1}.\overline{V-L}}{3r}$, according as the ages of the lives are unequal or equal.

PROBLEM IV.

To determine the value of a given ſum, payable on the extinction of the lives of A and B, ſhould they be the *firſt* that fail of the three lives A, B, and C.

SOLUTION.

Let Σ denote the value of S on the contingency of C's ſurviving B (by Queſt. XI. Note O), and the general rule, when the lives are *unequal*, will be $= \Sigma + \frac{S.s}{6a} \times \overline{HC - HBC} - \frac{S.s}{3} \times \frac{a.\overline{HK - HBK}}{2a} + \overline{AK - ABK} - \frac{2S.\overline{r-1}}{3r} \times \overline{AC - ABC} - \frac{S.s}{6ar} \times \overline{NC - NBC} + \frac{S.d}{3cr} \times \overline{AT - ABT} + \frac{s.\overline{NT - NBT}}{2a}$. ----If the three lives be *equal*, the Rule becomes $= \frac{S.\overline{r-1}}{3r} \times \overline{V - 3CC - 2CCC}$.

PROBLEM V.

To find the value of a given ſum, payable on the death of A, if his life ſhould be the *firſt* or *ſecond* that fails of the three lives A, B, and C.

SOLUTION

SOLUTION.

The value of the Reverſion, when the lives are *unequal*, will be $= D + E - M$.

When the lives are *equal*, it will be $= \frac{S.r-1}{3r} \times \overline{2V - 3CC - CCC.}$

PROBLEM VI.

To find the value of a given ſum, payable on the death of A, ſhould his life be the *ſecond* or *third* that fails of the three lives A, B, and C.

SOLUTION.

If the lives be *unequal*, the value of the Reverſion will be $= G - M$.-----If the three lives be *equal*, its value will be $= \frac{S.r-1}{3r} \times \overline{2V - 3C - CCC.}$

PROBLEM VII.

To find the value of a given ſum, payable on the death of A, ſhould his life be the *firſt* or the *laſt* that fails of the three lives A, B, and C.

SOLUTION.

In this caſe the value of the Reverſion will be $= G - \overline{D + E} + 2M$, if the lives be *unequal*, and $= \frac{S.r-1}{3r} \times \overline{2V - 3C - 3CC + 2CCC}$, if the lives be *equal*.

PROBLEM VIII.

To determine the value of a given ſum, payable on the death of A or B, ſhould *either* of them be the *firſt* that fails of the three lives A, B, and C.

SOLUTION.

Let Σ, as in Prob. IV. denote the value of S on the contingency of C's ſurviving B, and the value of the Reverſion, *when* C *is the oldeſt of the three lives*, will be $=$ S into $\frac{x}{3c} \times \overline{\frac{\beta.\overline{FK-AFK}}{2b} + \overline{BK-ABK} - \frac{\beta.\overline{FC-AFC}}{6b} + \frac{2.\overline{r-1}.\overline{BC-ABC}}{3r} + \frac{m.\overline{PC-APC}}{6br}} - \frac{d}{3cr} \times \overline{\overline{BT-ABT} + \frac{m.\overline{PT-APT}}{2b}} + \Sigma$.----But if A *be the oldeſt*, the value will be $=$ S into $\frac{\overline{r-1}.\overline{V-AB}}{r} - \frac{a}{3a} \times \overline{\frac{\beta.\overline{HF-HFC}}{b} + \frac{HB-HBC}{2} - \frac{\beta.\overline{AF-AFC}}{6b}} + \frac{2.\overline{r-1}}{3r} \times \overline{AB-ABC} + \frac{m.\overline{AP-APC}}{6br} + \frac{s}{3ar} \times \overline{\frac{BN.-BNC}{2} + \frac{m.\overline{PN-PNC}}{b}}$----And if the three lives be *equal*, the value will be $= \frac{2.S.\overline{r-1}}{3r} \times \overline{V-CCC}$.

PROBLEM IX.

To determine the value of a given ſum, payable on the death of A *or* B, ſhould *either* of them be the *ſecond* that fails of the three lives A, B, and C.

SOLUTION.

SOLUTION.

When the lives are of *unequal* ages, the value of the Reverſion will be $= \frac{S.\overline{r-1}.\overline{V-AB}}{r} D + \Sigma - 2Q$----(Σ denoting the ſame value as in Prob. IV. and VIII.) When the ages of the three lives are *equal*, the value will be $= \frac{2S.\overline{r-1}}{3r} \times \overline{V-3CC-2CCC}$.

PROBLEM. X.

To find the value of a given ſum, payable on the deceaſe of B *or* C, ſhould either of them be the *laſt* that fails of the three lives A, B, and C.

SOLUTION.

The value of the Reverſion, when the lives are *unequal*, will be $= \frac{S.\overline{r-1}}{r} \times \overline{BC-ABC} + R + W - M$, and when the lives are all *equal*, it wil be $= \frac{2S.\overline{r-1}}{3r} \times \overline{V-L}$.

PROBLEM XI.

To determine the value of a given ſum, payable on the contingency of C's ſurviving B, provided the life of A ſhall be then extinct.

SOLUTION.

When either B *or* C *are the oldeſt of the three lives*, the value of the given ſum will be = S into $\frac{x}{6c} \times \overline{\frac{\beta.\overline{FK-FKC}}{b} - \overline{BK-ABK}} + \frac{\beta}{3b} \times \overline{FC-AFC} - \frac{r-1}{3r} \times \overline{BC-ABC} - \frac{m}{3br} \times \overline{PC-APC} + \frac{d}{6cr} \times \overline{\overline{BT-ABT} - \frac{m.\overline{PT-APT}}{b}}$.

When A *is the oldeſt of the three lives*, the value will be $= \Sigma - \frac{S.x}{3c} \times \overline{\frac{\alpha.\overline{HK-HBK}}{a} + \overline{\frac{AK-ABK}{2}}} - \frac{S.\alpha}{6a} \times \overline{HC-HBC} - \frac{r-1}{3r} \times \overline{AC-ABC} + \frac{s}{6r} \times \overline{NC-NBC} + \frac{d}{3cr} \times \overline{\frac{AT-ABT}{2} + \frac{s.\overline{NT-NBT}}{a}}$ (Σ denoting the ſame value as in Prob. IV. VIII. and IX.)----When the three lives are *equal*, the value of the Reverſion will be $= \frac{S.r-1}{6r} \times \overline{V-3CC-2CCC}$.

In the further purſuit of theſe enquiries, I have diſcovered a very ſimple method of approximating to the values in the preceding Problems. But it would be improper to enter more fully into the ſubject at preſent, and therefore the publication of thoſe rules muſt be poſtponed to another opportunity.—I ſhall only obſerve here, that the ſolutions of thoſe caſes which involve three lives, and even of

of thoſe which involve two lives in the ſurvivorſhip, being formerly deduced from an erroneous hypotheſis, it was impoſſible to determine how far any approximations could be depended upon. By the aſſiſtance of the foregoing rules, which have been derived from the real probabilities of life, this point may now be aſcertained with the greateſt preciſion;—though perhaps it may not often be adviſeable to have recourſe to *approximations*, when the *exact values* can be obtained with ſo little additional trouble. Ed.

TABLE I. (a)

Shewing the preſent Values of an Annuity of 1 *l.* on a Single Life, according to Mr. *De Moivre*'s hypotheſis. See Vol. I. p. 2.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
8	19,736	18,160	16,791	15,595	14,544	12,790
9	19,868	18,269	16,882	15,672	14,607	12,839
10	19,868	18,269	16,882	15,672	14,607	12,839
11	19,736	18,160	16,791	15,595	14,544	12,790
12	19,604	18,049	16,698	15,517	14,480	12,741
13	19,469	17,937	16,604	15,437	14,412	12,691
14	19,331	17,823	16,508	15,356	14,342	12,639
15	19,192	17,707	16,410	15,273	14,271	12,586
16	19,050	17,588	16,311	15,189	14.197	12,532
17	18,905	17,467	16,209	15,102	14,123	12,476
18	18,759	17,344	16,105	15,015	14,047	12,419
19	18,610	17,220	15,999	14,923	13,970	12,361
20	18,458	17,093	15,891	14,831	13,891	12,301
21	18,305	16,963	15,781	14,737	13,810	12,239
22	18,148	16,830	15,669	14,641	13,727	12,177
23	17,990	16,696	15,554	14,543	13,642	12,112
24	17,827	16,559	15,437	14,442	13,555	12,045
25	17,664	16,419	15,318	14,340	13,466	11,978
26	17,497	16,277	15,197	14,235	13,375	11,908
27	17,327	16,133	15,073	14,128	13,282	11,837
28	17,154	15,985	14,946	14,018	13,186	11,763
29	16,979	15,835	14,816	13,905	13,088	11,688
30	16.800	15,682	14,684	13,791	12,988	11,610
31	16,620	15,526	14,549	13,673	12,855	11,530
32	16,436	15,367	14,411	13,553	12,780	11,449
33	16,248	15,204	14,270	13,430	12,673	11,365

(a) This Table is the ſame with Mr. *De Moivre*'s Table of the values of ſingle lives, publiſhed in his *Treatiſe on Life Annuities*, and carried as far as the age of 79, to three places of decimals, by Mr. *Dodſon* in his *Mathematical Repoſitory*, Vol. II. p. 169.

TABLE

TABLE I. continued.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
34	16,057	15,039	14,126	13,304	12,562	11,278
35	15,864	14,871	13,979	13,175	12,449	11,189
36	15,666	14,699	13,829	13,044	12,333	11,098
37	15,465	14,524	13,676	12,909	12,214	11,003
38	15,260	14,345	13,519	12,771	12,091	10,907
39	15,053	14,163	13,359	12,630	11,966	10,807
40	14,842	13,978	13,196	12,485	11,837	10,704
41	14,626	13,789	13,028	12,337	11,705	10,599
42	14,407	13,596	12,858	12,185	11,570	10,490
43	14,185	13,399	12,683	12,029	11,431	10,378
44	13,958	13,199	12,504	11,870	11,288	10,263
45	13,728	12,993	12,322	11,707	11,142	10,144
46	13,493	12,784	12,135	11,540	10,992	10,021
47	13,254	12,571	11,944	11,368	10,837	9,895
48	13,012	12,354	11,748	11,192	10,679	9,765
49	12,764	12,131	11,548	11,012	10,515	9,630
50	12,511	11,904	11,344	10,827	10,348	9,492
51	12,255	11,673	11,135	10,638	10,176	9,349
52	11,994	11,437	10,921	10 443	9,999	9,201
53	11,729	11,195	10,702	10,243	9,817	9,049
54	11,457	10,950	10,478	10,039	9,630	8,891
55	11,183	10,698	10,248	9,829	9,437	8,729
56	10,902	10,443	10,014	9,614	9,239	8,561
57	10,616	10,181	9,773	9,393	9,036	8,387
58	10,325	9,913	9,527	9,166	8,826	8,208
59	10,029	9,640	9,275	8,933	8,611	8,023
60	9,727	9,361	9,017	8,694	8,389	7,831
61	9,419	9,076	8,753	8,449	8,161	7,633
62	9,107	8,786	8,482	8,197	7,926	7,428
63	8,787	8,488	8,205	7,938	7,684	7,216
64	8,462	8,185	7,921	7,672	7,435	6,997
65	8,132	7,875	7,631	7,399	7,179	6,770
66	7,794	7,558	7,333	7,119	6,915	6,535
67	7,450	7,234	7,027	6,831	6,643	6,292
68	7,099	6,902	6,714	6,534	6,362	6,040
69	6,743	6,565	6,394	6,230	6,073	5,779
70	6,378	6,219	6,065	5,918	5,775	5,508

TABLE. I. Continued

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
71	6,008	5,865	5,728	5,596	5,468	5,228
72	5,631	5,505	5,383	5,265	5,152	4,937
73	5,246	5,136	5,029	4,926	4,826	4,636
74	4,854	4,759	4,666	4,576	4,489	4,324
75	4,453	4,373	4,293	4,217	4,143	4,000
76	4,046	3,978	3,912	3,847	3,784	3,664
77	3,632	3,575	3,520	3,467	3,415	3,315
78	3,207	3,163	3,111	3,076	3,034	2,953
79	2,776	2,745	2,707	2,673	2,641	2,578
80	2,334	2,309	2,284	2,259	2,235	2,188
81	1,886	1,867	1,850	1,832	1,816	1,783
82	1,429	1,411	1,406	1,394	1,384	1,362
83	0,961	0,955	0,950	0,943	0,937	0,925
84	0,484	0,483	0,481	0,479	0,476	0,472
85	0,000	0,000	0,000	0,000	0,000	0,000

TABLE II.

Shewing the Value of an Annuity on the joint continuance of Two Lives, according to Mr. *De Moivre's Hypotheſis*; computed by the Rule in Note (L). See Vol. I. p. 2 and 3, and Eſſay II. p. 308, &c.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
10	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9 822	9.059	8.391
	60	8.704	8.105	7.572
	65	7.417	6.980	6.585
	70	5.936	5.652	5.391
	15	14.574	12.860	11.478
	20	14.225	12 593	11.266
	25	13.822	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
15	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8,632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.361

TABLE II. Continued

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
20	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8 879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.333
	25	13.192	11.786	10.621
	30	12.794	11.468	10.367
	35	12.333	11.093	10.067
	40	11.776	10.655	9.708
25	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
	70	5.826	5.551	5.294
	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.514
	45	10 898	9.936	9.112
30	50	10.183	9.345	8 620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	5.254

TABLE II. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
	35	11.632	10.530	9.609
	40	11.175	10.157	9.291
35	45	10.622	9.702	8.913
	50	9.955	9.149	8.450
	55	9.156	8.476	7.879
	60	8.202	7.658	7 172
	65	7.066	6.662	6.294
	70	5.718	5.450	5.203
	40	10 777	9.826	9.014
	45	10.283	9.418	8.671
40	50	9.677	8.911	8.244
	55	8.936	8.283	7.710
	60	8.038	7.510	7.039
	65	6.951	6.556	6.198
	70	5.646	5.383	5.141
	45	9.863	9.063	8.370
	50	9.331	8.619	7.987
45	55	8.662	8.044	7.500
	60	7 831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
50	60	7.568	7.091	6.664
	65	6.623	6.258	5 926
	70	5.442	5.193	4.964
	55	7.849	7.332	6.873
55	60	7.220	6.781	6.386
	65	6.379	6.036	5.724
	70	5.201	5.053	4.833

TABLE II. Continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
60	60	6.737	6.351	6.001
	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
65	65	5.547	5.277	5.031
	70	4.773	4.571	4.385
70	70	4.270	4.104	3.952

THE END.

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